1506
UNIVERSITÀ
DEGLI STUDI
DI URBINO
CARLO BO

Dipartimento di Scienze Pure e Applicate
Corso di Dottorato: Scienze di Base e Applicazioni
Curriculum Scienze della Complessità – ciclo XXX

Ph.D. Thesis

Public Engagement, Storytelling and Complexity in Maths Communication

Supervisor
Prof. Gian Italo Bischi

Candidate
Prof. Andrea Capozucca

Dottorato in Scienze della Complessità
Ciclo XXX – A.A. 2016/2017
Settore Scientifico Disciplinare MAT/04 – SECS-S/06

To Francesca
who made all of this possible

Contents

List of figures

List of tables

Acknowledgements

6 April 2014. Saturday. A spring sunny day in Orvieto. After a pleasant lunch, I and my colleague Stefano Leonesi decided to take two hours off from the conference program to visit the wonders of the city. Waiting in line for a guided tour to visit the underground Orvieto, we started talking about mathematics. Pretty soon, our conversation turned to other issues. I told Stefano the vicissitudes that prevented me from continuing my studies after graduation addressing me towards teaching. Stefano looked at me and said: “A few days ago, a colleague of the University of Urbino told me that in the next few months there will be a call for two PhD posts in Science of Complexity. I really think you should try. Why don’t you?”

This is how it all began. And now I’m here, three years later, to write my acknowledgements at the end of this doctorate. Embarking on a doctorate course at 40, it was a disruptive, challenging and inspiring experience that required me to put myself out there and rearranged my life. So, my first thanks goes to Stefano Leonesi who planted a bug in my ear when everything seemed well-established, scientifically speaking.

Then, many thanks are due to my Supervisor Prof. Gian Italo Bischi for his wisdom, knowledge, expertise, passion, willingness and careful thoughts. He stood by me with patience and care, I dare say fatherly in his manner, leaving me the freedom to choose and the capacity to continuously improve my knowledge and skills. During our fruitful discussions, I learned the pleasure of looking at things with intellectual curiosity taking care of all the possible connections with other bodies of knowledge. In short, it wouldn’t be enough twice the pages of this thesis to give back all I received from him. I hope to continue our inspiring cooperation in the future.

This thesis was conceived, rooted and expanded by the guidance and questioning of several individuals, all of whom have offered valuable insights and inquiries along the way. First and foremost, I would like to thank Prof. Vincenzo Fano, the Coordinator of the PhD program in Science of Complexity, for his valuable suggestions to improve this research work. I’d also like to thank Unicam Science Outreach members, in particular Silvia Benvenuti, Mario Compiani and Alessandra Renieri. Their guidance, questioning and collaboration were so precious to tackle the complex science and maths communication matters. A sincere thanks goes to Mauro Comoglio who was, is and will be a helpful and inspiring point of reference in the world of education always open to dialogue and debate.

A special thank you to Marco Fermani who believed and fully involved himself in FOTCAB project from the very beginning. His background in physics coupled with his expertise in music composition and playing were a huge support to implement and develop the project. The time spent together has always been inspiring for me. Thanks to Simone Giorgini for his musical arrangements and recording of “Logistic Map Song”. A huge thanks to Diego Zallocco for creating “Music2Chaos” sonification program. And particular thanks go to Valentina Di Sante and Silvia Gambini for their creative collaboration in the dance part of the project and for their choreography of “Logistic Map Song” presented during Bridges 2016 workshop.

Then, I would like to thank Chris Budd, Kristof Fenyvesi and Erik Stern for having me during my visiting periods abroad. Above all, for their valuable communication tips, their essential and important support in the development of the theoretical structure of my research and in the design exploitation phase of the engagement and outreach activities we set up.

Many thanks to all my friends on who I was always able to count over these three years through times of happiness and times of sadness. Guys, you’re great.

Last, but certainly not least, thank you to my family for always supporting me in my endeavors. So, thank you Dad for being there quietly. Sincere thanks to Franco and Maria for their concrete support. Thank you Sebastiano for being simply “Maccio” and cause of great joy along the path. Thank you Silvia for putting up with a “father in computer” especially over the past year. Your smiles and hugs are the greatest gift of all, much more than everything I will ever able to write. A very special thanks to my wife, Francesca, without whom none of this would have been possible. I actually appreciate your sacrife, your love and your devotion. You forced me to try, supported me and believed in me more than I did. Even my most absurd explorations were conceivable in your eyes. And I owe this all to you.

And finally, thanks to my Mother who always follows and protects every step I take in every single moment of my life.

Andrea

Chapter 1

Introduction

All over the globe, every day of the week, mathematicians are carrying out research, providing new theorems, inventing new definitions, solving problems, posing new ones. They create a language and new models which become instruments for a deeper understanding of scientific, economic and social complexities. The vast majority of the public have no idea that any of this is happening.

It's part of the researchers' mission to raise the general public awareness of mathematics and science in general. This need is further emphasized by a survey of Eurobarometer 2010[1]: the society is strongly interested in science but, at the same time, is often scared by the risks posed by new technologies and the power given by the science to the scientists. Moreover, irrational attitudes towards science are often prompted by a broad scientific illiteracy. The result is a remarkable gap between the community of scientists and the society at large. The need for science communication, therefore, comes not only from the fact that the public wants to be informed, but also from the fact that it has the right to be so. In a "risk society", as defined by the sociologist Ulrich Beck, in which the image of science as a certain and reliable knowledge is hopelessly out-dated, the citizens can no longer accept top-down choices – albeit supported by the opinions of recognized experts – without being properly informed and involved[2].

Promoting math interest in Europe is becoming a crucial need, not only for the subject (e.g. to call more resources and funding by the governments and the European Commission), but more deeply for our society and culture. A basic knowledge in mathematics is required to understand and face current political and social challenges, and an appraisal of mathematical culture is an absolute precondition for laying the foundations of modern and complete European citizens. *"Mathematics is the key enabling skill for technological innovation and more sustainable development. Mathematics is the key for a deeper understanding of reality. However, even nowadays, this centrality of mathematics to modern life is not well recognized by laymen or our governaments, and it is a hard but fundamental task to try to change this common misperception"*[3].

---

[1] The Standard Eurobarometer was established in 1974. Each survey consists of approximately 1000 face-to-face interviews per country. Reports are published twice yearly. Eurobarometer surveys monitor the evolution of public opinion in all 28 EU Member States. The aim is to assess EU citizens' awareness of and support for the European Union's activities. Tracing public opinion trends helps the preparation of policy, decision-making, and the evaluation of the EU's work.

[2] AAVV (2010), *Science and Technology Report*. Special Eurobarometer 340 / Wave 73.1 – TNS Opinion & Social, p. 8-24

[3] Natalini R. (2015), *To Infinity and Beyond*. EMS Newsletter, n.97, September, p. 60-61

With few exceptions, as South Korea and Finland, international surveys are almost unanimous in denouncing the absence of scientific culture of the population, even in the most advanced countries. The European concern about young people lack of preparation in scientific disciplines, mainly in mathematics, has become matter of public debate. Ignored in the past, international surveys results on students qualification, such as those in PISA[4], are emphasized in the newspapers. Add to that the fact that sources of friction between science and society keep piling up because of the consequences of the introduction of new technologies, the choices which we are compelled by new opportunities offered, and the impact of new findings on belief and values on which our identity, culture and ways of thinking rely on.

In recent times, communicating mathematics is faced with a substrate of pervasive simplification and a sort of demonization of formal speeches, where almost anything is translated into poor and inappropriate terms. There's no perception that science is part of our lives and the feeling in respect of mathematics is becoming more and more negative. Moreover, many educated people think that they are unable to understand it, even broadly speaking, and it is a source of pride to them. Internal communication among mathematicians and scientists follows rules different from communication targeted at the general public. And today we can no longer afford to ignore how do people feel about science and mathematics outside our department or laboratory doors.

Researchers are often neither trained to communicate their findings and results to a non-specialized audience, nor they are interested in transmitting the beauty and importance of their field to the general public. This depends on many factors: the most important being probably the fact that so far dissemination and outreach have never played a relevant role in building academic career (at least in Italy)[5]. On the other hand, in a world in which science is increasingly specialized, journalists are often unable to understand, hence to communicate, the specific results of the various disciplines. We experience more and more the need for the "connection disseminators" which George Thomson[6] already imagined in his book *The Foreseeable Future* (1957). In his words: *"Dissemination should be greatly extended. It is not easy to do, and those who can do it successfully fully deserve a high place in the estimation as scientific researchers"*. Connection disseminators means new mass communicators with a strong technical expertise, but also with marked critical skills: persons who are able not only to understand the technical contents of a scientific or mathematical result, but also to frame it in the right historical, philosophical, ethical and social context.

For most of the 20th Century, mathematicians were free to pursue their subject essentially independently of the rest of human society and culture. In his celebrated book *A Mathematician's Apology*[7], G.H. Hardy wrote: *"It is a melancholy experience for a professional mathematician to find himself writing about mathematics"*. In Hardy's view, writing about existing mathematics

---

[4] Program for International Student Assessment (PISA) is an international survey brought by Organization for Economic Co-operation and Development (OECD).

[5] Something is changing after, in the last two years, the so called "third mission" has been introduced and evaluated in Italian Universities following a disposition of the Italian Ministery of Univeristy and Research.

[6] George Paget Thomson (1892-1975) was a well-known british physicist who proved the electron wave-particle duality together but regardless of the american physicist Clinton Joseph Davisson. Because of that discovery, they were awarded with the Nobel Prize in 1937.

[7] Hardy G.H. (1940), *A Mathematician's Apology.* Cambridge University Press.

paled into insignificance when compared to creating new mathematics. In many ways he was, and still is, right, but the two activities are not mutually exclusive. Moreover, as the 20th Century has given way to 21st, it has become increasingly vital for mathematicians to take steps to increase public awareness of their motives, activities, concerns and contributions. Such awareness has direct benefits for the mathematical enterprise, even if that is viewed entirely selfishly. Fortunately, many mathematicians are now convinced that writing about mathematics is at least as valuable as writing mathematics. To paraphrase Felix Klein, it would be pointless for mathematicians to invent new theorems unless the public gets to hear of them. Not the details, of course, but the general nature of the enterprise. In particular, that new mathematics is constantly being created, and what it is used for. The target of communication is not to transform the whole public of non experts in mathematicians. However, it’s a duty to strive to understand the intellectual and collective work and, more generally, the sense of the scientific enterprise that is the basis of the modern conception of the world. In 1994, the American astronomer Carl Sagan received the Public Welfare Medal by the National Academy of Science. With this award, he denied two strong misconceptions floating around among scientists who used to communicate with the general public: the idea that scientists doing it deprive valuable time to the research and the idea that a researcher is not able to be understood from others, as if his mental universe is so far from that of a layman to be in need of a “translator”.

Fortunately, in the last years these ideas are slowly disappearing. On one hand, here in Europe, we are realizing that it is important to encourage, also from the economic, political and career viewpoint, the communication activity of those researchers who are able to do it. As an evidence of this trend, just think about Cédric Villani, winner of the Fields Medal in 2010 and currently engaged solely in communication activities, providing with his work great benefits for the whole French mathematical community. Moreover, it is enough to observe the focus on dissemination activities within the Sixth Framework Program (FP6) and, more recently, within the calls of the Horizon 2020. On the other hand, we witness the proliferation of courses targeted to train science journalists and communicators able to dialogue with the scientific community and correctly report to the general audience. Therefore, science communication becomes a field of advanced research, which requires strong innovation and development.

The European institutions themselves, from the Royal Society to the Académie des Sciences, from the Max Planck Gesellschaft up to the recent introduction of the so called Third Mission for the Italian Universities, are inviting their members to communicate their research findings and raise the public awareness of mathematics and sciences in general. What was before seen as a waste of time is now outlined as a must. That requires to step outside the boundaries, also human, of specialisation and share their passions with others. Whatever purposes will be behind the decision for communicating, talking about science and mathematics, in addition to spreading knowledge, helps to raise awareness of the value of scientific and mathematical thinking, and of a reasonable and positive approach to the problems, even with those that have nothing to do with science. Furthermore, the effort of communicating mathematics sheds light on the process, clarifies ideas and unlocks complex stages. Paradoxically, science communication with the public can also help to inform other researchers of their activities and go beyond the barriers among different areas and fields[8].

Getting acquainted with science communication techniques may be useful in teaching. Although students have a specific reason to study and be applied, the ability to engage them and the strategies to keep the audience riveted are precious tools for every teacher. The school is the first place where we can make mathematics more attractive and interesting. It is essential to improve the public image of mathematics in our everyday lives by creating an appropriate language to put young people in touch with our scientific experience.

In the post-academic science era, the communication between the scientist and the public has become fundamental. Communication system gives a strong dynamic to the scientific process and contributes to science evolution. However, the system itself is evolving and changing across time. There are several public figures inside this system which contribute to take relevant decisions on the development of science in different forms, at different levels and in an extremely dynamic way[9]. These public figures don't have the sole referent in the scientist, but talk to each other. The emergent structure looks like an archipelago where all the islands are interconnected with bridges on which they can convey important information flows in both directions. In this archipelago there is no central control, but a number of centers with different decision-making power on the overall governance of the "city". Nor there is an outskirts, but a bit more peripherical set of islands. In addition, every bridge is unique because it connects different points of different islands in different ways. Therefore, we need to have both an analytical and a synthetic view of the archipelago, because the structure of science communication is a complex evolutionary phenomena with unexpected, unpredictable and emerging behaviours.

In this context, the science of complexity plays a central role, catering a unitary overview to the world and allowing us a holistic understanding of reality. The complexity approach takes into account the correlations among the different levels of reality of the science communication system and the circularity established among its components. Science communication will play a crucial role in the growth of future society. It's important to desing and developed actual communication strategies, targeted to the background and needs of the audience: from policy makers to potential industrial partners, from youth and school teachers to the general public. To achieve such an ambitious goal, a good deal could be to form an interdisciplinary team with all the skills and competences for a correct and sound scientific communication. It will feed the exchange of experiences and knowledge among research fields traditionally distant, comparing different methods and making use of tools and methods from non-linear dynamics and chaos theory.

The scientific communication can really support a unifying and interdisciplinary vision of science, overcoming any division into specific branches and specialization, according to the opinion that Stanislaw Ulam attributes to Stefan Banach in his *Adventures of a mathematician*[10]. Banach's vision deals with mathematics, but it can be easily extended to a wider setting, substituting below "mathematician" with "scientist". It describes a clever mathematician as one able to discover

[8] Carrada G. (2005), *Comunicare la scienza. Kit di sopravvivenza per ricercatori*. I Quaderni del MdS.

[9] Greco P. (2004), *Il modello Venezia. La comunicazione nell'era post accademica della scienza*. In: Pitrelli N., Sturloni G. (eds), *La comunicazione della scienza. Atti del I e II convegno nazionale*. Roma-Milano, Zadig, p. 11-38.

[10] Ulam S. (1991), *Adventures of a Mathematician*. University of California Press.

analogies between theories, while a genius is who's able to see analogies even between analogies, so as to unveil the most intimate and deepest roots and connections of science. A vision that shows how the complexity language, the emergent phenomena, and, in general, the non-linearity, could be usefully described with appropriate mathematical and communication tools recently developed in interdisciplinarity fields like cybernatics in the fifties, system dynamics in the sixties, financial market analysis in the eighties, up to the study of complex networks.

How could we make this happen? Finding new strategies and tools to communicate in a language accessible to the public the methods and the results of the mathematics within the complexity. Dino Buzzati (1906-1972), an italian writer and journalist, wrote this in a letter to the italian poet and essayst Leonardo Sinisgalli (1908-1981): *"The normal rule of dissemination is that the scientist falls. Here is the reader that rises.";* the scientist have to go down, but not too much, then asking a little effort to the reader/listener without exaggeration. The concepts, methods and results of mathematical complexity, and of mathematics in general, must be communicated in a simple, but not distorted, way without lowering the level and avoiding too spectacular tones. Even Albert Einstein told that most scientific tests intended for non-specialists try to impress on the reader rather than explain in clear and understandable terms the aims and the basic methods.

On the one hand, the new information and communication technologies (ICT) are key tools valid for a wide-ranging dissemination and communication of science. The major social networks like Facebook and Twitter, along with the blogs, are widely used to communicate new findings and advances of science, allowing you to keep in touch with a huge number of "followers". Massive Open Online Courses (MOOC) and Moodel-based platforms are opening new technological frontiers for teaching. The static nature of web sites has been almost entirely supplanted by the use of dynamic new platforms, as well as is very common, for example, the use of YouTube channels with videos, interviews, seminars and thematic lessons to achieve the public in a more widespread and effective way. On the other hand, the use of non-standard places and unconventional approaches for science dissemination and outreach allow to present science under different reading levels according to the culture and the interests of the different possible participants, becoming mine of ideas for thoughts and new insights.

Everything comes from the idea of giving back to mathematics those moments and situations from which it is artificially separated when encoded in an article or in a manual, enlightening the creative and dynamic aspects that distinguishes it. The aims are, first, removing the erroneous caricature of abstruse, barren, useless and far from the meaning of existence discipline, and then trying to reveal how mathematics is the very essence of the reasoning inherent in each of us and of how the things work in the world around us. "*It's worth making people aware that new mathematics is constantly being created. This objective is more important than explaining what that new mathematics consists of, and it is more important than explaining what mathematics actually is. Only when people recognise that mathematicians are doing something do they start to get interested in what they are doing. Only when they've seen examples of what mathematicians are doing do they start to wonder what mathematics is"*[11].

[11] Stewart I. (2006), *Mathematics, the Media and the Public*. Proceedings of the International Congress of Mathematicians, Madrid, Spain, p. 1631-1644.

However, the role of mathematics in maintaining society is seldom appreciated, mostly because it takes place behind the scenes. Although the role of mathematical sciences in civilization has been of central importance for centuries, the current trend to a global economy and a knowledge society has made information and innovation technologies increasingly dependent on scientific research, whose results and techniques are underpinned and driven by mathematics. *"It is a common interest of the entire mathematical community to outreach activities to make society and industry aware that mathematics is the common denominator of much that goes on in everyday life, activating the many sectors of society that can benefit from mathematics. Indeed, promoting such awareness will bring resources to all mathematicians"*[12]. Mathematicians or scientists must invest the necessary time and work in communicating mathematics, and the mathematical community should have the energy to make the outreach effort. A change of attitude of mathematicians towards public awareness of their discipline is vital. It is necessary to enhance the internal level of consciousness in the mathematical community because improving communication could be very useful for the future research. The main focus of a global policy to promote mathematics among the public should concern strategies and ideas to implement in order to have a deep impact on society as a whole. We have to reach a wider audience to show the real value of present and past mathematical results. We have to be more appealing always maintaining a sufficient standard of accuracy, avoiding excesses and cliches, and creating astonishing situations with the right mixture between science and entertainment, that is ultimately the heart of the communication strategy.

In an overall sense, mathematical thinking is, after speech, the most important human faculty. It was this skill especially that helped the human species in the struggle for survival and improved the competitive abilities of societies. I believe that mathematical thinking has a special place in evolution. By mathematical thinking I mean analytic and logical thinking in a very broad sense, which is certainly not independent of the ability to speak. Of course, the development of mathematics as a science is a cultural achievement but, in contrast to languages, it developed in a similar way in different societies. We can face the fact that the importance of mathematics for mankind has grown continuously over the centuries, regardless of the cultural and social systems. No modern science is possible without mathematics and societies with highly developed sciences are in general more competitive than others. Attaching this value to mathematics, one must conclude that society has the fundamental right to demand an appropriate explanation of mathematics. And it is the duty of mathematicians to face this responsibility.

Next to a mathematical vision is increasingly necessary a "philosophical" vision, in accordance with the original etymology of this word: philosophy as love to wisdom, and consequently to science, again meant in a global sense, intrinsically related to the real world and its cultural and civil progress. It's also equally important an "historical" vision of the science. Science communication can't forget the way in which revolutions, ideas and scientific methods are born, as much as the scientists who developed them, so as to be seen as men of their time, culture and society.

In this perspective, fruitful collaborations are possible if each person involved is willing to contaminate their specialization through learning the language and the specificity of the others. These contaminations are not synonymous with superficiality, dispersion and amateurism, but opportunities to create special connections, collaborations and synergies that lead to deeper and

[12] ESF Forward Look on "Mathematics and Industry" (2010), European Science Foundation, Strasburg. www.esf.org

original visions of what are generally obtained in an internal logic to the individual disciplines. There could be risks, but I agree with Leonardo Sinisgalli when states that *"it needed a symbiosis between intellect and instinct, reason and passion, real and imaginary; it was urgent to try a mingling, a graft, even if it means sacrificing the purity"*.

Within this landscape arises my research project whose main target is to find an attractive and effective way to communicate complexity theory to the general public and at the same time to use complexity theory to communicate science in general. Science of complexity is interested in the complex and unpredictable links among various systems among which the physical and social ones, and allows an holistic, less pretentious and more careful understanding of reality. It gives us a new way of looking at the system as a whole and a clear insight about its organization taking into account the relations among different levels of reality and the circularity that happens between the system and the environment. So, the science of complex systems is both multidisciplinary and interdisciplinary. It feeds off knowledge and experience exchange among research fields traditionally far apart, and comparison among different methods. Along these lines, a reference point is, for example, Santa Fe Institute, born in the 80s in New Mexico (USA) as a research center where physicists, mathematicians, computer scientists, biologists, sociologists, economists and others study the diversity of the complex phenomena searching for common origin and language.

The word "complexity" is too vague and generic. It has to do with interaction, interdisciplinarity, emergent behaviors and nonlinearity, and all too often is open to misunderstanding among common people. Complexity refers to interweaving of the fabric. While being constituted by different parts, it possesses features that the individual parts haven't got and can be explained only to a limited extent undoing it. Hence, the simple is no longer the foundation of all the things, but just an articulation among different complexities. Consequently, the complexity of the world requires thinking able to face and engage with it. In order to understand complex phenomena we have to accept that between causes and effect could be a circular relationship different from the basic idea classical way of thinking. We need a circularity in thinking that increases our ability to understand, without falling back on a vicious cycle. We can apply this kind of vision in various areas from social studies to economics, from biology to physics, from meteorology to functioning of the immune system, from neuroscience to teaching: all systems where dynamic order and organization emerge from an underlying simplicity across a self-organization process. Communication itself is a complex science.

To understand and look at the bigger picture of complexity we have to accept a counterintuitive and nonlinear causality which can often feel like unexpected and astonishing. We have to focus our attention on self-organization process which makes possible that highly organized behaviors could arise from relationships circularities in the absence of a project. This can include the language. In the process of interaction with the text by which we understand its meaning, we understand the phrase starting with the meaning of the words. But, at the same time, the meaning of the words sets out based on what emerges in terms of global meaning of the phrase[13]. This kind of self-organization could seem paradoxical: the system as a whole is more than the sum of its parts, precisely because self-organization emerges, but is also less than the sum of its parts, because it carries out just one of the possible organizations.

[13] Cerrato S. et al. (1996), *Caos e complessità.* Quaderni di divulgazione, Laboratorio interdisciplinare SISSA, Cuen srl.

Before I face the problem of how to communicate complexity, and how using complexity ideas to communicate mathematics, I have to deal with the complexity embodied in communicating mathematics. This has forced me to be familiar with the past and present masters of science and mathematics communication starting from Galielo Galilei via Michael Faraday until Ian Stewart . I had to learn how they did it in the past and do it nowadays. I wanted to understand why they started doing that and how communicating mathematics has changed over the years. I needed to know better approaches, methods and strategies they use for an effective communication. I wanted to analyze in detail the difficulties they face in communicating mathematics, the objectives they set out, the styles they use with different audience, what's worth communicating and the tools of the trade.

Early on, I realized to have a huge amount of literature on science communication and communication in general, but short of material on mathematics communication. So, I wondered how I can remedy this and the answer was a journey. That's the second step of my research project. I've been on a journey of knowledge, discovery and first-hand experience around Europe to find out scientific and organizational models that have led and are leading to special achievements in communicating mathematics. A direct result has been my personal meetings with some of the greatest experts in the field like Chris Budd, Alex Bellos, Andrew Jeffrey, Simon Singh, Kristof Fenyvesi, Andreas Matt, Eduardo Sáenz de Cabezon, Rogerio Martins, Cedric Villani and many others. Visiting periods, participation in conferences and events, direct cooperation and interviews have been key moments in my research path. The report of this journey has also become a series of articles entitled "Communicating mathematics in Europe" in the quarterly magazine *Lettera Matematica*[14] organized by Pristem Research Center and edited by Springer both in the italian and the international edition.

During this journey, I've also developed projects and events focused on communicating mathematics to the general public, presented workshops for students and teachers, written a book for kids from 9-years-old and up based on an innovative approach to mathematics, and started creating a show on chaos and complexity which involves music, dance and visual graphics.

Chapter 2 collects the first three articles in the series "Communicating mathematics in Europe" for the international Springer journal *Lettera Matematica Pristem*.

Chapter 3 provides an in-depth review of science and mathematics communication literature. In the first part, I start trying to answer questions like "Why to communicate?", "Who have to communicate?", "How to communicate?" and "Who is the public?". I reflect on the differences between the "deficit" model and "dialogue" model in science communication and how from them we have arrived to a three-pronged approach consisting of communication, consultation and participation. Then, I quickly retrace the historical evolution of science popularization to the present day. In the second part, I move from science communication to mathematics communication

[14] *Lettera Matematica* is a quarterly journal, that has the objective of discussing mathematics and the world that revolves around it, thereby embracing other fields of knowledge and engaging other scientific communities. The journal addresses topics related to mathematical research but also aims to discuss and reflect on society and its relationships to scientific culture, underlining the contribution that the mathematician – like all other intellectuals – can and must make to the growth and well-balanced development of society. It also provides common ground for those mathematicians who use their chosen discipline as a bridge to other worlds rather than working in isolation.

highlighting the main obstacles and challenges we have to face with, in particular those of language and rigour, and how we can reinforce the message we'd like to convey.

Public Engagement and Outreach are the key words in Chapter 4. After a brief introduction of the close link among engagement, outreach activities and communication, I analyse thoroughly three live or face-to-face communication events like science festival, math busking and science cafes. I start describing my first-hand experience in Bath Taps Into Science festival and Birmingham Big Bang Fair during my visiting period at the University of Bath and what I learned from that. Then, I present in detail some engagement and outreach activities I've designed, set up and/or taken part in: FermHAmente Science Festival, VereMath Street, Scienza in Vacanza and a series of Science Cafes. The last three have been carried out together with the research group Unicam Science Outreach in which I'm included since July 2015, and of which I outline the main goals. There is also a section dedicated to a proposal for a maths/science communication course drawn up during the Imaginary Conference held in Berlin in July 2016.

In Chapter 5, I tackle the issue of how communication can improve teaching. At the beginning, I explore the benefits teachers could obtain by using popularization techniques into a teaching/learning environment, how storytelling can be an effective tool to communicate and teach mathematics, and how laboratorial activities can enhance students perception about mathematics improving their reasoning skills, mathematical thinking and analytical capabilities. Then, I present the book "Il tranello e la soluzione matematica" by Andrea Capozucca and Barbara Cerquetti and the whole project behind it, and a laboratorial activity project involving mathematics and dance. In particular, I underline the idea of the book as an innovative tool for teaching mathematics in an engaging, interactive and interdisciplinary way, and the importance of a "whole body" learning approach to mathematics as I first-hand experienced with Erik Stern and his math-dance research group.

Chapter 6 provides an overview of the FOTCAB project, an interdisciplinary show/exhibition to communicate complexity and chaos theory to the public. After a description of the overall structure of the project, choices I do and methods I use, I go into detail as regards its main aims, its implementation and development. Then, I set out the project outline and present what has been carried out so far. The project is still work in progress.

Finally, in Chapter 7 I give a short summary of the whole journey reasoning on the evidence I have accumulated, a quick overview of the limitations I experienced and suggestions for future implementations and developments of the undertaken lines of research and projects.

Chapter 2

Articles in *Lettera Matematica Pristem*

This chapter collects, in chronological order, the first three articles of the series "Communicating mathematics in Europe" that I have written for *Lettera Matematica* edited by Springer and Centro P.RI.ST.EM Bocconi University of Milan. Each article is structured around an interview with an European master maths communicator, and deepens specific themes related to mathematics communication. Guests of these three episodes are Chris Budd, Alex Bellos and Andrew Jeffrey. All the articles are translated by Daniele A. Gewurz.

2.1 Article 1: Chris Budd

Lettera Matematica n.98 – ottobre 2016

Lettera Matematica International Edition, Springer-Verlag

DOI 10.1007/s40329-016-0147-z

Abstract. This article is the first in a series on the communication of mathematics in Europe. In this first instalment Andrea Capozucca visits with Prof. Christopher Budd, OBE. Some of the topics covered in the interview with Budd are the golden rules for an effective communication, the ways to improve skills and nourish passions, past, present and future projects, Bath Taps into Science, the events of the British Science Week, and much more. The result is a snapshot of the present state of the communication of mathematics, seen through the eyes of one of the leading figures in the field of public engagement and outreach. The author also experienced first-hand the methods and tools used in communicating maths in Britain, leading to a relection on the importance of the communication of science today.

Keywords. Chris Budd – Bath Taps into Science – Popular science – Maths communication - Outreach – Public engagement – Interdisciplinarity

Communicating mathematics

My doctoral research project is about the communication of mathematics, and especially the construction of new tools, original, interdisciplinary approaches to science outreach, and public engagement with mathematics, sciences and technology. The first problem in a new project is always the same: where to start? If we want to look beyond the horizon, we have to climb *"on the shoulders of giants"* ([7], p. 167). Thus, why not begin with a journey? A journey of knowledge, discovery and foundations. A tour around Europe to meet the protagonists and the main stages in the field of mathematical communication, to discover and experiment personally ways and methods used by those "giants" to build and improve the tools a good communicator must have in order to be effective and reliable, taking as a model the most advanced results and the most important projects in Europe. I had the same idea as Enrico Betti, Francesco Brioschi and Felice Casorati who, in September 1858, began a journey through France and Germany to meet the most important mathematicians of the time, to *"form contacts with the protagonists of the most advanced studies and 'photograph' mindsets, structures and organisations that made these great achievements possible"* [6].

The adventure begins

First stop in my journey: the University of Bath, to meet Chris Budd, who has been working for several years in the field of communicating mathematics (Fig. 2.1). He is a professor of mathematics at the Royal Institution of Great Britain, vice-president of the Institute for Mathematics and its Applications (IMA), the founder and director of the Bath Taps into Science festival, as well the recipient, in the last twenty years, of several awards for his work in scientific and mathematical education, including the Order of the British Empire (OBE), bestowed by Queen Elizabeth.

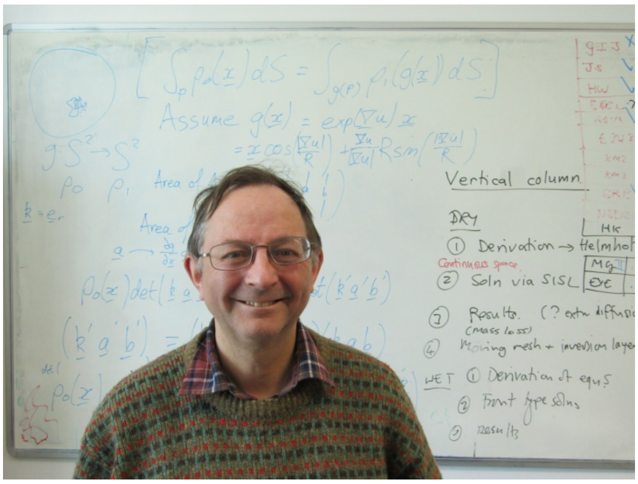


Fig. 2.1 – Prof Chris Budd in his office

And now I have arrived. *"I have an appointment with Professor Budd. I am Andrea Capozucca."* At my side sits a blonde girl with a book in her lap. The lift's doors open, and an unprepossessing man gets out; he wears a symmetrically patterned green wool jumper and has a cup of cofee in his left hand. He is Budd, I immediately recognise him. I am about to get up, but I realise that he is looking at the girl on my right. Without hesitating, as soon as he is out of the lift, he turns left and in a few steps he is in front of her. *"Andrea? Welcome to Bath!"*, he tells her holding out his hand. She looks up, surprised, and answers, *"Er… Actually, I am not Andrea. My name is Helen. I am sorry."* A chill descends on the entrance hall. Budd is incredulous. Now, having realised the misunderstanding, I get up and in a moment I am behind him. I tap him on a shoulder saying: *"Excuse me, Professor Budd. I am Andrea Capozucca. Pleased to meet you."* He turns and from his puzzled look I understand he did not expect to see a man, much less one as big as me. Our only contacts, in the last few months, had been by email; this was the first time we met in person. In a moment his initial perplexity turns into a wide smile that breaks the embarrassment. *“Andrea? I have a niece called Andrea. The name deceived me.”* Another smile, and he shakes my hand and shows me the way towards his oice, to plan together my stay there. As soon as we are in the lift, he looks at me and continues, *“It’ll be okay. Some arrangements will have to be changed, as you may imagine, but no problem. It will be 2 weeks full of work and meetings. Are you ready? Welcome to Bath!”*

“This is my oice. Settle down wherever you can find room.” Budd’s welcoming voice takes me back to the here and now. I go in and drop my backpack on the first empty chair. The other ones are around a round table almost completely covered by books, folders, posters and boxes. While Budd reaches his desk to get his laptop, I look around the room, curious. On the wall on my right is a blackboard chock-full of equations and notes about some mesh methods used to improve the accuracy of weather forecasting and climate studies; on the opposite wall, a coat rack keeps company with a second bookshelf, propped against which stays a curious green trolley case in which some mathematical puzzles can be glimpsed. But one thing in particular catches my attention: a wonderful cartoon in which I recognise Ian Stewart, Marcus Du Sautoy and other giants of the communication of mathematics (Fig. 2.2). *“Call me Chris”*, he says passing by me and noticing my interest in the cartoon. *“Can you recognise them? They are the champions of popular mathematics in the United Kingdom. In the foreground, Zeeman, Stewart and Du Sautoy, and behind them all the other ones. A beautiful cartoon we were given some years ago, in 2010, at the end of a IMA conference about public engagement, with all of them, and yours truly as one of the organisers”*, Chris remarks. *“And how come you are not in the cartoon, Chris?”*, I ask, intrigued. *“Because I got one all for myself!”*, he replies with a smile and points at another cartoon at a short distance: Chris in a bathtub with a double pendulum and some butterlies luttering about his head (hinting at the chaos theory, one of his research areas) (Fig. 2.3). So I understand my journey could not have started in a better place.

Fig. 2.2 – The cartoon with the British “champions” of the communication of mathematics
(in the foreground, from the left, Zeeman, Stewart and Du Sautoy can be recognized) on a wall of Prof Budd’s office

Fig. 2.3 – Prof Budd's cartoon

His calm but engaging style, the passion and mastery with which he tells what he does and his great ability to listen make everything so much easier. I ask Chris if I may interview him and he answers serenely: *"Why, of course, but not right now. We have lots of stuff to do before"*. And, with pen and paper, he begins to explain to me in detail the engagements and activities we shall be directly involved in the next 10 days, including taking part in the Big Bang Fair in Birmingham as scientific promoters at the IMA stand, together with the public engagement and maths communication group led by John Meeson, as well as organising and mounting the "Bath Taps into Science" festival in Bath. Afterwards, he says, *"That is all, for today. Let's meet tomorrow at 9, here, in my oice. I'll be glad to introduce you to Andrew Ross and the rest of the board of the 'Bath Taps into Science' festival, to adjust the last details of the programme"*. Andrew Ross is the festival project manager of "Bath Taps into Science" (Fig. 2.4). The other board members I met are Michelle Smith, Margarida Dolan, Bob Draper and Peter Ford (Fig. 2.5).

Fig. 2.4 – From the left Prof Budd, Andrew Ross and Andrea Capozucca

Fig. 2.5 – Margarida Dolan, who teaches science communication and public engagement to undergraduates and graduates, with one of her student

While going back to my lodgings, I think about the delayed interview and feel like one who goes to a lifeguard to be taught how to swim and, instead of receiving instructions, is directly thrown into water. A method more inductive than deductive taught me that the first rule of a good communicator is being able to put themselves on the line. A good communicator cannot stand still in a single, fixed role, but has to learn how to be flexible, assume multiple roles, and switch from one to another. Thus, immediately to work. Theory may wait, for now. As a consequence, the following days were a valuable and indispensable training ground to refine the essential abilities for an efficient and effective communication of mathematics. They were full and tiring days, but rich in important experiences and different roles to cover. I took part as an observer in a Royal Institution Maths Masterclass, where I learnt that the direct involvement of the audience in hands-on and interactive activities facilitates their understanding of the proposed mathematical notion and improves learning and self-esteem. I had the opportunity to observe and experiment in IMA's stand in the Big Bang Fair the effectiveness and validity of public engagement as a two-way process that engages attention and listening, with the goal of generating a mutual well-being in the participants. I lived from the inside the organisational dynamics of a science festival and understood the complexity and interconnectedness of its organisation and the perfect coordination of all its components, including the sponsors. I saw the surprised and intrigued faces of people of all ages attending the exhibits of Bath Taps into Science, and their amazement in front of something new, an amazement that is not just an aesthetic feeling, or a momentary curiosity, but the beginning of a process: it kindles the desire to enter into relations with the mathematical (and scientific) world, to know it. Moreover, living alongside Chris I also learnt to understand the greatness of a scientist who gets down from his pedestal to be at the side of his students, to observe them, even help them to tidy up tables and chairs, so transmitting his passion for his work. Lastly, I remember the tiredness, on Saturday afternoon, at the end of the Family Science Fair, the last engagement in our calendar, and my wonder when Chris suggests closing the day with a pint at the historical pub "The Raven" in the centre of Bath, *"I still have an hour before my train to Bristol. Let's enjoy a well-deserved pint at the Raven. Every mathematician who passed through Bath has been at least once at the Raven for a beer. So, now it's your turn"* (Fig. 2.6).

Fig. 2.6 – The pub *The Raven* in Bath

Ten days after our first meeting we are once again back in his oice. Everything is in its place, even the green trolley case that, during the Big Bang Fair, I discovered to be Chris's "Maths Magic Box", that is, a collection of games and hands-on activities of increasing difficulty, most of which are based on mathematical principles. They are intended for stands where people will not interact with the promoters for more than 5 minutes, and Budd also uses them to train students who will be personally involved in scientific promotion. Chris gives a course in communication of mathematics whose main goals are to give the students the communication skills required for maths, and soon after that to put these skills in practice in concrete situations, such as science festivals, masterclasses, popular articles, "math busking" and so on. A theoretico-practical course requiring a significant involvement and time investment, during which Chris puts at his students' disposal all the richness and depth of his experience, including the contents of his green trolley case.

A splendid framed photograph behind him shows him smiling together with his mother, wife and daughter (Fig. 2.7). *"It was taken last October at the Windsor Castle. It was an amazing day. I met lots of incredible people who'd done wonderful things. I was very honoured to meet the Queen herself and she seemed very interested in the Bath Taps into Science fair"*, he remarks happily behind me.

I take the opportunity to ask, *"What are the golden rules for a speaker to follow to catch the listeners' attention?" "There are really only two rules that I think you should follow"*, he replies, determined. *"One is that you should always imagine that you are in the audience. Think what it would be like for you to be sitting in that audience listening to the person that is giving his talk. So, be completely aware of your audience. Then, the second rule is connect with the audience and be very enthusiastic about what you're doing, be very positive. Show a lot of interest both in your subject but also in your audience. You should talk to your audience, not at your audience. These are really the only two rules, everything else is a detail. There are diferent styles and all styles would work provided you go by those first two rules"*.

Fig. 2.7 – Prof Budd is honoured with OBE at Windsor Castle
(from the left Sue, Prof Budd's wife, Prof Budd, his daughter Bryony and his mother Jillian)

At this point, it comes naturally to me to ask, *"Then, what are the golden rules to communicate maths?"* In a calm voice, he replies, *"Maths communication is particularly diicult because lots of mathematics really is beyond the audience. It's the sort of subject where a lot of background knowledge is often needed to understand something"*. A short pause for relection, then he resumes, conidently, *"The two things that I always like to think of are 'what do I want the audience understand at the end?' and 'how can I get there from what they know?'. And then lead them to that in easy stages trying never to lose them on the way, but also to make them aware of where we're getting and showing them the importance and relevance of what we're doing. But I also feel quite strongly that you should never show people mathematics in such a way that you leave apart the audience. If they are coming to hear maths, then give them some maths, but don't kill them with maths!"*, he concludes with a smile and sips from his ever-present cup of cofee. *"So, that's my rule: don't bring maths down to where the audience is, but bring the audience up to where you want, where you feel they should be"*.

Listening these words, I recall what Dino Buzzati wrote in a letter to Leonardo Sinisgalli, then the director of the journal Civiltà delle macchine, that in communication it is the listener that has to rise, in contrast with popularisation, where the scientist goes down. The term "populariser" is still used to denote the scientist who spreads technical or scientiic ideas to the public. Today, however, this notion, where "who knows" pours their knowledge onto "who does not know" in a top-down, one-way relationship in which the learner is completely passive, is outmoded and replaced by a broader notion of communication, where there is a direct involvement on the part of both the communicator and the audience in a two-way flow of information that includes listening, debating and interacting.

Returning to the communication of mathematics, I try to get into more detail: *"Is there any aspect you deem indispensable for an effective communication of maths?"*. *"As I said earlier"*, he replies, *"maths communication can be quite hard, because of the nature of our subject. But I think the two things that always we must bear in mind are: maths is a wonderful subject and, if you convey the kind of fun and puzzled and creative side of maths, that's very important; and, maths is a subject which has many, many applications. What I never want to do is to sell maths just for its* *applications or just think of maths as something without applications. You should try to put it in the context of people's lives to help the communication"*.

Chris is an applied mathematician and is known in the mathematical and scientific world for his important contributions in the field of non-linear diferential equations and their applications to industry. He often says, *"I am interested in every field to which mathematics is applicable, but we must not sell maths just for its applications"*. From his words a "unitary" mathematics emerges, not one that is separated, as often occurs, into its pure and applied aspects. Chris goes on, *"I've never thought that there has been a separation. If you look at the really great mathematicians like Euler and Gauss, Archimedes and Riemann, people like these saw no separation. They say, 'It's just maths'. Yes, there is a separation between purely abstract and applied, but that's fine because you can tell a good story in that way. A very nice story would be if you look at Fourier who developed the subject of Fourier series as an answer to a problem in conduction of heat. He started with that problem and then from that found the best way for solving it was the Fourier series. And then you can show that the Fourier series lead to a study of waves and the modern study of communication. So, this is where the power of abstraction was so important, because it showed how a mathematical tool, which was used to solve one problem, could then be useful to solve many, many other problems. That's a great story and Fourier himself was a very interesting guy who was for a while a magistrate in Napoleon's army in Egypt. This is in itself a story that deserves to be told. A story that enriches and completes the strictly mathematical one"*.

Listening to Chris is an ecstatic experience. His ability to speak to audiences of different kinds and absorb the listener are even more impressive in a face-to-face meeting. While I ponder this, my gaze falls on a set of flyers on the table: four brochures of different colours, with different images, all bearing the same title, "Living in a complex world". *"What's this about?"*, I ask. *"In 2010 I was the director of this exhibition, within the Royal Society Summer Exhibition, which was enormously successful. We built four different paths to offer the visitors a hands-on experience about the ways mathematics helps us understand the complexity in the world around us, from weather forecasting and chaos to energy production, from bouncing balls and sand to the behaviour of locks of birds and a crowd in motion. The main goal of the exhibition was to communicate that maths can be used not just to understand complexity, but also to see how to use this knowledge to improve our everyday life. We worked one whole year to create the exhibition! But it was worth it"*.

"Chris, do you believe that complexity science can give an answer to the separation between pure and applied mathematics?", I ask him with interest, since my doctoral program is on complexity science. *"Good question!"* he answers with exuberance. *"One reason it could be a very good answer is there's a lot of complexity science directly related to people's life. You can talk about crowds for example, or you can talk about the weather, or you can talk about economics. A possible risk with complexity science is that sometimes people make claims which are not really true. So we have to be quite careful to separate the real hard science from what I call the 'seedy' science. We have a lot to be careful about!"*

We take a short break, and between sips of coffee, Chris replies to some emails and yours truly begins to realise that his time in Bath is almost over. As soon as Chris is again at the table with me, I read him a passage from an article by Umberto Eco [4], in which the great Italian semiologist argued that scientific knowledge should be told through stories. I ask Chris what he thinks about it.

"I suppose it depends on the story", he replies while checking his watch. *"The story I always like to tell is how a mathematical idea can be relevant to someone's life or how it's important turned in an application. So, in math way, I think, yes, you can tell a story. However, you've got to be a little careful because of course some mathematics just is mathematics. If I'm talking about the formula for π, I feel good enough and it's all right, in that it doesn't need to be linked to a story. But one thing I do know is that children or, in fact, adults, like to know about the people behind the mathematics. And I suppose that's part of a story. So, not just something that you have in a book, but someone like Euler or Galois. Florence Nightingale*[15] *was responsible for some mathematics and you can tell the story about the person. And others that I found very effective"*.

In short, storytelling is fine, but with great care. Tales may be interesting, but another problem arises. When we talk about mathematics, we must keep in mind formal rigour: it is difficult to even imagine a non-rigorous maths, and it is quite legitimate to raise the doubt whether it is possible to communicate maths omitting rigour. On the other hand, non-mathematicians may by scared by even a single formula in a text, just as a sentence including strictly technical terms completely excludes those who don't know the meaning of the terms. The journalist and science writer Pietro Greco himself proposes, in this regard, an uncertainty principle for science communication that says *"I cannot express a scientific notion, simultaneously, with both a maximum of communicability and a maximum of rigour"* ([5], p. 19). This holds especially for mathematics. Chris nods and points out that this idea dates back to Michael Faraday, the great 19th-century communicator. *"He had exactly the same vision as Mr Greco"*, Chris says. After telling me how the Royal Institution's Christmas Lectures, started in 1825 by Michael Faraday who gave several of them, were the first attempt to communicate science to the general public, he adds, *"Obviously if I went in front of an audience and talk with full rigour about mathematics, I would lose everybody almost immediately. So, you don't do that, you have to be reasonable. It's actually the same with the undergraduate teaching. If you go with full rigour to an undergraduate, he would not understand. So, my plan with undergraduate teaching is always to give the basic ideas and then say to the students where they can look this up, then trying to get more details. I think the same holds for communication. If you get the basic enthusiasm, people can always follow you up if they need to. But you have to make compromises"*. As soon as Chris stops, I remind him about Mathscot!, the yellow character who sponsored the "MathsCounts" programme and who wandered about the stands in Birmingham to let himself be photographed with the visitors. And I tell Chris, *"Mathscot! looks like a perfect compromise!"* We both burst into laughter, then Chris remarks seriously, *"That is an example of how not to communicate maths!"*

I take the opportunity to thank him again for the wonderful experience I had in the IMA stand at the Big Bang Fair in Birmingham and we end by talking about his "creature", the Bath Taps into Science festival. I ask him when and how the idea of the festival was born, getting Chris started. *"Bath Taps started in year 2000. The reason we started then was we wanted to mark the new millennium and we thought this 'a great tangible sign would be to have a science fair'. So, I started quite a small fair that now has become quite a big fair. How do we choose the topics? We like to*

[15] Florence Nightingale showed that statistics could be used in an effective way as a basis to significantly change the social practices of the time [2].

have a bit of a theme: it might be citizen science[16]*, or this year we have a sort of astronomical type theme, but I've always found that the best fairs are ones which are led not so much top down by the organizers, but bottom up by the exhibitors. So, let people have full creativity about what they want and use that. But at the same time it's important to have a bit of structure. Now, Bath Taps has usually focused on the physical sciences with a bit of biology and a bit of psychology. We have quite a large maths contents which is unusual for a science festival, and I'm afraid this is me. But we're always trying to extend the scope, while at the same time keeping within the overall themes of science. Quite often a main talk that opens up Bath Taps is related to these themes"*[17]. If I were to describe Professor Budd as I saw him while busy those days, during the Bath Taps festival, I would say that he is a living example of public engagement. Catherine Attard suggests the following definition of "engagement": *"a multidimensional construct, consisting of three domains: operative, cognitive and afective"* [1]. The interaction of these three domains leads both parties to feel good and at ease, to think deeply, to take an active part in what is happening and to create a positive relationship between the parties themselves. This is exactly what I am experiencing during this interview. With this thought in my mind, I ask Chris, *"Why is engagement so important in communicating mathematics?"*

"My research area, where I do my serious research", he says while straightening his spectacles, *"is exactly in applying mathematics to other areas of work, particularly engineering, dynamical systems physics and also to problems in industry. So, this is where I work in my professional career. What's wonderful about public engagement work is that it brings me in contact with other disciplines. Now I'm working quite closely with artists and there's a lot in common between art and mathematics people. Mistakenly many say that mathematics is not creative and art is creative. In fact, they are both very abstract and involve creativity. So, I've been really enjoying recently working with artists exploring how we can use maths to help art and use art to help mathematics. A few weeks ago, I gave a talk titled 'The Mathematics of Mordor' where we looked at how the films about The Lord of the Rings required a lot of creative mathematics to make them work. So, I think very strongly that maths is the ultimate interdisciplinary subject!"*

I check the time and see that it is just after noon. Chris has several engagements with his students in the afternoon, and then has to go home, since tomorrow he will have to attend, as a speaker, a 2-day meeting in Manchester about science communication. I gather my stuff, put my sack on my back and follow Chris along the fifth-floor corridor that leads to the lift. But before saying goodbye, a last question, *"How about future projects in the works?"*

The lift doors slide closed and Chris begins, *"As for my research projects, I'm still very much working in the area of climate. I'm trying to build mathematical models which will help us to understand both our past climate and our future climate. I'm very passionate about this thing because it's so important for the human race that we understand climate. My other big area which is about to start is working more in communications with the big data, as things change because of*

---

[16] Scientific work undertaken by members of the general public, often in collaboration with or under the direction of professional scientists and scientific institutions.

[17] The 2016 lectio magistralis, titled "Big Bangs and Black Holes" was given by astrophysicist Carole Mundell, recently awarded a Woman of the Year Technology Award.

using mass of data. These are my big future projects and I've got some PhD students about to start with me doing that. So, I'm very excited about that".

In the meantime, the lift has arrived at the ground floor. *"In terms of communication, I'm writing three books at the moment. I'm writing my second Mathematics Galore! [3]. I'm writing a book that is a sort of picture book where you take photographs of things in real life which have a mathematics in them (think about a picture of a rainbow or of a cloud or of a crowd or of rocks). And the third book is on the climate, a sort of a guide about how mathematics can help us to understand climate. Oh, sorry, there's a fourth book I forgot, ready to be printed which is called 101 Uses of a Quadratic Equation. All started from the great success of an article that appeared in the online journal Plus (https://plus.maths.org/content/101-uses-quadratic-equation) and the followers' request to write more about the theme. I spent a long time writing"*.

This is how the interview ends. I realize that I have internalised two fundamental things about communicating mathematics: the first is that theory and practice have to be strictly connected, since the former would be empty without the latter and the latter would be blind without the former; the second is the effectiveness and power of public engagement. Here I am at my flat. Just the time to finish packing and off to the coach station. Next stop: London.

Who is Chris Budd

Christopher J. Budd was born in London on 15 February 1960. In 1982 he received his Bachelor's degree in mathematics at Cambridge University as the senior wrangler. He went on to study at Oxford University where, in 1986, he received his D.Phil. in mathematics with a study on numerical methods to solve nonlinear elliptic partial differential equations, written under the advisement of John Norbury. For the next 3 years he was a researcher at St John's College, Oxford working in numerical analysis at the Oxford University Computing Laboratory and as a fellow sponsored by the Central Electricity Generating Board (CEGB), developing numerical methods for third-order partial differential equations. From 1989 to 1995 he taught numerical analysis at the University of Bristol and in 1991 he was awarded the Leslie Fox Prize for Numerical Analysis. Finally, in September 1995, he was appointed Professor of Applied Mathematics at the University of Bath, where he works still today. In 1999 he was one of ten scientists awarded the title of "Scientist for the new century" by the Royal Institution of Great Britain, which the following year appointed him Chair of Mathematics.

Today he is the director of the Interdisciplinary Centre for Nonlinear Mechanics and of the Institute for Complex Systems in Bath, vice-president of the Institute for Mathematics and its Applications (https://www.ima.org.uk), holds the chair in Public Understanding of Mathematics at ICMS in Edinburgh and, since 2016, Professor of Geometry at Gresham College, where he is going to give a series of public lectures titled "Mathematics and the Making of the Modern World".

His research interests range from scientific computation, especially in the new field of geometric integration, to non-smooth dynamical systems, from partial differential equations to the subjects of mathematical modelling and industrial mathematics. A large part of his work in the last decade has focussed on meteorology, with a particular interest in numerical methods to improve the reliability of the weather forecasting and data assimilation methods, in close collaboration with the Met Office, the United Kingdom's national weather service. In 2012 he was awarded the Knowledge

Transfer Award for his progress in this field. Another research area cherished by Prof Budd is climate. He is actively involved in several international networks about climate modelling: he is a co-director of CliMathNet (https://www.climathnet.org) and an active member of the Maths of Planet Earth (MPE; https://www.mpecdt.org) programme. He also collaborates with energy, telecommunication, food and aerospace industries.

He is greatly involved in education as well as in communication of mathematics to the general public, especially to the young. He teaches in the national programme of interactive maths lecture shows for teenagers, Maths Inspiration (https://www.mathsinspiration.com) and is vicepresident of the Royal Institution Maths Masterclasses programme for Bath and Bristol. An occasional guest in radio and TV shows, he has been a member of the EPSRC Public Understanding of Science committee since 2000, the year when he created the Bath Taps into Science festival, which offers public engagement and science outreach activities as part of Britain's yearly Science Week. In 2010 he was the director of the unprecedented "Living in a Complex World" exhibition, part of the Royal Society Summer Exhibition. He was later appointed Education Secretary of the London Mathematical Society (LMS) and in 2011 he was one of the authors of Vorderman Report on the present and future state of mathematical education in the United Kingdom. In October 2015 Queen Elizabeth II made him an Officer of the Order of the British Empire (OBE) for his service to scientific and mathematical education over the years.

Bath Taps into Science

Bath Taps into Science (BTIS; http://www.bathtapsintoscience.com) is a prize-winning science festival with the goal of involving and intriguing the young and the general public about science, engineering and mathematics. The festival has been awarded several prizes, including the British Science Association Award for best event during Science Week. It was one of the reasons for Prof Budd being honoured with the OBE.

Launched in 2000, the festival is presently a staple of the scientific landscape of Bath. Even though it is organised and largely funded by the University of Bath, BTIS cooperates with several other organisations, in Bath and elsewhere, including many schools. The philosophy underlying the festival is simple: everyone may take part in it and have a closer view at science (which here includes engineering, technology, mathematics, medicine and social sciences), as long as it is presented in an absorbing and interactive way. There is no risk of debasing science; on the contrary, BTIS's goal is to put it in the spotlight. Since its birth, the festival has kept growing, and in 2016 its offerings included conferences for the families, science shows, the School Science Fair (for schools) and the Family Science Fair (for everybody).

More than 2000 primary-school students take presently part in the School Science Fair, which takes place within the University of Bath. The festival is a special opportunity to involve students and inspire them to study the sciences, as well as to meet researchers and private companies and to get in touch with real science (Figs. 2.8, 2.9, 2.10, 2.11). The Family Science Fair takes place in Victoria Park in the centre of Bath, and attracts more than 4500 people, both families and passers-by (Figs. 2.12, 2.13, 2.14). It is an occasion for everybody to become a scientist for 1 day. Experiments, interactive exhibits, public engagement, shows, games and much more, allow participants to realise how important and present in everyday life science is.

Fig. 2.8 – The Department of Engineering's stand about electricity at the School Science Fair within Bath Taps into Science

Fig. 2.9 – The Department of Biology's stand at the School Science Fair within Bath Taps into Science

Fig. 2.10 – The Royal Casino organized by the Mathematics Communication Group at the School Science Fair within Bath Taps into Science

Fig. 2.11 – A general view on some of the stands at the School Science Fair within Bath Taps into Science

Fig. 2.12 – Stands full of visitors in the central pavillon at the Family Science Fair

Fig. 2.13 – The stand of the Department of Natural Science at the Family Science Fair

Fig. 2.14 – The audience waiting for the beginning of Andrew Ross's "chemistry show" at the Family Science Fair

The general aims of the festival are to work in cooperation with schools to contextualise and make more effective the learning through science, technology, engineering and mathematics (STEM), engage and stimulate the general public's interest in scientific subjects, develop contacts between different worlds, provide an opportunity for researchers to show the impact of their research to the community at large and develop the communication skills of the university students who take part in these activities.

References

[1] Attard, C.: Engagement and mathematics. Journal of Professional Learning, Sem. 2 (2015). http://cpl.asn.au/journal/semester-2-2015/engagement-and-mathematics-what-does-it-look-likein-your-classroom

[2] Bradley, M.J.: The foundation of mathematics. Infobase, New York (2006)

[3] Budd, C., Sangwin, C.: Mathematics Galore!: Masterclasses, Workshop, and Team Projects in Mathematics and its Applications. Oxford University Press, Oxford (2001)

[4] Eco, U.: La bustina di Minerva—Ecco l'angolo retto. L'Espresso, (2005)

[5] Greco, P.: Modello Venezia. La comunicazione nell'era postaccademica della scienza, in Pitrelli, N., Sturloni, G.. (eds.): La comunicazione della scienza—Atti del I e II convegno nazionale, Zadig, Milano-Roma (2004)

[6] Guerraggio, A., Nastasi, P.: L'Italia degli scienziati. 150 anni di storia nazionale. Bruno Mondadori, Milano-Torino (2010)

[7] John of Salisbury: The metalogicon: a twelfth-century defense of the verbal and logical arts of the trivium, trans. Daniel D. McGarry. Paul Dry Books, Philadelphia (2009)

2.2 Article 2: Alex Bellos

Lettera Matematica n.101 – luglio 2017

Lettera Matematica International Edition, Springer-Verlag

DOI 10.1007/s40329-017-0198-9

Abstract. This article is the second in a series on the communication of mathematics in Europe. In this second instalment Andrea Capozucca visits with Alex Bellos. Some of the topics covered in the interview with Bellos are how to use storytelling for an effective communication, what maths stories are worth telling, the importance of the new mass-media in maths communication, past, present and future projects, the importance of having a proper language for every audience and much more. The result is a journey into the world of maths communication, seen through the eyes of one of the most brilliant and interesting maths communicator all around the world.

Keywords. Alex Bellos – Storytelling – Popular Science – Maths Communication – Math Puzzles

A meeting in London

"This is Kensal Green. Mind the gap", utters the voice, firm and calm, coming from one of the speakers of the Tube carriage that has housed me for almost half an hour. I have reached my destination. It's 20 min to noon, the time agreed for our meeting. *"Doors open"*. I get off the train and head for the exit, a steep flight of stairs along which I run into a mother struggling with her pram. *"May I help you?"* We climb the stairs together. I am in a hurry. The smile of the child who stares at me while I walk away is the best thank-you. Altruism is good for you. Finally I go out on the road. The setting is that of the classic London suburb with Victorian-style two-storey buildings that all look the same. I try to get my bearings. A bus starts slowly. I cross the one-way street reminding myself to look to the right. In the distance, framed by the well-kept greenery bordering a garden, I read "College Road". Bingo. With a fast pace I cover the road that separates me from Gracelands Café, our meeting place (Fig. 2.15).

Fig. 2.15 - Gracelands Café: outside

Inside, the anonymous facade suddenly gives way to an original atmosphere, pretty vintage, in which smells, furniture and music blend perfectly. I immediately feel that they are passionate about food and food lovers. I order something to drink to keep me company during the wait. A curious mix of lights raining down from the ceiling momentarily captures my attention. Then I move the eye to the clock hanging on the wall: it is 12 o'clock. Not even time to think "Here we are" and, from the sunny street, I see Alex Bellos arriving (Fig. 2.16).

Fig. 2.16 - Alex Bellos at Gracelands Café during the interview

This is how one of the compelling stories he likes to tell in his books might begin. Alex Bellos is currently considered one of the most brilliant and interesting communicators of mathematics in the world. Here's how Steven Strogatz describes him: *"Think of the best storyteller you know and the coolest teacher you ever had, and now you've got some idea of what Alex Bellos is like"*[18].

There is something magical about his ability to make us understand how mathematics can give meaning and shape to our vision of the world and to transform our perception of a discipline long dreaded by many. Just read one of his books to be convinced: each page provides insights, creates wonder, recounts complex mathematical topics in an accessible manner, tells fascinating stories and involves the reader into compelling narratives holding him hooked to the end.

"Hi, Andrea. Nice to meet you. How about ordering something to eat before we sit down?" We order a delicious vegetarian quiche, a house specialty. While we sit at the counter, Alex explains the philosophy underlying this café: cooking high quality courses with high quality ingredients, serving them fresh in a friendly and relaxed atmosphere. If you substitute "cooking dishes" with "writing books" and take numbers, shapes, theorems and patterns as quality ingredients, you will get the philosophy behind Alex's work. Between mouthfuls, recalling Strogatz's words, I ask: *"Do you feel more like a teacher or a storyteller?"*

[18] Steven Strogatz is Professor of Applied Mathematics at Cornell University, known for his studies on the synchronisation of dynamical systems, for his contributions to mathematical biology and the theory of complex networks, and for his tireless and prolific work of popularisation of mathematics. The quote appears on Alex Bellos's website, http://www.alexbellos.com/.

“Storyteller.” A curt answer. *“I definitely feel like a storyteller, not a teacher, because my training is in storytelling. I have never once had dry training in teaching or education. Wanting to become a teacher is a serious thing and you need to train and there are a lot of different ways about training. I might be a good teacher, but that’s not something I consider myself particularly. I consider myself a storyteller. My background is essentially in journalism, it’s all about storytelling. And I think that you make people interested by telling stories.”* The passion for journalism appeared clearly in his college years at Oxford, during which, in addition to studying mathematics and philosophy, he was the director of the student newspaper Cherwell[19]. *“I’m a journalist, I’m not a mathematician. It depends on what you mean by mathematician. Some people say, just by the fact that I went to university to study maths that means that I’m a mathematician. And say that if you study language, you are a linguist. And if you study history, you are an historian. Obviously I’m not a professional mathematician because I’m not attached to a research institution; I do no research. Sometimes it’s just easy to say I’m like the maths guy. It’s all a role playing: I might be the physicist or the astronomer, I might be the mathematician. It doesn’t mean that I’m a professional mathematician, that means that I am playing for the maths team.”*

In the introduction to his book Alex Through the Looking Glass, Alex defines mathematics like a joke: the mental process is the same. Jokes are stories with a premise and a punchline. In mathematics too there are such stories, and the punchline is called a theorem. *“Like the funniest punchlines, the finest theorems reveal something you are not expecting. ... With jokes, you laugh. With maths, you gasp in awe”* [3].

Alex explains mathematics with a great humour. He manages to combine cleverly fascinating stories of his travels around the world with insightful and challenging theories about how numbers, mathematics and culture are inextricably linked. He shows how the numbers have always been toys, as well as tools, without asking specific prerequisites from the reader. Interested and curious, I ask him: *“What’s your secret?”*

“What I’m trying to do is just to tell interesting stories”, he says, pausing for a moment to savour another bite. *“And, if the stories involve mathematics, you may need to understand mathematics to understand them; essentially they are great stories and interesting stories. Quite often the problem of maths communication, and lots of maths communicators, is they say things like: ‘Right, this is really interesting!’ If you’re saying something is interesting, people think it’s not, because, if it were interesting, you will never say that it was. Or some people go: ‘Yeah, this is really fun!’ So, if the thing’s really fun, you never need to explain it. Here, there’s the idea: don’t tell what can be shown. There’s probably the current of, also you know, playwrights’ or filmmakers’ ‘show, don’t tell’. It’s like in the movies when you have conversations to explain the plot. It’s terrible. You need to basically do something and then you go: ‘Oh, is he doing that? I get it’. So, it’s the same thing with telling stories of maths. Look into my books and inside my talks: I’m trying to find a story which combines a real adventure or a story, like a geographical or historical story, with the conceptual story and other striking stories. It’s quite difficult with maths sometimes and there’s quite a lot of maths.”*

[19] Cherwell (http://www.cherwell.org/) is one of the oldest student publications in the United Kingdom (it was first published in 1920); independent of any publisher, it has been the springboard for many a career in journalism.

Alex loves mathematics, but is also interested in music, cinema, literature and sport. Maths always proved easy to him, but, after graduation, love for journalism prevailed. After a period as an apprentice journalist, he joined The Guardian, first in the newsroom, then as a foreign correspondent in South America. During the 5 years he spent in Brazil, Alex wrote the book Futebol: the Brazilian Way of Life [1]: a portrait of Brazil seen through the lens of one of the most popular sports in the world. In this book a typical feature of Bellos as a writer emerges: that of the narrator/traveller. *"When you try to write a book, you can only ever really write the book when you are in the best place to write it"*, Alex says with conviction. Before telling a story, he wants to know personally the protagonists and the places that have seen its birth and its growth, empathise with the culture and the identity of those places. A sort of new Marco Polo.

Then, the return to London, which coincided with his return to mathematics. While Buddy Holly's That'll Be the Day is playing in the background I ask him, *"A coincidence, a falling back in love or a natural consequence of things?" "I had no idea of what to do, because I couldn't write about Brazil anymore, and I was a bit bored by the UK. It was a time of a big career crisis."* He pauses for a moment and then said, *"It was then that one of my friends, she's a literary agent, said: 'Why don't you write about mathematics?' 'I don't want to write about mathematics. Mathematics is boring'. And she said: 'Alex, Alex, just... let me show you that there's nothing else you can do'. So, she got me and I got some books at the library, because I remembered that I'd really enjoyed maths, but also left it. I remembered that they're all boring books, reading all these books and just falling asleep. It's quite easy to fall asleep even with a good book, but I was falling asleep and I was thinking that maths education would be boring... and then I had the realisation!"* From the tone of his voice and the light that illuminates his eyes, I perceive the magnitude of that realisation, a real turning point. *"When I woke up, I said: 'No, no. Maths is interesting. The fact that I'm bored by all these books is not a problem with mathematics, but is a problem with these books'. It means that no one actually tries to write a book in the way that others want to read that book. And I thought: 'Well, ok, why don't I just try to write a book? I want to try to make it more exciting, more interesting'. And I realised that I was in a kind of unique position, because I had written for fifteen years as a journalist of top level, but also had like top of maths training."* A privileged position indeed in those years. Among Bellos's generation, I think there really is none with that maths background and that journalism background. After all, that the mathematical language used in books is complicated was also argued by Vladimir I. Arnol'd, who ironically said that it was almost impossible for him to read things written by the mathematicians who were his contemporaries, who, instead of saying "Petya washed his hands," write simply:

> *"There is a $t1 < 0$ such that the image of $t1$ under the natural mapping $t1 \mapsto$ Petya($t1$) belongs to the set of dirty hands, and a $t2$, $t1 < t2 \leq 0$, such that the image of $t2$ under the above-mentioned mapping belongs to the complement of the set defined in the preceding sentence"* [11].

The German literary critic Marcel Reich-Ranicki was a firm supporter of the idea that *"a writer is somebody for whom writing is more difficult than it is for other people"*. Although many may think that writing just means constructing sentences or periods, observing grammar, having a style that differs from the other ones, writing actually means being able to build realities that allow the reader to enter them without difficulty, to love them, to also fear them, to want to explore them, as the writer did in the first place. The writer must identify with the reader and, conversely, gain his trust, arouse emotions, be familiar with them, know perfectly the mechanisms of communication. We share this thought and, soon after, Alex adds that, whenever he has to speak, prepare a talk or write about mathematics, it is as if in that moment he were the foreign correspondent of the mathematical world. *"Most of the other journalists who write about maths stay at home, they go to the library and they just write about what they know because they can't see it. For some people that works really well, but that's not me. I'm the sort of person who wants to plan to meet all the people. If you say something and the person wants to know what happens next, you have to say something which is not just interesting, but teasing to what you want to come up next. I think that often people think 'Oh, maths is really a bit boring' or 'Maths is something I never understand', so you have to manage to say something sparked and dressed ... One of the problems with the manuals of mathematics is the level of the mathematics. Either it's too slow (for the people who get it, too slow means boring), or it's too fast (for the people who need to go slow, it means too complicated)."*

This is a conundrum that a good communicator of mathematics has to solve. Alex solves it by telling stories. For instance, in the third chapter, the one on zero being invented in India, of Alex's Adventures in Numberland [2], *"you could just say, 'Zero was invented in India'. It is much more interesting to say, 'I flew to India. A man dressed in a robe came to me, among smells and music, and he said: "Zero was invented here"'. It sounds much more interesting"* (Fig. 2.17). It certainly sounds better and simultaneously captures the attention of both people who already know the story and of those who are preparing to meet it. Or, in the eighth chapter of Alex Through the Looking Glass [3], where he starts with an interview with Cédric Villani in his Parisian studio, whom he describes as surrounded by a heterogenous things, almost an Amazon rainforest, random things he needs to be aware. This is a great start to the chapter about a person who conceals a curious personality and pushes the reader to learn something more about him and the topics connected to him.

Fig. 2.17 - Alex in India interviewing the head of Vedic maths

As Jerome Bruner stated, separating science from storytelling was an error [10], since the narrative mode of thought is not alternative, but complementary to the analytic-paradigmatic one. The techniques of storytelling applied to mathematical writing may make communication more effective at different levels, even though doing so with mathematics is difficult. Communicating a theorem or a mathematical concept means simplifying without compromising either comprehension or rigour. Alex knows perfectly well the difficulty inherent in the communication of mathematics. *"With* *maths it's really difficult, because when you explain something in mathematics – a mathematical proof, for example – you've got to do the preliminaries: assume such a set of natural numbers, these are the symbols I need, assume the axiom of choice or not... It is boring, but you need it for the rigour. You have to know exactly what to use and what not to use. Sometimes I write things and people say: 'Ah, but what about...? You didn't... You ignored all the complex solutions', you know. Of course I did: it would be just a waste of time, you are just confusing people. You need to make your judgement based on the audience, the medium and the actual subject. So, it depends. For instance, how would you explain the sine curve? To a different audience you'd use different language and different terminology. You have to do as little preliminaries as possible. And as a journalist, you should be really concise anyway. Lots of people who don't know journalists don't realise that when you write a paragraph, you can probably eliminate 15% of all the words and not change the meaning and make it better. What's important is to make something as concise as possible. In storytelling you have to leave out something, not as in a proof."*

A glance at the plate confirms that the delicious quiche is almost finished, just like our time, but every time Alex speaks about himself, it is as if time has stopped. He is an extraordinary storyteller whom one never tires of listening to. Visiting his website (http://www.alexbellos.com), one immediately becomes aware that Alex is not only a successful writer and a brilliant journalist, but also an experienced speaker, guest of festivals, theatres, museums, large industries and institutions (Fig. 2.18). Also on the website we learn that he likes to communicate live beforein front of an an audience. I ask, then, what are the basic rules to be followed in this case. Alex's reply is quick in coming.

Fig. 2.18 - Giving a talk at the Glastonbury Festival

"You need to have total mastery of your subject. Then, you need to be confident. Certain people respond to someone who is confident even if they know the subject. You will not be completely good about that, but you can always do better. You need to plan and prepare the talk before going on the stage and just talk. Lots of people who give talks just want to either be funny or say something that is fun or kind of curious, something which is quite light. What I always want to try actually is to teach something, to say something that's surprising (someone afterwards has to say 'I learnt that'). Have some kind of structure so that you know where you're going, and maybe at the beginning say: 'This is what I'm going to do'; and when you're doing it, you say: 'This is what I'm doing'; at the

end you will say: 'This is what I've just done'. People really need quite a lot of that all day. And good visuals!"

As he finishes speaking, I look through the window at four boys sitting at one of the tables outside. While waiting for their food, they are bent over a smartphone, absorbed in a music video. The media have now pervaded every space–time part of our lives. We live in a society in which the combination net/digital technology/video language has greatly increased the creative power of the moving image, providing an enormous potential in many areas, including popular science. Just surf the Internet for a few minutes to discover the enormous amount of scientific video content. YouTube, in particular, has become an important tool in the communication landscape. I share this view with Alex who, besides being an author and a presenter of radio documentaries about mathematics for BBC Radio 4, is a member of the extraordinary team of experts of the "Numberphile" project[20], so I ask him: *"Do you think that the media and the social networks are useful in and for communicating mathematics?"*

"Yes, they're amazing!", Alex replies, beaming unequivocally. *"I am totally amazed by Numberphile. I mean, he has over one million of subscribers"*. (More than two million as of the time of this writing.) *"I think it's great. He has on average a couple of videos a week. And sometimes it presents some quite complicated maths, it's quite dry. He averages maybe 300,000 people per video. It's really amazing that 300,000 people take their time to go to watch something like this."*

The video entitled "The Scientific Way to Cut Cake" holds the absolute record of views, over 14 million! The project is supported by the Mathematical Sciences Research Institute, a guarantee of the quality of its contents, but there might be the risk of making mathematics just a source of entertainment or misrepresenting it. Alex is categorical on this: *"No, I don't think so. I mean, in a sense the more maths is a show the better because you're not trivialising it. As a maths communicator, you are trying to make maths entertaining and interesting. I might think that it could be a matter of science. I don't know if you have ever listened to 'The Infinite monkey cage'*[21] *on the radio. It's brilliant, hilarious. That's like a show. That's entertainment! It is really good"*.

Alex's relationship with the media goes beyond YouTube. He has a personal blog on mathematics and maintains a blog about puzzles for The Guardian, a fortnightly column on mathematical puzzles titled "Alex Bellos's Monday Puzzle" that is hugely successful. The notion of a blog was created in 1993, "invented" by Tim Berners-Lee in Geneva, as a personal journal, with an essential graphics and few lines of html. The name is a contraction of "web" and "log": hence "weblog". Today it is a

[20] *Numberphile* is a popular science YouTube channel created by the independent Australian video-journalist and filmmaker Brady John Haran. The project, with the clear goal of disseminating culture, is supported by the Mathematical Sciences Research Institute (MSRI) and funded by the Science Sandbox division of the Simons Foundation. See also http://www.numberphile.com/.

[21] *The Infinite Monkey Cage* is a comedy/popular science programme aired on BBC Radio 4. Hosted by physicist Brian Cox and comedian Robin Ince, the first series began in September 2009 and is now close to 100 episodes. Each episode is centred around a particular theme and, usually, there are three guests: two scientists with a direct interest in the topic of the episode and a comedian who plays the role of "facilitator", sometimes even asking "stupid" questions that the other guests might leave out. The programme won a Gold Sony Radio Award for the Best Speech Programme in 2011 and a Best Radio Talk Show in 2015. The name of the transmission is a clear reference to the "infinite monkey theorem".

strong communication tool, each with a structure, contents and a shape that faithfully reflect the identity of its creator. The virtual space has its own rules of communication, which need to be studied and understood in depth, lest it be ineffective. There are some important aspects to keep in mind for communication to be effective. *"Well, you need to understand the media you are using"*, Alex says, almost reading my mind. *"If you're doing something on a blog, if you want people to go and read it, you must have a really great headline or a really great picture. Otherwise, they are not going to read it. So, that's what's important."* Visual reality is very important, as well as taking advantage of trivial aspects of the language to create entertaining titles. For instance, the title of one of the latest Monday Puzzles by Alex was "Are you smarter than a British 13-year-old?" *"People really like that. So, I think about the medium and I think what is the story that's best suited to that medium."*

"How do you choose the topics of your posts?" I ask. *"I should really write more on my blog, but I've been busy the last few months"*, he replies expressing some regret. *"I'm a journalist, so I'm always looking at the music of the stories. I have a big file of things that I think I might turn into stories and suddenly, you know, it's like a garden: some of these things grow and become big. The problem you have with writing something on maths is that, unlike pretty much any other science, there is no way I would be able to report an advance in mathematics. It's too complicated, and they wouldn't even understand it. If you think about all the people here who are expert in their own subject, there are probably only 100 people in the world who would understand it. I want to understand and, as a communicator, I know that people would be interested. So, with maths, I think you have to essentially go to recreational maths, with things that are entertaining and fun. But this does not mean that you have to be silly."* For instance, one of the most popular posts of 2015 was about the discovery of a new type of pentagon which can tile the plane without overlapping and without leaving gaps, a topic that also includes a little research [6]. In 1985 Rolf Stein found the fourteenth pentagon that could tile the plane, after which it was believed that there were no others. Then, a month before the publication of the post, three students at the University of Washington Bothell found the fifteenth one with the aid of computers. *"And the story is a great story that started hundred years ago with the German mathematician Karl Reinhardt who found a few, then in the 1960s three more were found... until today. I prefer to write geometrical stories just because the blog is essentially a visual medium."*

Alex checks his watch and realises that we have gone far over our planned time. While we collect our things, I ask him about his future projects. *"Oh, I have lots of projects"*, he replies while donning his hat. *"I did the [mathematical] 'Colouring Book' [8] which someone in Italy didn't want. Germany, Spain, everyone got it. In America it's obtaining a great success and they want another one. So, I'm working on a second 'Colouring Book' [9]. I'm also writing a book on mathematical problems. The puzzle column in The Guardian does really well and that happens because in my blog last April I posted a story about Albert, Bernard and Cheryl and about the birthday problem. I put it on my blog and I had 5 million people, it was the most popular story I have ever written. Actually there are two stories: the first where I posed the question and the second where I gave the answer (45). At the end of the year, if you look at the most popular stories in The Guardian in the entire year, the 6th and the 9th were my blog. So, because The Guardian said, 'Wow! The puzzles come and get viral', and actually people love these adult challenges, they gave me a regular column, so that every two weeks I come out with these puzzles (on average*

100,000 followers). Faber, which is a publishing company here, do this thing with The Guardian: they'd like to turn the column into a book. So, they gave a contract to turn the column into a book. And it should come out at the end of the year [7]. That's the reason why I'm so busy. I have also got a little baby and I have to finish this book by June. I'm looking at the history of the puzzles I put into my book. A very big project."

We leave the café. Meanwhile, Alex continues: *"I'll also do many maths talks and I'm working on my elliptical pool table project*[22] *(Fig. 2.19). Maybe I'll actually sell it. I basically keep on promoting it now because it's a really good way to do maths. I'm going more into puzzles because now with my young family I'm not able to do much travelling and also I've done most of the maths that I think is explainable at a simple level in my first two books. So, I'm looking for new directions and I think that writing puzzles is quite a good way, I think puzzles are quite fun, but every two weeks I need a new puzzle for The Guardian. And that's good hard work"*.

Fig. 2.19 - Alex and his elliptical pool table

The background music drifts out from inside the café every time the door opens to let someone enter or leave. Traffic is not heavy and the sun, now low on the horizon, begins to outline the roofs. I greet Alex with a handshake and thank him for the precious time that he gave me. I see him walk away in the same direction from which I saw him appear just 2 h before. The air is still, my mind is in overdrive. I retrace Alex's answers one at a time, trying to connect them. A story emerges, that of Alex. To communicate is to tell a story, get into it, know it and revive it. Whether it's about a person or mathematics does not matter: what matters is that it's worth it. My phone vibrates. A message: *"Let's meet tomorrow morning at 10. Morton Hotel, Russell Square. Andrew."* The journey continues.

Who is Alex Bellos

Alex Bellos was born in Oxford in 1969. He spent his first 12 years in Edinburgh, before moving to Southampton with his family. In 1987 he returned to Oxford to study mathematics and philosophy at the Corpus Christi College. During his university years he was editor and director of the independent student newspaper Cherwell. As a graduate, he began working as an apprentice

[22] https://www.wired.com/2015/11/elliptic-pool-loop-round-billiard-table.

journalist at Brighton Evening Argus. Finally, he moved to London where he started working in the newsroom of The Guardian.

For 5 years, starting in 1998, he was the correspondent in South America based in Rio de Janeiro. During his stay in Brazil, in 2002, he wrote his first book, entitled Futebol: The Brazilian Way of Life, which was voted the book of the year by the British Book Award and was included among the books of the year by Publishers Weekly. The book was the result of a profound research that led the author to travel far and wide in Brazil and then to the Faroe Islands in the North Atlantic Ocean. In 2014, the book was updated with the addition of an extra chapter on the occasion of the FIFA World Cup. Bellos is also co-author of Pelé: The Autobiography, which topped the sales charts in the UK.

In 2003 he began appearing on TV and other media hosting the TV series Inside Out Brazil produced by the BBC. In 2006 he authored the documentary Et Dieu créa… le foot about football in Amazonia, for the National Geographic Channel. In the following years he was author and guest of radio documentaries on mathematics and science in general for BBC Radio 4. Since 2011 he has been a member of the team of the Numberphile project, a YouTube channel with videos on numbers and everything about them, about mathematics and its applications.

Fig. 2.20 - A book launch in Brazil for Alex's Adventures in Numberland

Fig. 2.21 - A prize ceremony (best non-fiction book at the British book awards for Alex's Adventures in Numberland)

Also in 2003 he returned to London and began writing about mathematics. In 2010 he published Alex's Adventures in Numberland, which remained for 4 weeks in the Sunday Times top ten (Figs. 2.20, 2.21). The book was short-listed for no fewer than three UK literary awards, including the prestigious BBC Samuel Johnson prize for non-fiction. In the same year, in the United States, under the title Here's Looking at Euclid, it won the Amazon.com prize in the Science category. Translated into more than twenty languages, in 2012 the Italian edition, titled Il meraviglioso mondo dei numeri, won the Premio Galileo for scientific books and the Premio Peano for popular mathematics books.

Bellos's interests then moved towards the links between mathematics and civilization, examining how maths helps us give meaning to the world, and how numbers affect our actions. In 2014 he published Alex Through the Looking Glass, which, like Alex's Adventures in Numberland, became a best-seller, receiving positive reviews from major newspapers including the Daily Telegraph and the New York Times. Its American title is The Grapes of Math.

His most recent books are Can You Solve My Problems? (2016), a compendium of almost 200 logical-mathematical puzzles with a historical-biographical background, and a book for kids using football to explain everything from mathematics to zoology, from English to fashion, titled Football School (2017), co-authored with Ben Lyttleton. He is also the author of two mathematical colouring books, Snowflake, Seashell, Star (2015) and Visions of Numberland (2017), in collaboration with Edmund Harriss.

Finally, since 2012, he maintains the mathematical blog Alex's Adventures in Numberland for The Guardian, which in 2016 has been chosen as the best science blog by the Association of British Science Writers. For the same newspaper, he is in charge of the puzzle blog Can You Solve My Problems?; a selection of the best puzzles from those collected in the book of the same title.

What is storytelling?

Storytelling is an art and a tool for portraying real or fictional events through words, images or sounds. It is a natural tool through which effective communication takes place: it involves content, emotions, intentions and contexts. Luisa Carrada argues that stories are the only thing that since the dim and distant past can unify information, knowledge, wisdom, emotions, care of themselves and others. And, quoting Ursula K. Le Guin, "There have been great societies that did not use the wheel, but there have been no societies that did not tell stories". Storytelling, then, has always existed informally. It belongs to fiction, popular tradition, classical literature and culture that sees man as the protagonist of a process that transmits values. Storytelling has now become a scientific and systemic discipline that is indispensable for anyone who wants to communicate to an audience. It is a new communication paradigm.

But communicating using storytelling does not just mean telling a story or knowing how to entertain the audience. Nowadays storytelling is the science of narrative applied to marketing, politics, sociology and even medicine. And, as a "science", it has some shared rules that every storyteller must respect:

- give the story a narrative sequence that might not reflect the chronological occurrence of the real facts, nor the contingency of cause-effect relationships;
- inspire the reader/listener's trust;
- develop the story emotionally using conflicts, solutions, tensions, mystery, curiosity, and so on;
- create a special relationship with the reader/listener allowing them to identify with the story;

- take care of the interweaving between the various parts of the narration and its entirety;
- remove anything that is unnecessary or is superfluous to narrative (minor events and characters, incidental mentions, etc.);
- highlight details that in reality may appear less significant;
- leave room for the reader/listener to apply his own moral to the story;
- put the readers/listeners in the best condition to fully immerse themselves in the story and to live through the experiences narrated in the first person;
- make sure the story is plausible;
- remember that the reader/listener evaluates the new stories by comparing them with the ones they already know.

However, storytelling is more than just respecting these rules. Behind the creation of a story there is an in-depth research work, the commitment of highly trained people who know the codes of communication and master the principles of rhetoric and narratology.

References

[1] Bellos, A.: Futebol: The Brazilian Way of Life. Bloomsbury, London (2002) (updated edition with a new chapter on the occasion of 2014 FIFA World Cup (2014))

[2] Bellos, A.: Alex's Adventures in Numberland. Bloomsbury, London (2010) (published in USA as Here's Looking at Euclid)

[3] Bellos, A.: Alex Through the Looking Glass. Bloomsbury, London (2014) (published in USA as The Grapes of Math)

[4] Bellos, A.: Can you solve the maths question for Singapore schoolkids that went viral? The guardian.com. https://www.theguardian.com/science/alexs-adventures-in-numberland/2015/apr/13/can-you-solve-the-singapore-primary-maths-question-that-went-viral (2015). Accessed 13 April 2015

[5] Bellos, A.: How to solve Albert, Bernard and Cheryl's birthday maths problem. The guardian.com. https://www.theguardian.com/science/alexs-adventures-in-numberland/2015/apr/13/how-to-solve-albert-bernard-and-cheryls-birthday-maths-problem (2015). Accessed 13 April 2015

[6] Bellos, A.: Attack on the pentagon results in discovery of new mathematical tile. The guardian.com. https://www.theguardian.com/science/alexs-adventures-in-numberland/2015/aug/10/attack-on-the-pentagon-results-in-discovery-of-new-mathematical-tile (2015). Accessed 11 August 2015

[7] Bellos, A.: Can You Solve My Problems? A Casebook of Ingenious, Perplexing and Totally Satisfying Puzzles. Guardian Faber, London (2016)

[8] Bellos, A., Harriss, E.: Snowflake, Seashell, Star. Canongate Books, Edinburgh (2015) (published in USA as Patterns of the Universe)

[9] Bellos, A., Harriss, E.: Visions of Numberland: A Colouring Journey Through the Mysteries of Maths. Bloomsbury, London (2017)

[10] Bruner, J.: Life as Narrative. Soc. Res. 54, 11–32 (1987)

[11] Zdravkovska, S.: Conversation with Vladimir Igorevich Arnol'd. Math. Intell. 9, 28–32 (1987)

2.3 Article 3: Andrew Jeffrey

Lettera Matematica n.102 – ottobre 2017

Lettera Matematica International Edition, Springer-Verlag – in press

Abstract. This article is the third in a series on the communication of mathematics in Europe. In this third instalment Andrea Capozucca visits with Andrew Jeffrey. Some of the topics covered in the interview with Jeffrey are how to use magic for an effective communication, teaching and communicating as mutually dependent activities, how to tell a story about mathematics into a magic show, the importance of training maths teachers, past, present and future projects, the importance of being able to handle space, time, motivations and expectations, and much more. The result is another journey into the world of maths communication, seen through the eyes of one of the most brilliant and "magical" maths communicator in Europe.

Keywords. Andrew Jeffrey – Magic – Teaching and education – Maths Communication – Engagement

> *Close to an old, half-abandoned inn, we saw three men arguing heatedly beside a herd of camel. Amid the shouts and insults, the men gestured wildly in fierce debate, and we could hear their angry cries:*
>
> *"It cannot be!"*
>
> *"That is robbery!"*
>
> *"But I do not agree!"*
>
> *The intelligent Beremiz asked them why they were quarreling.*
>
> *"We are brothers," the oldest explained, "And we received these 35 camels as our inheritance. According to the express wishes of my father, half of them belong to me, one-third to my brother Hamed, and one-ninth to Harim, the youngest. Nevertheless, we do not know how to make the division, and whatever one of us suggests, the other two disputes. Of the solutions tried so far, none have been acceptable. If half of 35 is 17½, if neither one-third nor one-ninth of this amount is a precise number, then how can we make the division?"*
>
> *"Very simple," said the Man Who Counted. "I promise to make the division fairly, but let me add to the inheritance of 35 camels this splendid beast that brought us here at such an opportune moment."*

At this point I intervened.

"But I cannot permit such madness. How are we going to continue on our journey if we are left without a camel?"

"Do not worry, my Baghdad friend," Beremiz said in a whisper. "I know exactly what I am doing. Give me your camel, and you will see what results."

And such was the tone of confidence in his voice that, without the slightest hesitation, I gave over my beautiful Jamal, which was then added to the number that had to be divided between the three brothers.

"My friends," he said, "I am going to make a fair and accurate division of the camels, which, as you can see, now number 36."

Turning to the eldest of the brothers, he spoke thus: "You would have half of 35—that is, 17½. Now you will receive half of 36—that is, 18. You have nothing to complain about, because you gain by this division."

Turning to the second heir, he continued, "And you, Hamed, you would have received one-third of 35—that is, 11 and some. Now you will receive one-third of 36—that is, 12. You cannot protest, as you too gain by this division."

Finally, he spoke to the youngest: "And you, young Harim Namir, according to your father's last wishes, you were to receive one-ninth of 35, or 3 camels and part of another. Nevertheless, I will give you one-ninth of 36, or 4. You have benefited substantially and should be grateful to me for it."

And he concluded with the greatest confidence, "By this advantageous division, which has benefited everyone, 18 camels belong to the oldest, 12 to the next, and 4 to the youngest, which comes out to—18 + 12 + 4—34 camels. Of the 36 camels, therefore, there are 2 extra. One, as you know, belongs to my friend from Baghdad. The other rightly belongs to me for having resolved the complicated problem of the inheritance to everyone's satisfaction."

"Stranger, you are a most intelligent man," exclaimed the oldest of the three brothers, "and we accept your solution with the confidence that it was achieved with justice and equity."

The clever Beremiz, the Man Who Counted, took possession of one of the finest animals in the herd and, handing me the reins of my own animal, said, "Now, dear friend, you can continue the journey on your camel, comfortable and content. I have one of my own to carry me."

And we traveled on towards Baghdad [1].

A quiet place in Russell Square

I am in Morton Hotel, in Russell Square, still in London, the same city where I met Alex Bellos [2].

I'm sitting at "The Library"[23], steps away from the bustle of Bloomsbury, the area where I was immersed just a few minutes ago. The ambience is special: the old and the new are intertwined, creating a timeless, almost magical, atmosphere. While I wait for Andrew Jeffrey (see Appendix 1), I sit in one of the cosy armchairs in the back, in front of a magnificent bookshelf. My gaze lingers on one book in particular: *The Man Who Counted* by Malba Tahan[24]. I still remember the astonishment that caught me the first time I read the tale of the camel division. I was fascinated as before a trick of a skilled magician. Surprise and wonder arise whenever we see something that appears to be inconsistent with our model of reality. Surprise is often followed by the curiosity that makes the impact of wonder an active process, a cognitive path. Meanwhile Andrew has arrived, has sat down on the sofa to my right and signals to me that he is ready. He has returned two hours ago from one of his annual business trips to Singapore[25] and tomorrow he will have an important meeting here in London, but despite his tiredness he found time for our chat. His smile confirms his happiness about our meeting. With the combination surprise-curiosity turning around in my mind, I ask him, *"The purpose of magic is to surprise. The purpose of the communication is to excite curiosity. Your main goal is 'to bring the magic of mathematics to everyone'. But what does magic have to do with communicating mathematics?"*

"It's a personal decision", he answers with a smile, *"because I grew up being engaged with mathematics and I grew up being engaged with magic, but they were parallel lines for me. They came together very recently. I am a professional magician and I was for many years a professional teacher and now I teach teachers, which sometimes involves magic, sometimes doesn't. I mean 'magic' in the broadest sense of the word, as in the 'ah's, the moments of amazement and astonishment, the wonderful moments when you realise that every prime number is exactly one away from a multiple of six, when you realise that if you join the midpoints of any quadrilateral, you get a parallelogram. Those 'wow... that's magic!' moments. It's not magic like a rabbit coming out of a hat. But I think if we can present maths to children in a magical way, we can present maths to children as 'This is interesting? Look at this! Why has this happened?' or 'How come if I add the first odd numbers together, I keep getting square numbers? Why?' or 'Why if I add two consecutive triangular numbers together, do I get a square number? How come? Is that why it works? Can I be sure?'. This is for me what the magic is about."*

Mathematics is like magic, but without the lies. And if in magic the appeal lies in not knowing where the trick is, in mathematics it is quite the opposite: the wonder emerges precisely when we understand the secrets that lie behind the numbers. Andrew Jeffrey is a "mathemagician". The term was probably first coined in honour of Martin Gardner, one of the greatest popularisers of mathematics of the 20th century, as well as a respected illusionist. Since then, mathemagician is a title also used for other mathematicians/magicians like Arthur T. Benjamin, Persi Diaconis, Colm

[23] "The Library" is a lounge bar located within Russell Square's Morton Hotel, inspired by Charleston House, the country home of the Bloomsbury Group, a group of artists and intellectuals including Virginia Woolf and John Maynard Keynes, who met in the private homes of the area in the early 20th century to give vent to their creativity and spend some time with friends.

[24] Malba Tahan is the pseudonym of Brazilian mathematician Júlio César de Mello and Sousa (1895-1974), the author of several books animated by a great desire to popularise the mysteries and delights of mathematics.

[25] Together with Liz Gibbs, also a trainer with international experience, he gives seminars for teachers about the "Singapore method" for teaching/learning mathematics. This method leaves out the teaching of calculation, procedures and memorisation to make room for problem-solving.

Mulcahy and Shona Jean McKay. Diaconis suggested that the reason why many mathematicians are also magicians is that *"inventing a magic trick and inventing a theorem are very similar activities"* [3]. This is a combination also reflected in the name of the company of which Andrew is director and founder: Magic Message Ltd (see Appendix 2). *"Where does the name come from?"*, I ask him, almost seeking a confirmation.

"I needed a company name. When I set up my company, I knew that it was going to be about communication. So, I had two strands to this. The first thing I want to do is to support teachers with maths. For me that's my cool thing. But the other part of my business is about doing magic and I believe magic should have some kind of story, some kind of purpose. I know they're excited by 'You choose a card... ok... it was this one!' No, that's just a fairly boring plot. You can make it exciting, but I realised that magic could be used to communicate ideas. So, that's why I went for the name Magic Message." A moment's pause, then he continues, *"If I had another chance, I'd choose 'Magic Message Maths' or something. I'd get the word 'maths' in it because in the long run up to what it is now, I think it's become more about maths. What's actually happened is that most of the calls come from schools, so I focussed on that."*

The main concern in teaching mathematics is to increase the level of knowledge and develop the skills of each individual. On the other hand, raising awareness among the public has more to do with increasing the enthusiasm for the discipline, providing the basis for a possible deeper study, which will occur in different times and ways. So, while there is a substantial difference between teaching and communicating mathematics, the two activities can benefit from each other. A good teacher is often that (also) because he or she is a good communicator, and a good communicator may be able to teach many things. The premise is that teaching is above all a form of communication and educational interaction is always a relationship mediated by communication. As Angela Spinelli, a researcher in "general teaching methodology" at the University of Rome "Tor Vergata", writes:

> *Teaching has its own peculiarities that revolve mainly around the message and, specifically, around the code. ... The relationship between the teacher and the learner consists of an intentional, designed, communication, possibly modified during the interaction and made of different languages: words and gestures, gazes and expressions, space management and physical proximity* [4, p.44-5; our transl.].

If we reread these sentences by replacing teaching with communication, teacher with communicator/populariser, and learner with the public, the meaning would be preserved, but at a different level. By "communication", I mean the attempts to transfer significant ideas about mathematics to an intelligent, non-specialist audience, so as not to scare them but still take advantage of their interests. I say "attempts" because success can be elusive. Andrew and I reflect aloud on the issue. *"Of course maths has be a mandatory part of the curriculum"*, he asserts strongly. *"However, people may have a point when they say that no one understands what mathematics is, and that is related to standardised testing. The premise is, we have to be realistic. We test what is testable. I don't think we are very good at testing children's mathematical abilities. I think we are very good at testing their mathematical fluency. In other words: 'Here are some sums, can you do them? Here are some equations, can you solve them?' But not 'Will I be able to get that car into that garage round that corner?' Now, that's a good test of all your mathematical* *trigonometry or areas or your shapes, your space, your geometry. I don't think it is sums that make up mathematics. That's like saying that language is about letters, the study of letters. It's not!"* From the firm tone of his voice I perceive that he takes this subject to heart. *"Mathematics isn't the study of numbers. It's the study of what you can create with them. We need to understand that mathematics is about creating things, but you've got to learn to use the tools first. If you only learn to use the tools, you'll never make anything. For example, Michelangelo is not the guy to whom the thing is the hammer and the chisel. He couldn't care less about them. But because he was so fluent with them, he could create something beautiful. And so we ought to be fluent with calculations, but also to use it in a creative way: 'What am I going to do with this piece of maths?'"*

Amazement, fun and dedication are three key ingredients in the recipe for knowledge. And magic can be used in schools to stimulate students' curiosity, as writer, magician and "wonder injector" Mariano Tomatis[26] says:

> *For the cognitive experience to have an educational significance, it is necessary to reflect on the relationship between the wonder and the nudge towards scientific inquiry. Conjuring and magic can be considered forms of marvel engineering. After a moment of astonishment, it is natural for our mind to activate itself to "solving" rationally the violation of expectations: this activation paves the way to reflection for which we have to take into account some amount of intellectual effort* [5; our transl.].

This "chain of wonder" follows a model proposed by Raphaël Seth Taylor in which the initial stimulus may generate paths at different heights and produce different results, ranging from mere frustration to a real "paradigm shift". In Seth Taylor's words:

> *Different people seek different paths through this chain. Magicians strive to bring people from expectation violation into wonder and cut off any further processing. Their goal is to foster that emotion and encourage its experience. Teachers on the other hand are focused on learning, and occasionally paradigm shift. Their goal is to have the right amount of relevance lead to the right amount of effort, resulting in education. To[o] little effort may not engage the students enough to create lasting memory, and too much effort may turn them off to the process of learning. For them, [it is important] being able to provide interesting experiences of expectation violation to steer students through the path of wonder* [6, p. 45].

To paraphrase Martin Gardner, there must always be an interchange between seriousness and "frivolity": the latter keeps us alert, while the seriousness makes the play worthwhile. At the end, the student may even be surprised by the amount of non-trivial mathematics he has learnt without even meaning to.

Andrew Jeffrey, who had made this approach the centre of his message well before Seth Taylor adopted it, highlights two other aspects that he considers fundamental: engagement and confidence. *"First thing, engagement. If you're not engaged, forget it. So we have to get some kind of motivation. Secondly, confidence. I don't mean confidence that you are right. I mean a different*

[26] http://www.marianotomatis.it/en/

type of confidence. Confidence to know that it is ok not to be right. And the truly confident mathematician is not the one who says, 'I know how to do this!' He's the one who says, 'I don't know how to do this yet. How exciting!' And what that takes is easier said than done. So, engagement is crucial. Confidence pulls in your ability. John Holt said, 'The true test of intelligence is not how much we know how to do, but how we behave when we don't know what to do' [7]. Now that's really profound, it had a really impact on me."

In Italy, as in other European countries, the public perception of mathematics is negative and full of prejudice often related to how it is taught. Some consider it a waste of time, others see it as far away from real life or believe it is stuff for geniuses. Considering Andrew's more than twenty years of experience in the field, I ask him, *"How can we overcome this prejudice?"* Smiling and slightly shaking his head, he leans forward and answers, *"It makes me laugh. I think Italians have all the great art and don't know what use is it? Italian should say, 'Was I a part of this?'. This is in itself beautiful. And it is important because I consider it beautiful and it may teach me something about the human condition. Mathematics is in itself beautiful. Now, I may think so, you may not think so. It doesn't matter. Like art. I might say 'This is divine, this is very nice' and another can say 'Pfff... I don't think so'. It doesn't matter. There is an art and there is a beauty inside the mathematics. I will look at it, I will learn it because it underpins everything we do in life. And the more we understand it, the more we can spot patterns. The more we can spot patterns, the more we can start to make predictions on what will happen. How to improve our life? How to keep people alive? We could make predictions about weather, temperature, medicine. All of this is done because people learn maths. However, you can teach it as an abstract subject if it is enjoyable. For example, most children who like arts don't ask the teacher 'Why will I need this?' Most of them like sports, like football. If they'd learnt football at school, most children wouldn't end up with 'Why will I need to do this in my life? Football is useless'. So, mathematics is useful because it teaches you to think, in a way that nothing else teaches you to think, and to become a problem solver. If you are ten years old, who knows what the problems of the world will be when you are fourteen? The mathematics we do today might solve tomorrow's problems, just like the mathematics we did two hundreds years ago solves today's problems. We used to think the Earth stood still and the Sun and all the planets turned around us. And then Copernicus said, 'Hang on a moment. I've done some mathematics and I'm not sure' and everyone said 'Uh! Are you crazy?' If children in school aren't shown how interesting it is, then who can blame them for thinking it's boring, dusty and dry?"*

It is clear from Andrew's words that there is great work to be done to raise awareness among students and, more generally, the public towards the idea of mathematics as a vital and dynamic discipline, constantly changing and an integral part of our culture, just like music and literature (Fig. 2.22). The opportunity to make up for what has happened and, consequently, consolidate and develop mathematical knowledge is partly in our hands. We must go back to talking about mathematics. Discussing maths is an important step on the way to his understanding and possibly to thinking mathematically. To present mathematics as a fascinating, fun and free experience, it is often necessary to give it back those moments and situations from which it has been artificially separated when it was codified in an article or a textbook. Behind every theorem there is a story worth telling.

Fig. 2.22 – Andrew Jeffrey with some students at the end of a show

"I think it's maths returning to history", Andrew remarks. *"If you go right back through history, the first mathematicians were the people with stories. They said, 'Look in the sky. Do you see that big yellow thing? I'm sure it was over there this morning'. It's through mathematics that such stories was resolved. I think that mathematics is a story, a dialogue, a form of communication; mathematics is about the explanations and the connections between things we know of the world and those we don't understand. And these beautiful things in the world are fused with mathematics. Even in the spiritual texts, there's mathematics going on. So, it's not just an assonance of things. And for me the universe is incredibly beautiful, not just to look at it saying 'Oh, it's just pretty', but looking at the numbers and go 'That's amazing, those numbers! If one of them changed there'd be no life on this planet'. And that's mind-blowing. So, I don't wish to separate mathematics from the rest of life. It should definitely be part of literature because it is a story. It should be part of us virtually."*

Unfortunately, not many people agree with Andrew. There is a clear separation between literature and science, a naive and false dichotomy. *"Literary intellectuals at one pole—at the other scientists ... Between the two a gulf of mutual incomprehension"* [8, p. 4]. So wrote C.P. Snow (1905–1980) back in 1959, and the current situation is not much better than in the past. It is also true that around the same time, in Italy, Leonardo Sinisgalli (1908–1981) argued that mathematics is a language alongside other languages, and that its themes and characters can become topics to talk, write, draw, compose verses about. *"For me, mathematics infuses everything I'm doing"*, Andrew remarks, and adds with a touch of humour, *"It's the language the universe uses to explain itself. Galilei said something like this. So, he must be copying me"*.

To achieve the goal of effectively communicating mathematics and reducing the distances between the involved parties, we need good and competent communicators. To get to the heart of our chat I ask, *"Which aspects do you consider to be fundamental in the communication of mathematics?"*. Using an ever more engaging tone, Andrew immediately goes to the essence of the matter, *"Firstly, the ability to take something complex and make it simple enough for someone to understand, but not too simple. Albert Einstein said, 'Everything should be made as simple as possible, but no simpler'. So, don't patronise people, but show them something physical. Like a painter who amazed me last year, showing me the 3D net of a 4D cube and my brain was like 'Wow! That's incredible!' And he told me that he had inserted in his paintings doodlings on mathematics he got from Brown University. Incredible! He'd taken a really complicated idea and he'd just shown it to me in a cardboard. So, I think a real skill is choosing a resource (a simple piece of paper or cardboard,* *dice) and making it a piece of maths that is concrete, relatable, physical. Because then we internalise it"*.

While Andrew speaks, I carefully observe a painting behind him and think that a good communicator should be like a good painter. He has at his disposal a canvas (the public), a brush and a palette (the tools), colours (the concepts) and technique (the communication and mathematical skills) to represent reality from his point of view and make it shared. I continue by asking, *"Which are the unescapable rules followed by Andrew Jeffrey as a communicator?"* In a friendly and relaxed way, he expounds his *vade mecum*.

"Golden rule number one: always check your flies. That's the number one more than anything else", he answers with his typical humour and we both burst out laughing heartily. *"Golden rule number two: remember that you do not have a right to be listened to, whoever you are. Everybody has to earn the right to be heard. I may think that I'm quite important because I'm the speaker. But I'm the least important person there, because if I don't learn anything, no one cares. Earn the right to be heard by respecting your audience, doing some welcoming research on who they are. You've done a lot of research on me, it's clear, so I told myself 'This man has prepared these questions and it will be an honour to speak with him'. Golden rule number three: be engaging. Don't lecture; if I hear myself lecture, I stop to go back on an analogy. Golden rule number four: have a good stock of appropriate examples and analogies because people relate to things in their life. So, know enough about them. You don't have to learn their day of birth. Just know if they've been teaching for a long time, which country they're from, where do they work, just enough. So start to talk with them and address them. Start from where they are, not from where they should be. Golden rule number five: look smart. It sounds like a stupid thing to say but you have to dress in a way that suits your persona. Not just physically or clothes: I think about the watch I'm going to wear, if I wear cufflinks, if my shoes are clean... Just little things like that. Each of these things is very small, but if you're a communicator people are looking at you and going: 'Shall I listen or not? Shall I believe you or not?' They're thinking that for the first 20/30 seconds, so the first 20/30 seconds have to be fantastic, clear, high-contact. Golden rule number six: even walking the room with your eyes is important. I don't have to explain this but... so, take this camera"*, he says pointing my camera. *"Let's suppose that it is our audience, and that we have an awesome view of these places. The wrong way to torture yourself is to say, 'Point 1, point 2, point 3'"* and he moves his hand in cadenced manner from left to right. *"Now the right feel for your brain is to go, 'Point 1, point 2, point 3'"*, moving his hand in the same way, this time from right to left. Then he asks me, *"Don't you feel better? I did exactly the same thing but the other way. And for some reason I feel better because the brain works left-right. Your western brain works left-right. If I'm talking to an Arabic group, I might go that way."* Andrew is unstoppable. *"Golden rule number seven: make your point and don't talk to fast. I often talk too fast"* and he covers his mouth with his hand. *"Golden rule number eight: know your subject. Know more than you plan to say. Know much more than you going to say, because the people are attending to listen to an expert. Don't say you are an expert, because if you are an expert you don't need to say that you are an expert. And, golden rule number nine, be humble. I'm pretty sure about this, but I always repeat this. Golden rule number ten: be honest, just be honest. And be passionate. If you are passionate and knowledgeable, you can get away with a lot. These are the kind of things I think are really important."*

Ten simple, effective rules that every communicator needs to constantly train and study in depth, on pain of loss of effectiveness and credibility. It is important for communicators to continually put themselves to the test, especially when developing new activities to communicate mathematics, different from a conventional conference and requiring a greater interaction with the public. You have to think creatively: how can I present that particular mathematical concept or idea in order for it to be challenging and, at the same time, engaging for the audience? This could mean an appropriate use of the so-called "emotional hooks" that may be of interest to the audience in front of you, such as *"novelty, incongruity, curiosity, humour, imagination, choice, control, empowerment, involvement, challenge, complexity, comprehension, social interaction and relevance"* [9, p. 97]. If you want to involve the audience, you need to reach them emotionally as well as cognitively, regardless of whether they are students, teachers or the general public. These emotional hooks may be created in ways that are slightly unusual and, if you add your own creativity to the process, the possibilities are endless. Andrew agrees and adds, *"One more thing that I think really helps is inviting some students to come up and be in the front with you, because there's nothing more exciting than seeing your friends upon the stage. Suddenly it's quite real because it's like the speaker on the stage is a bit distant from you and I can only relate to someone who is not distant from me. So, as soon as my friends are up there, maybe shaking hands and saying 'Hi', then suddenly it's more real for me and I'm more interested. Always a great idea! For example, I tell the story in my show of how Carl Friedrich Gauss added up the numbers from one to one hundred by using patterns. To introduce that, I get twenty students upon stage, ten boys and ten girls, and everyone is holding a number (boys from 1 to 10 from left to right and girls from 11 to 20 from right to left). So, I have that coupling them two by two I have ten couples whose sum is 21. Then, they'll see the pattern and then you say 'What about one hundred?' It's a very good popular section of the show because I choose children coming from different parts (left, right, up,...) and I make jokes and I talk about pairs (it's funny to couple and say, 'Your partner...' 'Oh, noooo...'). And if you use some gentle humour, it's always engaging and they'll remember it because they are part of the show."*

Telling mathematics in a show is not a simple thing. It requires mastery of the discipline and, at the same time, communicative and artistic techniques. There has to be a great deal of preparation that involves the selection of the most appropriate topics, management of stage times and methods of direct and indirect interactions with the audience. With mathematics, everything becomes more complicated. For instance, one of the causes of confusion, distress and passivity for those who have been away from mathematics for a long time is the heaviness of symbolism and terminology, which for a mathematician are commonplace. So, it can make sense to drastically reduce the technical terms introduced. Indeed, it is better to tackle a subject, discuss an idea, even without mentioning the exact terms involved. Turning around the situation and illustrating signifieds without signifiers can be instructive and enlightening, precisely because it overcomes one of the psychological barriers. Following this thread, I remind Andrew of the *Maths Inspiration* project[27] that saw him among the protagonists, together with Rob Eastaway[28], of one of the six interactive show-talks for teenagers entitled *What's the trick?* recorded in July 2013 at the Greenwich Theatre in London. It is

[27] *Maths Inspiration* is a national programme of interactive show-talks about mathematics that aims to inspire British teenagers to pursue the study of mathematics to a higher level. Since 2004, over one hundred thousand boys and girls aged 14 to 17 have attended the shows. For more information: http://www.mathsinspiration.com.

[28] The founder and director of the *Maths Inspiration* programme.

an interesting and motivating project, which clearly shows how a transfer of knowledge is more effective when different skills are able to interact. *"By transforming mathematics into a show, is there no risk of distorting or excessively simplifying it?"* Andrew replies instantly, *"Absolutely, it could be a risk. I'll have to be sure to consider very carefully whether the magic idea overshadows the mathematics. Here's an example. I do a trick where I borrow maybe ten euros and I fold it in half and I say, 'So, what fraction do I have?' 'One half.' And I fold it again and again. Then, I unfold and turn it into one million dollars. I talk about folding and unfolding as inverse operations, but I'm not sure that the message always gets through because I get, 'Aaaahhh! One million dollars!!! It's just a trick'. So, you have to be very careful because magic has such a power. And because of the power of it, you deduce what the danger is. Sometimes you have to choose very carefully, otherwise it just becomes a joke. I feel the same way about chemistry shows. I have a friend who does chemistry shows. And he does what people love, like explosions and colours and smoke. I enjoy those shows but I wonder, 'Am I learning any chemistry?' If I can see this in someone else's performance, I am hypercritical about myself. 'Can I see the maths or is the performance hiding it?'"*

Andrew begins to feel the tiredness and the jet lag. Despite this, he does not miss the opportunity to make me feel comfortable and, after a short pause, he asks me to continue. *"So, to what extent could performing arts be of help in communicating mathematics?"* A small pause for reflection. *"It depends, you've got to be very careful. I don't think that just because you can do something, you should. I think there is a danger and we need, in the presentation, to make sure that what we do doesn't overshadow the message we're trying to get across. Otherwise we could think that it is just fun. I don't want to do it this way because it is fun. And the trouble then is, 'Well, if it is not fun?' So, I don't need maths to be fun all the time. I think it can be enjoyable, but it doesn't have to be fun, it has just to give me a key. So, I'm aware there is a body of people out there who present maths in this way. It's a thin line, between presenting enjoyable maths and presenting funny maths. If I'm honest with you, Andrea, I've crossed the line. I know I've done something because it's really magical and the maths was minimum. Sometimes I've done something different that's more than a metaphor for maths as well. So, for example, I have a trick where I have a tiny piece of blank tissue paper and I say, 'This is how I feel with my brain when I'm stuck, you know, it feels quite small and it feels quite blank. And this is a teacher and the teacher is like a cigarette lighter'. I say, 'If the teacher can shed some light here, it catches fire and turns into money or something'. And that's not maths at all, but it's a metaphor. It's like I'm saying, 'In a world where I'm stuck, if I add something it's getting better'. So, it's ok to be stuck. But these are two different type of amusement."*

In short, you have to be very careful with mathematics, because the risk of getting the wrong end of the stick is always around the corner. Becoming an effective communicator of mathematics requires a long journey, strewn with obstacles large and small, including the fact that one must not underestimate the scepticism of a part of the mathematical world itself. However, in an increasingly technological society like today, it is important to avoid mathematical illiteracy. John Allen Paulos, in his book *Innumeracy* [10], emphasises the fact that people are totally ignorant of the mathematical mechanisms in the society that surrounds us. This is why the communication of mathematics is becoming a collective commitment, even within the mathematical community itself. *"Now, in England, we love science. I don't know why, but not so much maths"*, Andrew says with some regret. *"It could be a result of the efforts UK sustained after the 1985 document on Public*

Understanding of Science[29]*. We have an elite in communication. I think they realised that you can do even the right thing for the wrong reason. For example, funding. Your funding depends on how many students want to study your subject. So, you better get them interested in your subject, which is the most terrible thing to say. But this is reality. Italians love art, maths is an art, so start with Leonardo da Vinci. Start asking 'Who are you?' and then you begin to compare. George Bernard Shaw said, 'If you have an apple and I have an apple and we exchange these apples then you and I will still each have one apple. But if you have an idea and I have an idea and we exchange these ideas, then each of us will have two ideas'. Really lovely stuff. What do you need to think to keep changing an idea? We need to keep communicating no matter it's maths or science or all the things that you're talking now. If we stop communicating, then we stop learning other things. As a community, that may mean that some [of the scientists] should stand up on stages with silly suits and talk to people, other ones though should sit in the lab and look through a microscope, and work out stuff and type it up. And someone else needs to read that and tell them, so they can go to others. It's not for an individual. It's for the body to do. So, it's not for everyone to be a communicator of a subject, I think."*

The clock on the wall reminds us that our time is over. Andrew has to eat something, rest to overcome the jet lag and prepare his talk for the *Inspire Maths Conference*, to which he has invited me on the following day. We both agree that this meeting has been a mutual enrichment, especially for yours truly. Surrounded by the beauty of "The Library", we seem to have relived for a moment the magic I imagine was felt during the creative meetings of the Bloomsbury Group. Here it is a magic that has opened for me a window onto the broad and fascinating panorama of the communication of mathematics in the United Kingdom, the far from trivial connections between communicating and teaching mathematics, the long way still to go and the richness and the risks inherent in the cross-pollination between mathematics and performing arts. This magic would not have happened without Andrew's essential contribution. We say goodbye in front of the reception desk. It is time for me to resume my journey. I go back to the noisy traffic of Russell Square. I put on the earphones and press "Play" on my mp3 player. I am astonished as soon as the song begins: "It's a kind of magic...". God save the Queen, and Queen as well.

Appendix 1. Who is Andrew Jeffrey

Andrew Bruce Stewart Jeffrey was born in Ely, a small English town in the East Cambridgeshire district, in 1966. From the earliest age, he has been attracted to the beauty, logic and magic of mathematics. At first he wanted to be a lawyer, but after a year of volunteer work in a school before beginning university studies, he grew passionate about teaching. The teaching of mathematics, the subject of his degree thesis, was his specialisation, and he became a teacher.

In parallel with this, he developed a passion for magic. As a child, he had received a magician kit.

---

[29] This is a report prepared by a select group of experts chaired by Dr W.F. Bodmer on behalf of the UK Government, which should have stimulated better communication and perception of science in the United Kingdom, and should have remedied the gap that had arisen between a society increasingly conditioned by the introduction of new scientific and technological knowledge on one side and the world of research being increasingly isolated in its attempt to explain its role and goals on the other. Just seventeen years later, *Science* published a famous note by a group of British researchers entitled "From PUS to PEST" [Science 298, 49 (2002)], which definitively ratified its crisis and subsequent failure.

Later, he entered a magic club and at the age of twenty he became a professional magician (Fig. 2.23). At the same time, he discovered the Fellowship of Christian Magicians (www.fcm.org) and in 1998 became a European champion. In addition, Andrew Jeffrey is a highly prized and prominent conjurer in the UK as well as a great mentalist. He is currently the only magician to win the prestigious Sussex Magic Circle Competition (www.sussexmagiccircle.co.uk) for four years in a row from 2003 to 2007 in the Magic of the Mind and Stage & Cabaret categories (Fig. 2.24).

Fig. 2.23 – Andrew Jeffrey during a magic show

Fig. 2.24 – Andrew Jeffrey during the 2007 award ceremony of the Sussex Magic Circle Competition

In 2000 he realised the huge potential of combining magic with mathematics. The trick of the "Grey Elephant in Denmark"[30], well known today, was his favourite trick of that time. He began being invited to schools and conferences to present his new approach to mathematics and to help teachers make maths captivating and engaging. In 2007, while continuing his profession of magician, he faced a choice: to continue to be a teacher or to become a teacher trainer. He chose the latter and created Magic Message Ltd (see Appendix 2), a training company specialising in the production of stimulating resources for the professional development of teachers, motivational workshops and shows for students.

For some years now, the activity of Magic Message Ltd has been oriented less to magic and more to the training of teachers and the design of ad hoc school programmes. He is currently one of the experts of the large Oxford Owl project[31] created by Oxford University Press, designed to help support children's learning both at school and at home with direct involvement of parents. In Andrew Jeffrey, three souls coexist: the teacher, the magician and the researcher. Thanks to his ability to amaze, entertain and communicate, he is now an experienced speaker in the UK and abroad. Just in the last six months, he has travelled between Scotland, Switzerland, Ireland and Brunei with the specific goal of helping teachers make mathematics a magical experience for students.

Appendix 2. Magic Message Ltd

Magic Message Ltd was created in 2007 with a specific goal: to bring the magic of mathematics to everyone: students, teachers, and the general public. The philosophy of the company and its

[30] http://www.magicmgmt.com/gary/something/

[31] By subscribing to the www.oxfordowl.co.uk website, it is possible to take advantage of the support of experienced teachers and trainers, and to access a large amount of online resources, including "Storytelling Videos", a library of e-books, materials, tips and ideas for activities to be offered in class and home and useful links for teachers and parents.

founder, Andrew Jeffrey, is that everybody is fundamentally a mathematician. Someone just needs some more help and encouragement than others, whether they are learning, teaching or approaching mathematics for the first time.

From the very beginning, the connection between magic and mathematics has been clear, as well as the desire to create shows and projects aimed at mathematical learning at different levels. In recent years, Magic Message Ltd has especially concentrated its activity in schools, providing support for educational projects for single classes or entire schools, and high quality assistance through Continuing Professional Development (CPD) programmes, Inset days[32], talks, workshops and performances.

His professional experiences as a teacher and a magician, coupled with a strong propensity for research, allow Andrew Jeffrey to translate his ideas into interactive workshops and shows, motivational and highly engaging. A first example is his show “The Magic of Maths!”, a journey through mathematics that aims at changing the attitudes of the audience towards the discipline (Fig. 2.25). The show is a comedy/cabaret, during which Andrew Jeffrey offers the audience one challenge after another full of mathematical tricks.

Another interesting example of his shows is “Your Magical Mind”, a motivational show for students in secondary school and beyond, in which he demonstrates how the human mind can be trained to perform extraordinary mental feats such as lightning-fast calculations, memory games, reading thoughts, teleportation, correct forecast of future events, and more. During the show, Andrew Jeffrey explains and emphasises how every individual in the audience is fabulous and rich in talents, and also teachees them how to perform their own memory feats.

For more information about Magic Message Ltd, visit www.andrewjeffrey.co.uk.

Fig. 2.25 – Two images from Andrew Jeffrey's Magic of Maths! Show

[32] An Inset day, originally an acronym for “IN-SErvice Training Day”, also known as a Teacher Development Day or sometimes a Baker Day, is one of five days during which British schools suspend lessons to allow the entire school staff to attend training courses or to complete administrative tasks. Inset Days were introduced in 1988 by minister Kenneth Baker as part of a series of reforms that included the introduction of the National Curriculum.

References

[1] Tahan, M.: The Man Who Counted. A collection of mathematical adventures. Leslie Clark and Alastair Reid, English transl. W.W. Norton & Company, New York (1993)

[2] Capozucca, A.: Communicating mathematics in Europe - Episode 2: Alex Bellos (London). Lettera Matematica International Edition **5**, 223–230 (2017)

[3] Albers, D.J.: Professor of ~~Magic~~ Mathematics. Math Horizons, **2** (3), 11–15 (1995)

[4] Spinelli, A.: La partita aperta tra didattica e comunicazione. Rivista Scuola IaD **1**, 44–66 (2008/2009)

[5] Tomatis, M.: Un laboratorio di magia e matematica. http://www.marianotomatis.it/research.php?url=laboratorio. Accessed 5 October 2017

[6] Seth Taylor, R.: The Wonder of Magic: Eliciting Wonder and Analyzing its Expression. MPhil thesis, Massachusetts Institute of Technology (2007)

[7] Holt, J.: How children fail. Pitman, London (1964)

[8] Snow, C.P.: The two cultures. In: The two cultures and the scientific revolution. Cambridge University Press, Cambridge (1961)

[9] McCrory, P.: Developing interest in science through emotional engagement. In W. Harlen (ed.), ASE Guide to Primary Science Education, new ed. ASE, Hatfield, pp. 94–101 (2010)

[10] Paulos, J.A. Innumeracy: Mathematical Illiteracy and its Consequences. Hill and Wang, New York (1988)

Chapter 3

From Science Communication to Mathematics Communication

Before embarking on a journey, you need to know where are you going, the culture and traditions of your place of destination, habits, language and way of life of those who live there. Then, you have to choose how to behave and what to carry in your bag accordingly. Lastly, you have to plan your trip in detail. It is essential to avoid going off the rails and not risk to get lost along the way. Science communication and mathematics communication are two worlds worth visiting, but they are relatively new and deeply complex. They both have a rich history related to long traditions and eminent scientists, but different styles and habits. They both have shared paths, but separate at times. They present similar methods and approaches and are like two sides of the same coin. John M. Ziman (1925-2005), a British physicist and humanist, argued that the first phase of the scientific activity, the one producing new knowledge, must be strictly connencted with the second "public" phase. Without the communication on the newly-acquired knowledge, there is no science. Communication is the real connective tissue for a scientific society[33].

That's the reason why I spent most of my first two years of doctorate studying these two worlds to understand their dynamics, motivations, evolutions, techniques and tools. A study that has given rise to questions, thoughts and discussions, and has provided a broaden view of the scientific communication horizon and deep insights into the problems inherent in communication. This chapter summarises this view. It would be a snapshot of this vast horizon in which I try to define its edges as precisely as possible. At the same time, it could be a useful map to anyone wishing to start down the road to science and maths communication.

3.1 Why to communicate?

There are four cultural factors that have influenced the degree of separation of science from society[34], resulting in an increased need for communication: the loss of expertise and authority of

[33] Latour B. (1987), *Science in Action: How to Follow Scientists and Engineers Through Society*. Harvard University Press.

[34] Science for All (2010a), *Report and Action Plan from Science for All Expert Group*. London: Department for Business, Innovation and Skills. Available from http://interactive.bis.gov.uk/scienceandsociety

scientists, a change in the nature of knowledge production[35] (research developments are frequently achieved in a more interactive and interdisciplinary manner than in the past), improved communications and a proliferation of sources of information (for example, the change in teaching styles from a "chalk talk" to an interactive multimedia approach), and the democratic deficit (political-scientific decisions are increasingly made outside of the public arena).

Bringing together these cultural factors, a range of key motivations for science communication can be identified at institutional and national levels. Osborne and Monk[36] provide an overview of the most common suggestions, identifying four key arguments:

- The *utilitarian argument* - The people involved will gain technical skills and knowledge that will be useful to them in their wider lives.
- The *economic argument* - Advanced societies require a technologically skilled workforce; science adds significantly to the overall output of a country (or region).
- The *cultural argument* – Science represents a "shared heritage" and should be recognised as a wider part of our culture.
- The *democratic argument* – Science affects most major decisions in society, therefore it is important that "publics" are able to interpret basic scientific information. The goal is to go towards a social and ethical science communication that can improve citizens' participation.

In addition to these, there is the *altruistic argument* which motivations, such as sense of duty, raising public awareness of the discipline and even transmitting the importance of science, often predominate over motivations that are strategic or political (in a broad sense), such as fighting the irrationality of the public, justifying the use of public funding, attracting students to my discipline or generating additional funds[37].

There is often an inability to communicate within the scientific community that doesn't help to improve the public image of science and mathematics. This could lead to a lack of knowledge in the receiver of the message that would leave the playing field wide open to personal purpose manipulation of information and, even worse, to pseudoscience.

3.2 Who have to communicate?

Why scientists and mathematicians should become involved in communication? Because part of the responsibility for this situation is theirs. People don't often appreciate the power of a mathematical idea or a scientific result on the progress of science and technology. This means inadequate financial support to our disciplines and research and fewer jobs in the industry. For example, mathematics could be better redesigned in education if people have a more widely accepted

---

[35] Benneworth P. (2009), *The Challenges for 21st Century Science – A Review of the Evidence Base Surrounding the Value of Public Engagement by Scientists*. Univesiteit Twente.
[36] Osborne J., Monk M. (2000), *Good Practice in Science Teaching: What Research Has to Say*. Open University Press.
[37] Kreimer P. et al. (2010), *Popularization by Argentine researchers: The Activities and Motivations of CONICET scientists*. Public Understanding of Science, Sage Journal, p.42.

perspective on where to direct students careers. In many countries, fewer and fewer students choose to carry on studying scientific subjects.

A very clear message should go out that a mathematician as well as a scientist is a human being. Like a child, he/she constantly asks him/her-self questions. Through his/her deep understanding of the discipline, he/she could instill the real "taste" of it. But I think that not all mathematicians and scientists must be involved in communicating science and mathematics. This practice needs a passionate personal involvement that is not necessarily shared by all. But, on the other hand, by taking part into popularization and outreach activities, train our communication skills could help us to improve the quality of our teaching allowing us to place our discipline within a wider framework, to introduce connections with disciplines scientific and not, and to show practical applications.

3.3 What types of communication?

Various categorisations have been developed to distinguish among different types of science communication with the public. The fundamental concept shared by all the models is the distinction between "deficit" and "dialogue" approaches. In the former, the assumption can be that the audience members lack necessary knowledge about scientific concepts, and therefore communication from scientists to a public audience is required[38]. Two assumptions often underline this concept are that public opinion and political decision-makers are misinformed about science and the issues raised by its development. This situation is seen as being exacerbated by poor training in basic science and a general lack of interest among the institutions and the cultural intelligentsia[39]. For this perception a need arises for initiatives bridging the gap between experts and the general public, reversing public attitudes towards science and technology or at least attenuating their hostility. Such emphasis on the public's inability to understand the achievements of science, according to a model of linear, pedagogical and paternalistic communication, has warranted the label of "deficit model" for this view of the public understanding of science[40].
Conversely, a "dialogue" approach (occasionally known as a "contextual model") involves a two-way exchange of information between scientists and "publics". Dialogue came to be presented as the acceptable alternative to the deficit model from the late 1990s. A frequently cited report of the House of Lords (2000) in Britain acknowledged the limits of science communication based on a top-down science-public relationship, and detected a "new mood for dialogue". In many countries and at the European level, funding schemes and policy documents shifted their keywords from "public awareness of science" to "citizen engagement", from "communication" to "dialogue", from "science and society" to "science in society"[41]. But it has been argued that some dialogue methods are not genuinely two-way or symmetrical, in that the original sponsors of the communication

[38] Gross A.G. (1994), *The Roles of Rhetoric in the Public Understanding of Science*, Public Understanding of Science 3(1), Sage Journal, p.3-23.
[39] Snow C.P. (1959), *The Two Cultures*. Cambridge, Cambridge University Press.
[40] Ziman J. (1991), *Public Understanding of Science*. In: Science, Technology and Human Values, 16 (1), p.99-105.
[41] Bucchi M., Trench B. (2016), *Science Communication and Science in Society: A Conceptual Review in Ten Words*. Tecnoscienza, Italian Journal of Science & Technology Studies, 7(2), p.155-156.

(generally scientific or policy institutions) stay in control and the citizens taking part have no significant influence on the final outcomes[42].

Some authors have extended this further. For example, Rowe and Frewer[43] describe a three-pronged approach consisting of:

- Communication (information flowing from the "sponsor" – scientific organisation – to public representatives);
- Consultation (direction of travel of information from public representatives to the sponsor);
- Participation (two-way communication between sponsor and public representatives).

Bucchi also outlines a similar approach involving Transfer, Consultation and Knowledge Co-Production[44]. Starting from these multi-model framework, a more nuanced approach has recently been developed by the Science for All Expert Group in the UK, consisting of the "Public Engagement Triangle"[45] (Fig. 3.1).

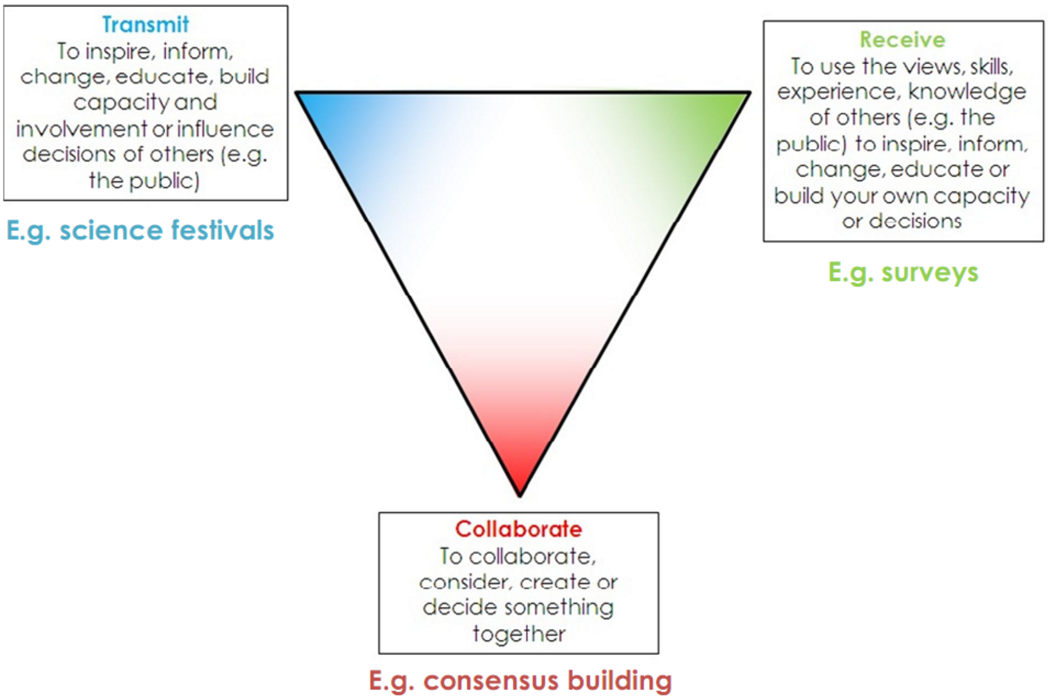


Fig. 3.1 – Public Engagement Triangle

[42] Davies S. et al. (2008), *Discussing Dialogue: Perspectives on the Value of Science Dialogue Events that do not Inform Policy*. In: Public Understanding of Science, 18 (3), p.338-353.

[43] Rowe G., Frewer L. (2005), *A Typology of Public Engagement Mechanisms*. Science Technology Human Values 30. DOI: 10.1177/0162243904271724

[44] Bucchi M. (2008), *Of Deficits, Deviations and Dialogues: Theories of Public Communication of Science*. In Bucchi and Trench (eds), *Handbook of Public Communication of Science and Technology*. London and New York, Routledge.

[45] Science for All (2010b), *Public Engagement with Science and Society – A Conversational Tool*. London, Department for Business, Innovation and Skills. Available from http://interactive.bis.gov.uk/scienceandsociety.

This "triangle tool" identifies three key communication approaches: Transmit, Receive and Collaborate. However, the strength in the triangle approach is that it recognises that these approaches do not work in isolation, but instead as the apexes of various spectra. Any science communication activity is likely to involve a blend of these three approaches according to the needs of the audiences and the scientists involved.

3.4 How to communicate?

There are a vast range of approaches to science communication. Mesure[46] identified over 1500 active initiatives within the UK alone. In the following chapters, I present a series of maths communication projects, activities and events I designed and presented or in which I directly took part in inside and outside Italy to provide a flavour of the variation and creativity involved in different successful approaches and evaluate their impact on the audience depending on contexts, contents, methods and communication styles.

Broadly speaking, there are three main forms of media used in science communication: traditional journalism, live or face-to-face events, online interactions[47]. Each approach has its associated pros and cons, as outlined in the Table 3.1.

MEDIUM	ADVANTAGES	DISADVANTAGES
Traditional Journalism (both print and broadcast) • Newspaper • Magazines • TV • Radio • Books • ...	Large potential audiences (potentially millions of people) High quality due to being overseen by professionals (e.g. journalists) Traditionally recognized as agenda setting Possible audience selection through appropriate choice of publication/programme	Scientists partial or complete lack of control of how the media covers their work Tends towards one-way communication Frequently provides a limited or superficial focus
Live or face-to-face events • Public lectures • Science Centres and Museums • Debates & dialogue • Science Busking • Sci-art • Science cafes	More personal (involves a direct interaction between scientists and "publics") Scientists are able to better control the content Engenders two-way	Limited audience reach (tens to thousands of people) Resource intensive, leading to low sustainability of activities Can be criticised for only attracting audiences with a pre-

[46] Mesure S. (2007), *The CreScENDO Project: final report for NESTA*. London: National Endowment for Science, Technology and the Arts.
[47] Bultitude K. (2011), *The Why and How of Science Communication*. In: Rosulek P., ed. *Science Communication*. Pilsen: European Commission.

• Science Festivals • ...	communication Can involve partnering with other external organizations with complementary expertise	existing interest Risk of sensationalizing science
Online interactions • Internet sites • Blogs, wikis and podcasting • Facebook, twitter and other social media • Citizen Science • ...	Large potential audiences (potentially millions of people) Can allow direct interaction between scientists and "publics" Initial content can be controlled by the scientists Caters for both one-way and two-way communication, depending on audience's preference Always accessible; suits the audience's time preferences	Can encourage superficial or "jokey" interactions Difficult to control how the content is picked up by others Requires regular attention to maintain profile Requires key communication skills that may not be immediately apparent

Table 3.1. Adapted from Bultitude K. (2010), *Presenting Science*.
In: Brake M.L., Weitkamp E. (eds.), *Introducing Science Communication*. London, Palgrave MacMillan.

3.5 Who is the public?

Talking about science communication, we use expressions like "public understanding of science", "raise public awareness" or "public engagement". The word "public" is widely used but seldom clarified. Many scientists consider the public to be anyone who is not a scientist, others think that public is synonymous with non-expert people or, more generally, the typical, randomly chosen person on the street. So, who is the public?

The public is not an homogeneous group of clones: there is no such thing as "the general public". Each person in your audience will have their own interests, prejudices and concerns, and it is important to take these into account when planning your activity. In particular, in advance of our activity, it could be useful to get a feeling for who our audience is likely to be, as well as what they will have in common. We can do this by consulting with the event organisers, or reflecting ourselves on who is likely to attend and what their backgrounds are likely to be. By segmenting our audience into recognisable demographics (for example age, educational background, socio-economic group and/or existing levels of interest in science), we can draw on the factors that our target audience will have in common. By appealing to their interests, we are much more likely to properly engage them with scientific concepts. For example, within the UK six key *audience clusters* have been identified according to their general attitudes towards science: the "concerned", "indifferent", "late adopters", "confident engagers", "distrustful engagers" and "disengaged sceptics"[48]. Members of these groups share common factors, which means that it is possible to identify specific mechanisms and/or content that is most like to appeal to members of that group.

Perhaps it could be more appropriate referring to the public in the plural as "publics". This plural form has become common in discussion and study of science in society, indicating in shorthand that "the public" is multi-faceted, even fragmented. Because it is not a common word, "publics" often carries quote marks around it that draw attention to its deliberate use. Adopting the plural form is an important part of recognising that generalisations about the public – specifically in terms of its deficits – are very rarely valid, and often seriously misleading[49]. Referring to "publics" has been associated with the proposal of a contextual model of communication, according to which the communicators inform themselves about, and are attentive to, the various understandings, beliefs and attitudes within the public. The plural-publics approach has been supported by the accumulation of evidence on the widely varying interest, attention and disposition towards scientific matters by the populations of individual countries and, comparatively, across countries and continents. A strong focus on "publics" is almost standard now in the training of scientists for public communication; short courses offered to researchers by research councils, universities, professional organization and others very often start by asking: who are the "publics" you want to communicate with, and why?[50]

This implies researching the cultural contexts – scientific, artistic, every day, and other – of such public. The increasingly blurred boundaries of communication contexts should also force us to explore with more courage conceptual affinities and potential inspiration in the humanities, arts and culture, largely neglected by science communication scholars, despite the growing science/art practice. For example, concepts such as style may be relevant to understanding variety in science communication as well as addressing the challenge of quality. This resonates with long-standing invitations of Lévy-Leblond to *"put science into culture"*[51], emphasising its connections with other domains rather than its separation from society and culture, as expressed in models and visions of knowledge translation and transfer. It also invites us to recognise the importance of a broader culture of science in society that goes beyond familiarity with technical contents to include an awareness of its role in society, cultural implications, intercultural contamination, relations with fashions and cultural tendencies, aims, potential and limits. We have to look at science communication not only as a means to achieve certain objectives but as a central space to understand (and partecipate in) the interacting transformations of both science and public discourse. In this perspective, communication is not simply a technical tool functioning within a certain ideology of science and its role in economic development and social progress, but has to be

[48] Ipsos MORI (2011), *Public Attitudes to Science 2011*. Department for Business, Innovation and Skills, p.71.

[49] Einsiedel E. (2014), *Publics and their Partecipation in Science and Technology: Changing Roles, Blurring Boundaries*. In: Bucchi M., Trench B., (eds.) *Routeledge Handbook of Public Communication of Science and Technology*. London and New York, Routeledge, p.125-139.

[50] Miller S. et al. (2009), *Can Science Communication Workshops Train Scientists for Reflexive Public Engagement?*. In: Science Communication, 31 (1), p.116-126.

[51] Lévy-Leblond J-M. (1996), *La Pierre de Touche – la science a l'épreuve*. Paris, Gallimard.

recognized as a key dynamic at the core of those co-evolutionary processes[52], redefining the meanings of science and public, knowledge and citizenship, expertise and democracy.

3.6 Popularization

The term "popularization" has the longest tradition among those used to describe a wide range of practices in making scientific information accessible to general, non-expert audiences. The near equivalent terms in other languages, including *vulgarisation* (French), *divulgazione* (Italian), *divulgación* (Spanish), also have long and continuing histories and carry similar connotations. One of the founders of modern science began the tradition following up previous little-known attempts. From *Sidereus Nuncius* (1610) through *Il Saggiatore* (1623) to *Dialogo sopra i due massimi sistemi* (1632), Galileo Galilei (1564-1642) was a pioneer of science communication. Starting with the use of the vulgar Italian language instead of latin, the aim of Galilei's communication strategies was to give the widest possible dissemination of scientific ideas of his time. He managed to get the debate on Copernicanism outside the limited scientific scholar community. After all, it was Italo Calvino who said that *"Galileo uses language not as a neutral tool, but with a literary awareness, with an ongoing expressive and imaginative partecipation and lyricism"*[53].

However, if we go back in time before Galileo Galilei, there are two great authors that can be considered earlier science popularizers: Dante Alighieri (1265-1321) and Luca Pacioli (1445-1517). Both of them adressed their efforts to set up and spread all the knowledge of their time outside the wise inner circle. Of course, we clearly note this by looking at Dante's *Divina Commedia* (1304-1321) where we can find careful descriptions of the natural phenomena (from the movement of the stars to lunar spots, from geological events to animal behaviors, from relative motion to multiple manifestations of light) as well as the use of *"volgare eloquenza"* in rhyme to promote the verbal transmission among common people. It is something that also emerges from Calvino's words: *"...in the same way Dante, in a different cultural horizon, built an encyclopedic and cosmological piece of work; as Galilei, he was also trying to construct an image of the universe through his literary work. This is a serious vocation for Italian Literature that goes from Dante to Galileo: the literary work as a map of the world and all human knowledge, and the art of writing driven by discovery that is sometimes theological, speculative, witchy, encyclopedic, of natural philosophy, of transformational and visionary observation."*[54] In Dante's *Convivio* (1307), there is even talk of the social role of science. In his Treaty, he claimed that all the people have the desire to know and actually *"la scienza è l'ultima perfezione de la nostra anima"*[55] ("Science is the last perfection of our soul"). Pacioli, instead, was attempting to promote mathematics in particular. He was the first to use mathematics as a powerful form of entertainment. In his *De Viribus Quantitatis* (between 1498 and 1509) there are mathematical games with numbers and knots, sleights of hand, amazing experiments and riddles: valuable material for engagement activities "ante-litteram". Pacioli's

---

[52] Jasanoff S. (2005), *Designs on Nature: Science and Democracy in Europe and the United States*. Princeton, Princeton University Press.

[53] Bischi G.I. (2015), *Matematica e Letteratura: dalla Divina Commedia al noir*. Alice & Bob 42, Egea.

[54] Calvino I. (1968), Interview on "L'Approdo letterario" magazine. In: Calvino I., *Una pietra sopra*. Milano, Mondadori.

[55] Alighieri D. (2005), *Convivio*. Garzanti Libri.

purpose was *"ponere et demostrare li admirandi e stupendi effecti che de ditta quantità procedano"* in everyday life and spare time. The key points of his action were to glorify practical and theorical relevance of mathematics, and present the wonderful and magic effects arising from its applications[56].
Other early examples of popularization, though not named as such at the time, include Fontanelle's *Entretiens sur la pluralité des mondes* (1686), a series of conversations between a philosopher and a marquise. During the 18th century, then, science popularization gradually defined itself as a distinctive narrative genre, often targeting in particular female readers as supposedly ignorant and curious as in Algarotti's classic *Newtonianism for Ladies* (1739) or de Lalande's *L'Astronomie des Dames* (1785).
In the 19th century, the *Christmas Lectures*, hosted by the Royal Institution of Great Britain, are another interesting effort of science popularization. They were a series of lectures on a single topic that presented scientific subjects to a general audience in an informative and entertaining manner. Apart from a short break for the second World War, they have been running continuously since 1824, the year of the first Christmas Lecture held by John Millington about Natural Philosophy. In the early days, Humphry Davy and Michael Faraday introduced innovations to the public lecture format to greater improve their popularity, which included targeting "juveniles", people aged 15 to 20, and incorporating demonstrations and practical experiments that wowed the audience members[57]. Since 1966, in addition to live audience, the Christmas Lectures have been televised annually, reaching international audiences.
Further channels of popularization emerged later, with scientific discoveries frequently featured in the daily press, science museum, public lectures and great exhibitions and fairs that showed visitors the latest marvels of science and technology. Particularly during the second half of the 19th century, popularization and popularisers profited from changes in the publishing business and the increasing reading audience to become influential voices, but their success also testified to the increasing relevance of science as a cultural force. The sales figures of Brewer's *Guide to the Scientific Knowledge of Things Familiar* (1852) are impressive even by contemporary standards. Through their books and public lectures, popularisers like J.H. Pepper and J.G. Wood in England or Paolo Mantegazza in Italy became public celebrities of their time[58].
In the following century and particularly after World War II, the new global and policy landscape redefined popularisation in conceptual and even ideological terms, particularly in US and Western Europe. With science's social and political role significantly captured by the metaphor of the *"goose laying golden eggs"* – e.g. delivering economic wealth, social progress and military power if appropriately fed – popularisation was expected to "sell science" to the broader public to strengthen social support and legitimation[59]. The goose metaphor was coined by Vannevar Bush,

---

[56] Montebelli V. (1998), *I giochi matematici nel "De Viribus Quantitatis"*. In: Giusti E. (ed), *Luca Pacioli e la Matematica del Rinascimento*. Petruzzi Editore, p. 313-330.
[57] James F.A.J.L. (2007), *Christmas at the Royal Institution: an Anthology of Lectures*. Singapore: World Scientific Publishing Co. Pte Ltd.
[58] Lightman B. (2007), *Victorian Popularizers of Science. Designing Nature for New Audiences*. Chicago, University of Chicago Press.
[59] Lewenstein B. (2008), *Popularisation*. In: Heilbron J.L. (ed), *The Oxford Companion to the History of Modern Science*, Oxford, Oxford University Press, p.667-668.

scientific advisor to the US government during World War II, and author of an influential report[60]. The approach he, among others, proposed fueled the development of popularisation strategies and channels, including interactive science centers and partnerships between science institutions and Hollywood studios.

When a new phase of critical reflection on the role of science in development and (more broadly) in society opened, spurred by environmentalist, anti-war and anti-nuclear movements, the concept of popularisation also came under criticism as embodying a paternalistic, diffusionist conception of science communication[61]. More recent conceptualizations have reappraised the term, considering it suitable to describe specific types and contexts of communicative interactions among science and the public.

3.7 From science communication to mathematics communication

Reinhold Remmert[62] strongly claimed that *"it is not possible to popularize mathematics"*. This statement about the popularization of mathematics has a point. However, I'd like to distinguish between popularization and communication. While his statement may apply to popularization as I try to explain further on in this chapter, it does, in my opinion, not apply to the communication of mathematics, even if we have to face many difficulties and obstacles on the way. First of all, I'd like to ask why it is not possible to communicate mathematics. What is different in mathematics compared to other sciences? As I see it through my own experience, I have the feeling that mathematics falls into a special category if we compare it to the other sciences. There are two significant structural reasons why it is so difficult to communicate or even popularize mathematics.

The first reason is that objects in mathematics are often abstract creations of human thought. Except for very simple ideas, like natural numbers or elementary geometrical figures, mathematical objects are not perceived, even if one can argue that they are not independent of perception. For example, objects like groups, vector spaces, or curved spaces in arbitrary dimension cannot be experienced with our five senses. They need a formal definition, which does not rely on our senses. Gaining an understanding of mathematical objects and relations is only possible after a long time of serious theoretical consideration. Another reason is that mathematics has developed its own language, more than any other science. This is necessarily a result from the previous point that mathematics cannot be experienced directly. Therefore, each mathematical term needs a precise formal definition. This definition includes further terms that must be defined, and so on, so that finally a cascade of terms and definitions is set up that make a simple explanation impossible. But even in ancient times, the

[60] Bush V. (1945), *Science: the Endless Frontier, a Report to the President*. Washington, United States Government Printing Office. Available at https://www.nsf.gov./od/lpa/nsf50/vbush1945.htm

[61] Hilgartner S., *The Dominant View of Popularization: Conceptual Problems, Political Uses*. In: Social Studies of Science, 20 (3), (1990), p.519-539.

[62] Reinhold Remmert (1930-2016) was a German mathematician known for his two books on number theory and complex analysis containing a huge amount of historical information together with references on important papers in the subject.

abstraction from objects of our perception has always been a decisive part of mathematics, which made it difficult to comprehend. As Menecmo (380c.-320c. BC) commented to his royal pupil Alexander the Great (356-323 BC) asked for a shortcut to geometry: *"O King, for travelling over the country there are royal roads and roads for common citizens; but in geometry there is one road for all"*. In other words, there is no royal road to geometry.

The language of mathematics requires an extremely compact presentation, a symbolism that allows replacing pages of written text by a single symbol. The peak of mathematical precision and compact information is a mathematical formula. But mathematical formulas frighten and deter. Stephen Hawking wrote in 1988 that *"[...] each equation I included in the book would halve the sales"*[63]. These structural reasons support the thesis of Remmert that the nature of mathematical research cannot be popularized. And all mathematicians engaged in research would agree, that it is nearly impossible to feasibly illustrate to a mathematically untrained person the project one is currently working on. Actually, this experience applies not only to mathematically untrained people but even to mathematicians working in a different field.

But, in an overall sense, mathematical thinking is, after speech, the most important human faculty. It was this skill especially that helped the human species in the struggle for survival and improved the competitive abilities of societies. Mathematical thinking has a special place in evolution. I fully agree with Gert-Martin Greuel when he says: *"By mathematical thinking I mean analytic and logical thinking in a very broad sense, which is certainly not independent of the ability to speak"*[64]. Of course, the development of mathematics as a science is a cultural achievement but, in contrast to languages, it developed in a similar way in different societies. It's a fact that the importance of mathematics for mankind has grown continuously over the centuries, regardless of the cultural and social systems. No modern science is possible without mathematics. In addition, there is the essential ethic of mathematics: the intellectual honesty, the critical rationalism, the respect for the reality, the international cooperation, the effort to avoid prejudices. In general, I would say mathematics is an essential part of our culture and it deserves along with science to be on the forefront of public life. This is important for the creation of a general culture that respects mathematics and science and tries to educate the citizens accordingly.

Attaching these values to mathematics, one must conclude that society has the fundamental right to demand an appropriate explanation of mathematics. And it is the duty of mathematicians to face this responsibility[65]. However, if mathematicians want to make their science easier to understand, it will be at the expense of correctness. And that's a problem for mathematicians. All their professional training is necessarily based on being exact and complete. Mathematicians simply abhor to be inexact or vague. But in order to be understood by society, they will have to be just that. The value of a fundamental science like mathematics cannot be measured by its applicability but only by its quality. History has shown that in the long run, quality is the only criterion that matters and that only high-quality research survives. It is worthwhile to emphasize again that any kind of mathematics, either science driven or application driven, can be of high or low quality. So, we need

[63] Hawking S. (2011), *A Brief History of Time: From Big Bang to Black Holes*. Transworld Publishers Ltd.
[64] Greuel G-M. (2015), *Mathematics between Research, Application and Communication*. EMS Newsletter 97, p.44-51.
[65] Greuel G-M. (2014), *Mathematics Communication for the Future. Mission and Implementation*. Berlin, Imaginary. Available at www.imaginary.org/background-material

a high quality mathematics communication. To reach this hard but fundamental goal, I've tried to understand better which are the main difficulties mathematics communicators have to deal with to achieve that goal.

Marianne Freiberger and Rachel Thomas, editors of Plus magazine (plus.maths.org), argued that *"there is an audience for all types of mathematics, be it pure or applied, easy or hard"*. Some people are drawn to the beauty of mathematical ideas or philosophical questions, others to practical applications, or hands-on problems, and others can be drawn in by "stealth" through revealing the mathematics that hides behind seemingly un-mathematical ideas. Even, or perhaps especially, the hardest mathematical concepts can find a large audience, as long as they are presented accessibly and in a non-patronising, engaging way. Communicating mathematics to the society at large does not mean that the public has to be invited to the University, where a mathematics professor explains mathematics at the blackboard. Indeed, it need not involve a university, nor a professor, nor a blackboard. This is a practice which requires considerable thought on the nature of terms and language which are dependent on dynamic behind their choice, and of the message we would like to convey. Furthermore, the way we communicate plays a key role in communicating mathematics and traditional forms of communication have changed and evolved throughout time.
When I say "society at large", I mean the people who go to exhibitions, those who read the science pages in the daily newspapers, the readers of the "Arts and Letters" section of the same newspapers, pupils, students, teachers, parents, but also those who are not necessarily mathematics oriented. Hardly any format can reach all of them. My belief is that we can't teach much mathematics to the society at large – the teaching has to be done in the schools. However, it is essential to work on the public perception of what mathematics is about, about what mathematicians do, about news and challenges, images and jobs.

3.7.1 Symbolism: a facilitation or a complication?

As the old wisdom of Laozi claimed, *"a journey of a thousand miles begins with a single step"* and I think it could have been the same for mathematics communication. Mathematical thinking has evolved through bifurcations, uncoupling of contiguous paths, producing a wide range of mathematical disciplines without taking account of an overall unity that has become an important goal only after the Congress of Paris at the beginning of the 20th century. The work of the Italian mathematician Giuseppe Peano[66] was essential in this direction. He outlined four-points mathematics communication plan based on two different directions called "horizontal" and "vertical".

Scientific and mathematical communities are based on the right and duty of communicating internally and should increasingly engage a meaningful dialogue across disciplines boundaries. This right and duty is set out in horizontal communication whose aim is to relate all the actors regardless of their spoken language and academy membership. Vertical communication, instead, is to translate concepts and models for whom is outside the mathematical community per se. The need to translate arises from the desire to enhance the cultural and educational level in society, from the requirement

[66] Kennedy H. (1983), *Peano. Storia di un matematico*, Torino, Boringhieri.

to convince governments to fund research and studies, and by the need of broadening the circle of those who know about mathematics.

Looking at those two indipendent dimensions, we can reinterpret Peano's four-points plan to promote mathematical knowledge universalization: the "Formulario Mathematico", a volume where Peano tried to collect all mathematical knowledge; the symbolism designed to express mathematics; the five postulates on which natural numbers system was based on; the latin language "sine flexione" to allow all the people to communicate[67]. In a letter of 24 February 1929 to Gaetano Scorza, Peano wrote: *"A collective work that we can do is to publish a new edition of the Formulario Mathematico, whose 1908 fifth edition has now ended. This Formulario is a mathematics encyclopedia, a collection of all mathematical propositions written in symbols with their history and proof. The use of symbols has the advantage of shortness; moreover many propositions that seem different in common language, turn out to be identical; and the propositions assume a correct form much more than with the common language."* Today, more than one hundred years later, the idea of collecting all mathematical knowledge in a volume is naive and impossible. The real success of his project is the symbolism which enabled mathematicians of different times and places to achieve mutual understanding, but at the cost of emphasizing the abstractness of that language in the eyes of a beginner.

Stella Baruk[68] emphasises how *"mathematics is still an obscure and meaningless language for too many students, a "non-sense" waiting for clarification. Unlike foreign languages, here the situation is more complex: there is a "formal non-sense" that may concern a simple letter whose role is not clear, and a "background non-sense" that affects the general feeling (it is not interesting or not necessary) towards mathematical world. These two "non-sense" are not equivalent: the perpetuation of the former permanently jeopardises the opportunity to overcome the latter"*. Peano designed his symbolism to give an overall sense to mathematics, but in the long term it has become a curtain that hides the real sense of mathematics to most.

So, the question is: what is the last open issue in mathematics understanding and communication? I think it is essential to understand the terms around which mathematics is structured. Mathematical essays, books or papers always contain the explanation of all the terms used. A vertical communicator should have the same attitude. This doesn't mean that he has to explain and prove all mathematics again for the layperson, otherwise it would be impossible for him to tell something about fractals or topology of the surfaces. A good communciator must remain true to the reader expectation by ensuring mathematics comprehensiveness and self-consistency, and avoiding the massive use of leaps of faith. Sometimes the reader is willing to take our word for it. It's time like these we have to make clear and justify what we claim by giving up on generalization and being confined specifically to a special case. Communicating mathematics aims to make aware people of an idea or a process, not to develop a complete theory. In doing so, we have to communicate in a rigorous way and show how we develop our reasoning.

[67] Gouthier D., Pitrelli N., Pupolizio I. (2001), *La lingua perfetta e i matematici: il caso di Giuseppe Peano*. Jekyll.comm, 1, SISSA.

[68] Stella Baruk is an influential French teacher, mathematician and educationalist of Iranian origin. Since the middle of the seventies, she introduced major innovations in teaching of mathematics inside French school. Among her bestselling books, there is noteworthy the *Dictionnaire de mathématique élémentaires* (Seuil, 1992).

3.7.2 How much rigour?

The need for rigour in communicating mathematics may raise non-trivial problems, when the target of this communication includes people with little mathematical background. Is it possible to keep some kind of rigour also in informal way of communicating maths? How should we keep track of this rigour in mathematical communication for people who are not mathematicians, to what extent and with what concerns?

Mathematics is strictly linked with rigour. On the other hand, mathematical rigour, and in particular rigorous mathematical language, is a thorough enemy of mathematical communication as I said before. So, these two requirements may seem to be deeply antagonistic, and partly they are. Pietro Greco proposed the existence of a constant K and a "theorem" asserting that the product of rigour and communicability will never be greater than K[69]. There is a great demand and a great need, both in schools and in society, for the informal communication of mathematics. So, far from thinking that there is a contradiction, I think that it is necessary to learn how to communicate mathematics in an informal way, whilst trying to maintain, as much as possible, all the essential features of mathematics and, among them, rigour.

Firstly, we have to distinguish between rigour in mathematics and rigour in communicating maths and in raising public awareness activities. The latter activities very rarely requires proofs and, when it goes into mathematical content, it mainly involves a description of mathematical objects or phenomena or, when justification of the results is required, it aims at convincing that the result is reasonable. Then, another aspect which requires clarification concerns what we call the popular perception of rigour, that is, what the general layperson thinks is rigour, mathematical rigour in particular. People working in maths communication activities need to be aware of these opinions and try to change them. Unfortunately, this misunderstanding, of associating rigour with the usage of technical mathematical language, can be quite common and extends also to mathematics classes in schools. It is of course an important and positive matter that students master the proper usage of a few technical terms, but there is a borderline, beyond which this insistence on formalism is too strong and it becomes pedantry[70]. Sometimes, it can be even worse: it can become a mask which just hides the lack of real mathematical content. The borderline we are referring to is related to the capacity for transmitting mathematical ideas, which gives meaning to the concepts introduced. As long as the meaning is clear, students are not at risk of repeating empty phrases, while, when the meaning is not mastered, the insistence on the usage of technical terms is not only useless, but it may even risk producing negative effects.

Another element we should be aware of, regarding the public perception of mathematical rigour, is what Peano called the “fear of rigour”, which is extremely common in people who have not mastered elementary mathematics. With the words of Peano: *"someone who does not master the basic elements of any part of mathematics always remains hesitant and with an exaggerated fear of*

[69] Greco P. (2004), *Il modello Venezia. La comunicazione nell’era post accademica della scienza*. In: Pitrelli N., Sturloni G. (eds), *La comunicazione della scienza. Atti del I e II convegno nazionale*. Roma-Milano, Zadig, p. 11-38.

[70] Dedò M. (2012), *Rigour in Communicating Maths: a Mathematical Feature or an Unnecessary Pedantry?* In: Behrends E. et al. (eds), *Raising Public Awareness of Mathematics*. Springer-Verlag Berlin Heidelberg.

rigour"[71]. How to overcome this fear? A possible solution could be to simplify the language used. We should be able to remove almost completely technical terms: a mathematical term lives by its unique well-defined meaning. In the ear of those not concerned with mathematics, this poignancy gets lost and the discourse shades into a non mathematical context by diluting that rigour so essential to develop our thoughts.

Inside the mathematical world there are habits that enable an effective and quick communication. Every person in a specific field, from housewife to calculus expert, takes over a specific language. This language and its terminology become a vital communication tool for exchanging direct and unambiguous information. Unlike the common language, every term of a special language is characterized by monosemy, monoreferentiality, uniqueness, ability to relate with different terms, precise location in the linguistic regime. In other words, a much stricter system takes place[72]. Of course, it is an impediment to any attempt of communication outwards. To communicate outside the pure horizontality means to necessarily give up this practice. The mathematical language is an enormous achievement of human mankind. But things get simpler if and only if the new level of abstraction is taught when the person feels the need for that abstraction: otherwise, it is just an empty formalism.

Another thing to note is that mathematical language, which is precious when used by people who have mastered it, may become a dangerous instrument when used by people who have not mastered it. This is particularly risky in a situation, like an exhibition or an activity for general public, where we are speaking to "everybody". Sometimes we are tempted to use mathematical language, we may think that by saying something in a formal way we save our consciences. However, we should always keep in mind that a communication is something with two ends, with the message going from one to the other. And, in order to judge the correctness of a message, it is not enough to judge how correct the message is when it starts its journey, but it is necessary to judge how the message arrives at the other end and how it is interpreted, and understood, by the person who receives it. When we use normal language, we lose much of the precision: things are so much easier with the language of mathematics when it is mastered and especially, it is so much easier to check if a phrase is correct or not.

When we abandon formal language and we try to communicate the ideas of mathematics informally, there are some benefits, and we should learn how to profit from the best of these. The main negative aspect of "normal" language compared to mathematical language is its potential ambiguity: but the same ambiguity can also lead to a richness in many respects. First of all, because it allows us to explore regions that would surely be inaccessible for non-mathematicians if we used formal language. Another possible advantage of an ambiguous message is that ambiguity may enhance mental associations, and mental associations are a precious instrument in transmitting ideas and building up one's own mental images and abstract concepts. Of course, if we decide to accept ambiguity, and sometimes we deliberately play with it in a particular type of mathematical communication, but at the same time we don't want to completely give up rigour, we should be conscious that we are accepting a risk, and that we are walking a very narrow path: so we should also increase the controls we use in order to check how the message can be interpreted. An obvious

[71] Peano G. (1959), *Opere scelte.* Vol.III, a cura di U.M.I., p.273.

[72] Gouthier D. (2002), *Termini e linguaggio per comunicare la matematica*. JCOM 1 (2), June.

consequence of this is that we need to listen, as much as possible, to the reactions of the public to the different parts of an informal communication: this is a fundamental point in judging whether a mathematical concept arrives at the other end of the communication in a substantially correct way.

Although it is certainly true that in an informal communication we should abandon mathematical language, we should also try to maintain rigorous mathematical thinking, at least as much as possible. When we say that rigour should be maintained "as much as possible", we are also implicitly asserting that our method of understanding rigour is dynamic; we cannot just say that rigour is there or is not there, but the problem may be more complicated than that and we have to build rigour, through a continuous process, by going through different levels of approximation and complexity. This opens a big, not trivial problem, as regards "measuring" whether a given communication is rigorous enough. The rigour in an informal communication should be "measured" in a different way to a formal one, with special attention paid to the context and to the way the message may be interpreted by the given recipient.

3.8 How to reinforce the message?

3.8.1 The language

Mathematicians may not be good vertical communicators; of course there are exceptions who are able to accomplish the role of claryfing ideas and concepts, be familiar with the informal approach to mathematics and inspire confidence. Communication should be done by people who know what they are talking about and who know how to talk about it. That’s the reason why a mathematician can bring to mathematics communication a rigour and a point of view no good journalist or anyone else can. But this means to train carefully communication skills and study in detail communication methods.

The first step towards an effective mathematics vertical communication is to be provided with terms that are not equal to those used in horizontal communication, but still have the uniqueness and the conceptual rigour even if in a revised form. It is not enough to create a mathematics communication vocabulary to solve this problem. I rather think that we must begin a thorough examination of the tools in order to make terminological choices guided by our relation with the public and what we’d like to inform it. A mathematician that experiments vertical communication feels betrayed by the fact that the public doesn’t have the same communication automatism. Moreover, she/he is totally overwhelmed and confused by the lack of shared cultural benchmarks. The stance adopted by many good maths communicators is proof of this: some of them use sideshow tricks to encourage the audience; others need to reconstruct the lecture microcosm. The risk is that maths communicators tend to overestimate the amount of new information that the public is able to soak up at once, particularly if the audience is not familiar with the subject. Authors has a substantial advantage compared with speakers accordingly. Readers that lose track of an argument or a topic can read the tricky passage more than once, go back to the previous paragraphs to restate a definition or a keyword, and even use an external source to get the proper knowledge to go further. Listeners can’t usually do anything of this. Mathematical language is a stumbling block for a wider dissemination.

Mathematical statements are supposed to be clear, correct and unambiguous compared to spoken language. The meaning and the position of every word or symbol makes all the difference. William Thurston (1946-2012), a mathematician awarded the Fields Medal for his contribution to the study of 3-manifolds, expressed the difference between mathematics and other subjects in this way: *"Mathematicians attach meaning to the exact phrasing of a sentence, much more than is conventional. The meaning of words are precisely delimited. When I read articles or listen to speeches in the style of the humanities [...] I have great trouble concentrating and comprehending: I think I try to read more into the phrases and sentences than is meant to be there, because of habits developed in reading mathematics"*. Mathematicians similarly should realize that words that they use routinely can echo in unexpected ways into the minds of their listeners or readers, particularly in ways that reflect different experiences and contexts. Such words include acute, base, chaos, chord, composite, concurrent, coordinate, degree, dimension, domain, exponent, factor, graph, group, linear, matrix, mean, network, obtuse, order, power, prism, proof, radical, range, relation, root, series, set, vector, and volume. Each has a precise mathematical meaning; each also has multiple alternative meanings. On the other hand, the word "fractal," coined by mathematician Benoit Mandelbrot, is a noteworthy example of a term that works in both a mathematical and a popular context. Mathematics should use more such words. People are genuinely curious about mathematics, despite the overwhelming fear of the subject that many may feel. Mathematicians who pay particular attention to how they express themselves and connect with their audiences through a common, nontechnical language can make important contributions to the public understanding of mathematics[73].

3.8.2 Popularization

The second step towards an increasing awareness of mathematics is to popularize. Science popularization is better established than popularization of mathematics in popular culture. For example, science centers and science museums exist in many places, but mathematics centers and museums are rare. Among current trends for the old and difficult process of science popularization, mathematics, in all its scope, is not only the one with the most recent activity but it also has special features, not only because of its role in the general education process but also because of its history and applications that are a natural link between the humanities and technology. In contrast with the usual activity of mathematicians when they are creating new mathematics or when they are teaching it to students, a process that it is sometime referred as "doing mathematics", the popularization of mathematics is a very different activity associated with talking, writing or just communicating "about mathematics"[74]. Ian Stewart defines popularization as *"attempts to convey significant ideas from or about mathematics to intelligent, mostly sympathetic non-specialists, in a manner that avoids scaring them silly and exploits whatever interests them"*[75]. He says "attempts" because success can be elusive. The level of exposition can range from humorous short puzzles to books on hot research topics.

[73] Peterson I. (1991), *Searching for New Mathematics*, SIAM Review 13, p.37-42.

[74] Buescu J., Rodrigues J-F. (2012), *The Importance of Useful Mathematics: On Tools for its Popularization, from Industry to Art*. In: Beherends et al., *Raising Public Awareness of Mathematics*. Springer-Verlag Berlin Heidelberg, p.313-330.

[75] Stewart I. (2006), *Mathematics, the Media and the Public*. Proceedings of the International Congress of Mathematicians, Madrid, Spain, p.1631-1644.

Taking a step backwards in history, one of the first successful examples of mathematics popularization was the letters by Euler on different subjects in natural philosophy addressed to a German princess[76]. The first letter, written from Berlin on 19th April 1760, concerns “Of Magnitude or extension” and is followed, three days later, by a second one with the aim *“to unfold the idea of velocity, which is a particular species of extension, and susceptible of increase and of diminution”*. They may be considered a masterpiece of science popularization of the Enlightenment. As Condorcet wrote at the end of the 18th century, *“they have deservedly been considered as a treasury of science”* since *“they convey accurate ideas respecting a variety of objects, highly interesting in themselves, or excite a laudable curiosity”*.

The popularization of mathematics has a longer history than one might realize. There are other two examples that are worth mentioning. The former is *The Ladies ' Diary*, published from 1704-1841 by the Company of Stationers (London). It advertised itself as *"Containing new improvements in arts and sciences and many entertaining particulars "*. The "particulars" included mathematics problems and letters about them. The well-known latter is the story of the Phoenician princess Dido and the foundation of the city of Carthage. This example of the maximization of the area that could be enclosed by an oxhide is quoted in most introductory courses on the calculus of variations as the oldest applied isoperimetric problem, solved by Dido in an empirical form. It is however less known that this story was popularized by the British mathematical-physicist William Thomson (Lord Kelvin) in a conference[77] at the Royal Institution of London on 12th May 1893.

Popularization of mathematics brings our discipline to public attention in contrast to formal education which goal is to teach. Many mathematicians and scientists are afraid of popularization because they identify it with oversimplification. I have nothing against simplification, that’s what we are always doing in life. But we have to be careful and avoid mistakes. The error is just around the corner. Scientific societies should promote research, education and popularization. *“For most of us, to do science writing necessarily harms research and teaching. It’s true that these activities are also complementary. Sometimes, research helps us with ideas for writing to the public, and teaching can give us ideas on ways to explain things”* said Nuno Crato, a Portuguese mathematician and science writer, in an interview after receiving the “Raising Public Awareness of Mathematics Prize” for one of his articles in 2003. All the three goals are important and interconnected.

Some people are drawn to ideas, some to patterns and forms, some to mathematics as a universal language, and others to puzzles. In terms of popularization, the multi-faceted nature of maths should really give it an edge over other fields, as it provides so many ways to engage a non-expert audience. Communicating mathematics to the general public through popularization does not necessarily mean that we can teach mathematics, such that understanding would be the ultimate goal. Indeed, there are many occasions where teaching is impossible, unnecessary, or undesirable, but where we have the chance to report about mathematics, to tell stories (success stories, for example), to portray the field, the persons, to report about the setting and the impact of current

[76] Euler L. (1768), *Lettres a une Princesse d’Allemagne sur divers sujets de Physique et de Philosophie*. Petersburg. A third edition of the English translation was published in Edinburgh in 1823 and is available at http://www.euelrarchive.org.

[77] Thomson (Lord Kelvin) W. (1894), *Popular Lectures and Essays, vol.II*. London, Macmillan and Co., p.571-592.

research, etc. In this case, as Ziegler claims, *“the main goal after a public lecture or event, or after people read your article or book would not be that they understand more mathematics, but rather that they understand more about mathematics”*[78].

The secret for an effective dissemination is to give the public a map by deciding how detailed it should be and where to make it more comprehensible. The risk could be that every map creates an illusory kind of understanding. There is much more in the landscape than in the map, and in the latter could be any distorting effect of which the public is not aware. However, it is essential that the map properly reflects the main features of the landscape to avoid wrong conclusions. For example, New York MTA Subway Map allows me to move properly from place to place with clear information about travelling times and main junctions. If I liked some more information about my way around, I should use a more detailed map of New York road network and conformation of the land. If I liked even more details, I could use technological devices and applications to have a 360-degree view, real-time traffic conditions and route planning for travelling by foot, car, bicycle or public transportation. It could be the same with mathematics. Along these lines, it is acceptable to create an illusory comprehension when communicating mathematics. The presentation should allow the public to travel properly among mathematical main ideas. That depends on who’s talking or writing to decide on what topics focus attention and how much burrow into the abstract theory. Perhaps even more important, our audience should be aware that what they’re watching is just a map of the reality with all the limitations that this entails. In this domain, “understanding”, the “illusion of understanding”, or the feeling that they “understand what it is about” may be plainly the same thing. And in many public or literary settings or occasions, this is also all you can hope for. A full “understanding the mathematics”, as we know, is often only achievable via years of serious study.

Then, popularize means to go to people where they are — watching television, reading a newspaper or shopping for clothing — rather than expecting them to come to us. A program, as well as an event, a workshop, a book or an article, must be attractive to draw participants, as participation is voluntary. The primary attraction may not be mathematics, but rather something else such as music, humor, or physical activity. Without willing participants, without an audience, there is no possibility of success, no matter how worthwhile the mathematics. It is important that the satisfaction in the experience will include pleasure in the mathematics and encourage a favorable attitude and a readiness to consider more. The effects of any one experience are often slight and diffuse, but popular activities are repeatable. One can revisit a museum, watch a film again, follow a television series, and so on.

3.8.3 Informal communication

Looking around, a good and effective mathematics communication makes use totally or partially of informal communication strategies. This is not a necessary, or even sufficient, condition to succeed in popularization. But it is a way of any interest to bring together the two communication actors, mathematics and the public. On the one hand, people try to build their own role model (experts that are credible also outside their mathematical experiential space); on the other hand, there is a blindly

[78] Ziegler G.M. (2010), *Communicating Mathematics to Society at Large*. Proceedings of the International Congress of Mathematicians, Hyderabad, India, p.721.

faith in maths expert brilliance that is super smart by definition. It's clear the self-destructive substance behind this belief and it undermines the ability that anyone may able to figure it out. Both in terms of user and mathematics, more than other sciences, informal communication sets in motion a fruitful process that clears out confusion, anxiety and resignation against mathematics, i.e. creates conditions for a common ground.

3.8.4 Teaching

As most people have a bad experience with mathematics in school, we need to improve the school situation in order to hope for significant change in the popular view of mathematics. For who are far away from maths and for the support of today's students, we must create opportunities to learn about and to practice mathematics outside school. Programs to popularize mathematics among a broad audience serve this purpose.

Although the popularization and teaching of mathematics are related they are different components of the communication of mathematics and it's clear that there are important differences between them. In their aims, the purpose of popularization is to raise awareness, not just to educate, and the criterion of success is not only an increase of knowledge, but also a change of attitude[79]. However, new tools and new media provide new possibilities and raise new challenges in the communication of mathematics to different audiences, from pupils to teachers, requiring innovative ideas and interdisciplinary collaboration. Some countries such as Singapore, USA and Australia are now making serious attempts to redefine mathematics curriculum in the context of new demand created by a technology-pervasive society. But when I look back now, I feel that the negative image of mathematics has not changed much. Most people and even many mathematics teachers in primary and high schools think that mathematics is a collection of rules, formulas, rituals, and routines. There is enough literature which indicates that many students, parents, and teachers have negative attitudes, myths and phobia about mathematics.

There are some core questions that teachers should ask themselves and from which they may start thinking about how a good communication could improve their way to teach:

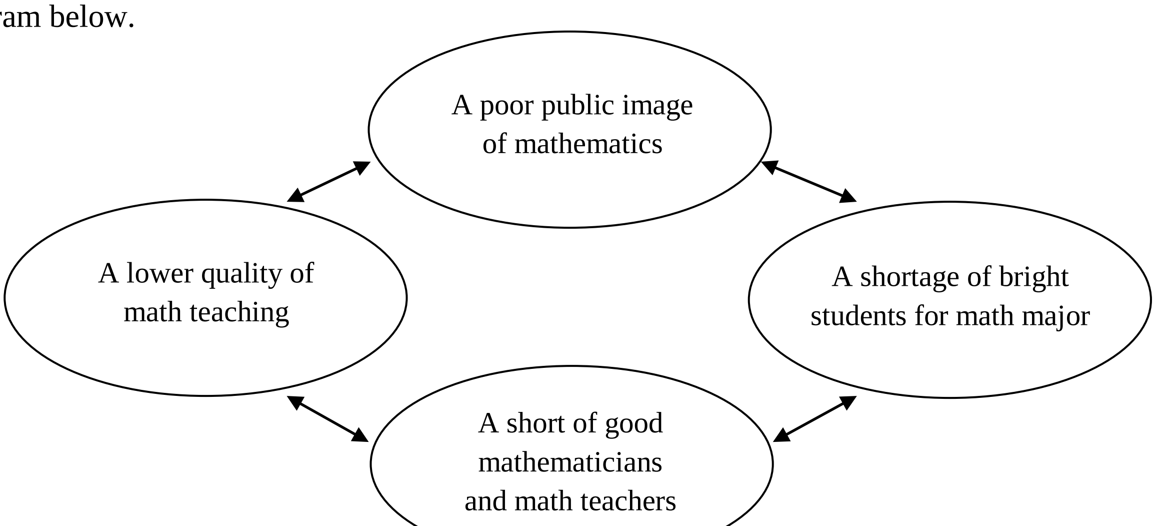
- How could we change people's negative attitudes and myths about mathematics?
- What could we do to arrest the feeling of discouragement and boredom in doing mathematics?
- Could we provide more fun with mathematics and where?
- How could we transmit to students that mathematicians have interests beyond mathematics as well?
- How could we make teaching and learning of mathematics more meaningful, exciting and effective?
- What could we do to popularize mathematics and combat mathematics anxiety among our students?

[79] Steen L. (1990), *Mathematical News That's Fit to Print*. In: Howson A.G., Kahane J.P. (eds), *The popularization of mathematics*, Cambridge University Press, ICMI Study #5, p.194-206.

Two possible answers to these questions are to discuss the need for communicating mathematics in every nation, and describe how mathematics can be popularized before, during and after school education so as to develop a culture of mathematics. There are four components of a "mathematically cultured" person: knowledge of elementary facts and methods, the development of a certain way of thinking and of approaching problems, some knowledge of the history of mathematical concepts and theories, and some knowledge of recent developments in mathematical sciences. An additional component concerns interdisciplinarity, or as mathematics is closely linked to other subjects studied at school, to how mathematics intersects with sciences, but also with literature, philosophy, history and so on. Too often, mathematics seems totally isolated and unrelated to the other subjects both in terms of content and approaches to learning. The school curriculum should not be implemented within the departments, but be interdisciplinary by involving other teachers. Unfortunately, most schools make attempts to prepare their students for only the first of them. So, other objectives, essential for developing a mathematical culture, have to be developed elsewhere.

Before we establish a need to communicate mathematics while teaching, it is important to first acknowledge some well-established facts. First of all, everybody should be mathematically literate in this information era. In other words, "mathematics for modern citizens". This means a meaningful, effective and relevant mathematics education for all. Then, mathematics should be considered as a way of thinking, more than just a collection of formulas. This means how to formulate a problem, how to look for a solution, how to convince or demonstrate. Next, mathematics incubates among people the values which are essential for the development of the personality and attributes which form the basis of a strong and mindful citizen. Lastly, all countries, developed or developing, need more students in mathematics as well as in science. To achieve this aim, all players involved should enjoy mathematics. We have to be clear in our minds that children will enjoy mathematics if their parents appreciate the power and importance of the discipline, students will enjoy mathematics if their teachers also enjoy the subject and help them to understand the power and value of mathematics by developing in them positive attitudes towards it, and the teachers will enjoy teaching mathematics if they are familiar with current developments in mathematics because otherwise it may be difficult for them to popularize the subject in the schools.

A negative public image of mathematics has serious implications on teaching and learning of mathematics. Consequently, a bad public image sets a kind of vicious cycle as illustrated in the diagram below.



The purpose of popularization of mathematics is to break this vicious circle. Therefore, communicating mathematics plays an key role in teaching and we must create synergy between popularization and teaching. It is a slow process, but essential to be implemented. Some misconceptions, supplied from the socio-cultural environment, contribute to cause difficulties to most of the students. Media also deal with mathematics inadequately, and too often exhibit behaviors that outrage the scientific method and the essential values conveyed by science: failure to listen to opinions of others, inability of maintaining a peaceful and constructive debate, use of consensus-building arguments, constant provocations and search for amazement at any cost rather than search for true, shallowness in dealing with complex themes, and dishonesty with which this complexity is addressed and resolved. So, every teacher has a great responsibility in communicating mathematics. Among the objectives of popularizing and teaching mathematics, we should include that the public should be made aware that, to be functionally literate in the 21st century, everybody must be able to read, write and do basic mathematics up to a certain level of competency. We have to raise awareness that mathematics is useful in everyday life and every child can and must master basic mathematics. Moreover, we should help students to reduce maths anxiety, maths phobia, and maths avoidance, and let them do new positive mathematics experience. Then, we should help mathematics teachers to appreciate more the value and power of mathematics and be aware of recent developments in mathematical sciences.

But nevertheless, schools and a large percentage of teachers are still reluctant to address a serious discourse on mathematics communication[80]. Different kinds of language coexist in the classroom: the formal language of mathematics, written and oral declarative languages of teacher and student, etc., and the language used to communicate between teacher and student, and among students. Maths teachers can afford to change continuosly their own speech pattern because they have clear in mind the mathematical object. Instead, student often risk to confuse abstract concepts with their representations. This is the most common educational mistake when certain topics are proposed and adopted without any dialogue[81]. That's why, in my opinion, it is increasingly necessary to communicate mathematics not only in both traditional directions (teacher that explains to the students and student that answers to teachers' questions), but in a new direction of dialogue between equals. Students have to talk to each other dealing with communication of mathematics: they should write about mathematics as well as they do with literature by incorporating drawings and diagrams spontaneously as effective as possible. The use of appropriate communicative strategies between equals to exchange messages and information serves in particular to reduce problems of misunderstanding between mathematical objects and their possible representations. Therefore, communication of mathematics comes with its dual role: a goal to be achieved as an essential component of teaching/learning process on one side, and then an effective tool to remove or reduce as much as possible problems of misunderstanding.

If the students are involved with mathematics, get excited by what they find out, enjoy themselves in pursuing their study and communicate mathematics to others, then it will follow that mathematics will be popularized. Also, if the students go home excited, talking about what they are doing in their schools, then so on, the image of mathematics will improve. Pyne, Bates and Turner have found that

[80] Radford L., Demers S. (2006), *Comunicazione e apprendimento. Riferimenti concettuali e pratici per le ore di matematica*. Bologna, Pitagora.

[81] D'Amore B., Pinilla M.I.F. (2007), *Modi di comunicare*. La vita scolastica, vol.61, n.16, p.20-22.

it is possible to change people's negative attitudes to mathematics if the teacher uses an investigative approach as teaching style, organizes continuous assessment, and provides support to students in groups or individually[82]. One of the most meaningful proposals is mathematics laboratory: a place where students can learn and explore various mathematical concepts and verify different mathematical facts and theories using varieties of activities and material. The use of mathematics laboratory helps to integrate theory and practical work in mathematics teaching/learning. There are various advantages of using mathematics laboratory which include display mathematical information, avenue for experimentation through practical work, pool of storage of mathematical materials for easy access, and removing abstractness and increasing effective teaching /learning. In Chapter 5, for example, I present in detail a project I run with school teachers and students where I investigated through a first-hand experience the extent to which the use of mathematics laboratory enhance the achievement of mathematics students, and effectiveness of combining this method with whole-person learning approach[83].

3.9 Tips for mathematics communication

To conclude this overview on science and mathematics communication, I find it helpful to complete it with some practical tips that a science communicator has always to bear in mind obtained from mixing basic principles proposed in the professional literature and my own personal experience. In this section, I use the term “the public” to express equally audience, readers, visitors and all the other who benefit from science communication.

All stages of the communication process are important and very rarely there is just one way to talk about a topic. Before starting to plan a communication activity, whatever it is, we have to reflect on some essential elements: objectives, public and the message we’d like to convey (see also Appendix A). Only later we will be able to set up spaces, time, resources, media and environment. So, the first question to be asked should be “why I do this”. By defining our objectives, we understand who we talk to and what change we’re going to get from them.

If communication is a two-way process, we have to know very well what we are up against. A master communicator, as well as a master teacher, must know the public in advance and doesn’t postulate anything. This is one of the first rules in communication. We have to be informed about public’s difficulties with the subject, address to each person without forgetting anyone and avoid to lose them on the way. Can’t communicate with the public is worse than not to be understood. The public is highly heterogeneous characterized by different levels of complexity. So, every public needs a message, a medium and a language “ad hoc”. To achieve this goal, we need to know as much as we can about our public: gender, age, socio-economic and cultural background, interests, needs, reasons, expectations and so on. The more we know, the more our communication will be effective and powerful. Once we know the public, we need to clarify what we would like to get

[82] Pyne C., Bates V., Turner W. (1995), *Is it Possible to Change People’s Negative Attitudes to Mathematics?* Mathematics Teaching, vol. 15, p. 8-10.
[83] Yorks L., Kasl E. (2002), *Towards a Theory and Practice for Whole-person Learning: Reconceptualizing Experience and the Role of Affect*. Adult Education Quarterly, vol. 52, n.3, p.176-192.

from it. Communication targets fall within one or more of these categories: information transfer, awareness, dialogue and persuasion. We must be realistic and not give in to the temptation of trying to explain everything. Unless we are writing a book, it's good to be focused on a single topic.

The public generally has no incentive to pay particular attention to what is told. We have to capture public's attention, otherwise every effort is useless. Be able to communicate means be able to transform what I'd like to say in what the public wants to know. A basic rule is choosing a topic which is both scientifically significant and interesting, challenging and intriguing at once, appropriate to convey a strong message, connected with public interest themes and everyday life. So, the next step consists of identifying emotions on which to prey. Often the validity of the argument is not enough. It becomes crucial to make the topic more appealing, spectacular and amazing where possible (but not only, and not too much). It is important not only to challenge our audience, but also challenge ourselves when developing new science communication activities. This could include appropriate use of "emotional hooks" that are likely to be of interest to our audience, for example *"novelty, incongruity, curiosity, humour, imagination, choice, control, empowerment, involvement, challenge, complexity, comprehension, social interaction and relevance"*[84]. It is important to reach the public affectively as well as cognitively if you truly want to engage them; likewise, you will get more out of the activity if you feel an emotional attachment to being involved. Our enthusiasm is infectious. The public will be more likely to become interested in a topic if they see that we are passionate about it. Remember that science communication is a two-way process where we should be learning and engaging as much as members of the public.These emotional hooks could be delivered through a slightly unusual format, a new demonstration or discussion topic, or involvement of other external people. In any case, if we bring our own creativity to the situation, then, the possibilities are endless.

The human mind seems to be made for building narratives that have always been our most natural way of knowing. Mental images, created by telling stories, can be valuable cognitive background. Thus, regardless of the media, format, purpose and content, communicating science or mathematics to the public means being able to turn it into a story. But communication requires the use of shared and common languages and to give meaning and relevance to a scientific argument. I think that there are some key points on which reflect to make the story more effective. Symbolic pictures inside a story, for instance, could give to the public an extra-mathematical fil-rouge that helps them to recover a piece of reasoning even in difficult stages. Repetition of a concept is a technique that serves to fix the meaning of a concept, determine its uniqueness, and can effectively substitute the rigour of a definition. A further opportunity is to recur to similarities. In order to be effective, a similarity has to refer back to concepts within the context or so classic that we are confident that the public already knew them. Then, a workaround for vertical communication could be that of passing judgments. This practice, absolutely unacquainted with horizontal communication, can be a strong tool to point out where to keep attention alive. Exaggeration of certain aspects could help to focus public attention on major hubs. Testament to this is the fact that talking about mathematics to several kinds of audiences, from seventh graders to senior citizens, contrary to common belief, I noticed that people do appreciate even abstract proofs and formulas whenever they are presented in an appropriate context. It seems to me that one of the best ways to do so is to slightly follow the

[84] McCrory P. (2010), *Developing Interest in Science Through Emotional Engagement*. In Harlen, W. (Ed) *ASE Guide to Primary Science Education* – New Edition. Association for Science Education, P.97.

historical development of the particular subject one is talking about. But, there's a risk. There could be the common tendency to oversimplify history, in part out of laziness, and consequently to grotesquely deform the evolution of some mathematical ideas.

Finally, whilst it is important to "take ownership" of any science communication activity that we deliver, be aware that other practitioners may have been involved in delivering similar events for many years. Just as we wouldn't publish a scientific paper without first seeing what had been published previously, it is worth exploring what learning has already been achieved in the area of science and maths communication that we are interested in. Moreover, in line with learning from others' experiences, it is crucial to reflect on and evaluate our own activities throughout the process of delivering them. This should include both monitoring (measuring how many people attended, plus basic quantifiable aspects of your event) as well as impacts (what did the participants – public groups, scientists, event organisers or public in general – get out of it).

Chapter 4

Public Engagement and Outreach

Terms such as *outreach* and *public engagement* are very familiar for persons working in science and mathematics communication. Science outreach is an umbrella term for a variety of activities by research institutes, universities and institutions such as science museums, aimed at promoting public awareness and understanding of science and making informal contributions to science education. Whilst public engagement describes intentional and meaningful interactions that provide opportunities for mutual learning between scientists and member of the public. It is more than just meeting an audience and telling them about your research and field of study. Mutual learning refers not just to the acquisition of knowledge, but also to increased familiarity with a breadth of perspectives, frames and worldviews. In the case of young people, public engagement is an effective way of stimulating interest in a subject and encouraging young people to consider research careers and to become more skilled and engaged citizens. In addition to these considerations, goals for science outreach and public engagement with science and mathematics include civil engagement skills and empowerment, increased awareness of the cultural relevance of science and maths, and recognition of the importance of multiple perspectives and domains of knowledge to scientific endeavors.

Successful outreach actvities and excellent public engagement with science and mathematics builds on a foundation of clear, concise communication. When communicating with public audiences, however, we need to shift to the way we would speak when "off the clock". Typical scientific communication includes a lenghty background, and then describes the methods and process used with great precision. In a paper, for example, it is not until the very end that the results or conclusions are reported. Public communication flips this approach on its head: the bottom-line is the lead, followed quickly by the "so what", and then the supporting details, as appropriate. The situation is highlighted in Fig. 4.1.

In a crowded communication environment, most people don't need to read or listen to content that is not immediately compelling with them. Being proactive about the way we phrase our key ideas to speak directly to an audience's interest, or even embedding a discussion of science in places where less motivated audiences might discover them accidentally, will help us more deeply engage with the audience, which will likely increase the odds of achieving our engagement goal.

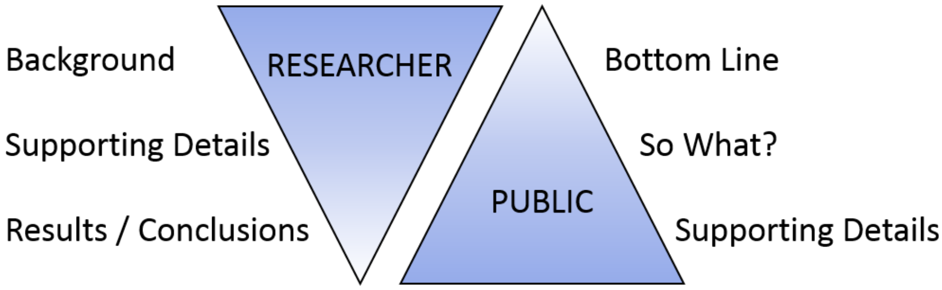


Fig. 4.1 - Scientists and the public have different communication styles. While scientists often start by placing a particular topic or their research in a historical context, the public wants to know the key point at the start. Photo from https://www.aaas.org/page/communicating-engage

With specific regard to communicating mathematics, the constructs of engagement and motivation are often used together and are very much connected. However, they are different. When we use the term "motivation", we refer to the ways in which people, and students in particular, choose to behave, their self-confidence in their ability, and their ability to overcome obstacles and challenges. People's motivations determine whether or not someone will engage in a particular pursuit and whether or not those motivations exist as part of one's belief about what is important[85]. Engagement, in its broadest sense, can be defined as a multi-faceted construct which operates at three levels: cognitive, affective and behavioral. Cognitive engagement involves the idea of recognition of the value of what I can learn and a willingness to go beyond the minimum requirements. Affective engagement includes people reactions to the proposal and the environment, influencing their willingness to become involved in. Finally, behavioral engagement encompasses the idea of active participation and involvement in the activity. So, when viewed through a mathematical lens, engagement occurs when mathematics is a subject people enjoy involving in, people see its relevance in their own lives now and in the future, see connections with their interests outside that specific context.

Taking into consideration the three main forms of media used in science communication (traditional journalism, live or face-to-face events, on-line interactions) and the will to learn more about science outreach and public engagement, I focused my attention on live or face-to-face events almost entirely. Particularly in my research project, I studied, first-hand experienced and analyzed public engagement in science festivals, science cafes and math busking, and subsequently I developed maths communication projects.

4.1 Science festivals

Science festivals are a relatively new type of dialogue-based engagement that appear to be rapidly growing across Europe. Festivals generally bring together temporary exhibits, museum-type activities, scientists, art and public organizations, students, and member of the general public, often attracting thousands of visitors. They are a reference point for science popularization by now and a great opportunity to meet researchers, passionate people, schools and families, but also to bring

[85] Eccles J.S., Wigfield A. (2002), *Motivational Beliefs, Values and Goals*. Annual Review of Psycology, 53 (1), p. 109-132.

back the science in society. Furthermore, science festivals are the biggest events for a dissemination of scientific culture. Within them, we can tell stories about mystery, discoveries, success, failures, experiments, dilemmas and wonders of the scientific world. We can also think about programs suited to any age group or level of knowledge in order to stimulate the interest of people through exhibitions, workshops, interactive educational modules, talks, round table discussions, theatre and music performances, films and so on.

To provide an in-depth understanding of this form of science communication, I spent an intensive and fruitful two week visiting period at the University of Bath in the United Kingdom from 8 to 23 March 2016, to work side by side with Chris Budd, one the best worldwide experts in mathematics engagement and outreach activities, and his communication team in preparing for and running 2016 Bath Taps into Science[86] Festival in Bath, and taking part into the Big Bang Fair[87] in Birmingham at the Institute of Mathematics and its Applications (IMA) Stand together with John Meeson and PhD mathematics students from all over the country. There have been precious occasions to train my maths communication skills and be familiar with a huge and professional science communication framework such as the UK science and mathematics community that is at the forefront in the field of public engagement with science and technology.

4.1.1 Bath Taps into Science

Bath Taps into Science is a week-long science festival which takes place in Bath. With about 60 events over 7 days during British Science Week, the festival reached over 8.300 people across schools and the local community. The aims of Bath Taps into Science are:

- work in conjunction with schools and colleges to contextualise learning of their students through STEM (Science, Technology, Engineering and Maths);
- engage and stimulate the general public with STEM;
- promote the STEM undertaken across Bath;
- develop links across the wider STEM community;
- provide an opportunity for researchers to show the impact of their research to the wider community;
- develop University of Bath undergraduate and postgraduate students' Science Communication skills.

In the months before the festival, 46 events are run for schools across the region ranging from workshops in schools to a large science fair which takes place in the last day of the festival week. Workshops are offered by University of Bath researchers and students and included STEM topics. The Schools Science Fair is run at the University of Bath. The venue allowed them to have 42 exhibition stands from the University, local companies and 8 secondary schools. Each school had 2

[86] See Ch.1, ref. *Communicating Mathematics in Europe. Episode 1: Chris Budd*.

[87] The Big Bang UK Young Scientists and Engineers Fair is the United Kingdom's largest celebration of STEM (science, technology, engineering and maths) for young people and is the largest youth event in the UK. The fair takes place annually in March. It is led by EngineeringUK in partnership with over 200 organizations across goverment, industry, education and the wider science and engineering community. Since 2012, the Big Bang Fair was hosted in the National Exhibition Centre in Birmingham.

hours in the fair interacting with an average of 9 stands. During the festival week, 14 events are run for the public (6 just for adults). Events included the ever popular Family Talk, which every year is given by an awarded scientist to a ticket reserved audience, lectures, musical performances and a comical theatre production. The festival also hosted a high profile discussion around water quality, part of a NERC funded project across Bath, Bristol and Keynsham. The festival week ended with the Family Science Festival in Victoria Park, Bath and with more space, activities and food/drink on offer. Despite the weather, the fair attracts in the region over 4,000 visitors who visit the 34 interactive stands on offer from the University of Bath and local companies. It is noteworthy that a good 20% of all the activities submitted is math-oriented unlike other festivals. That’s probably because of the active and valuable presence of Chris Budd and his communication group within the organization of the festival.

Many of the presenters at the Festival were from the University of Bath. In collaboration with the Public Engagement Office at the University they run a session for those wanting to take part in the festival for the first time and another drop-in session to support researchers with ideas and the mechanics of the exhibitions and logistics of the days. Funding is provided through a number of sources including sponsorship from local companies. The University of Bath provided most of the funding paying for staff to coordinate and run the festival.

4.1.2 Big Bang Fair

The Big Bang UK Young Scientists & Engineers Fair is the largest celebration of science, technology, engineering and maths (STEM) for young people in the UK. It is an award-winning combination of exciting theatre shows, interactive workshops, exhibits and careers information from STEM professionals. The main aim of the festival is to show young people (primarily aged 7-19) the exciting and rewarding opportunities out there for them with the right experience and qualifications, by bringing classroom learning to life. Having grown from 6,500 visitors in its first year (2009) to over 70,000 in 2017, The Big Bang Fair is made possible thanks to the collaborative efforts of over 200 organisations. The Fair also hosts national competitions, inviting young people from across the UK to compete, amongst other things, for the title of UK Young Scientist and UK Young Engineer of the Year. The Fair gives young people the chance to hear from inspiring engineers and scientists from some of the UK’s biggest companies and find out about the opportunities available in science and engineering.

Within this huge fair, there is the Public Engagement IMA Stand with Mathematics. It looks like Cindarella compared to the sponsor stands (engineering and science stands loom large), but it is the most visited stand in the last three years. All the outreach activities presented are coordinated by John Meeson. The aim of IMA Stand is to engage the curiosity of the visitors in simple activities that explained mathematical principles or utilised mathematical skills in an unthreatening and enjoyable way that challenged the stereotypical views that unfortunately often adhere to mathematics. Prof. Meeson is ably assisted by a number of PhD students who find that they enjoy the experience of maths outreach. The IMA team bewitch visitors with card tricks, maths puzzles and eight large hands-on activities of the Large Maths Outreach and Carrers Kit: Double Pendulum, Galton Board, Harmonograph, Soap Bubbles Wires (Fig. 4.2), Travelling Salesman Problem, Trebuchet, Aerodynamics and The Penrose Tiles. The kit was developed by the Institute of

Mathematics and its Applications as part of the National HE STEM programme and created in response to request for “hands on” activities for use in schools outreach events, careers fairs and science days.

Fig. 4.2 – Visitors and me struggling with the Soap Bubble Wires at the IMA Stand, Big Bang Fair Birmingham

4.1.3 What I learned

My first-hand experience in IMA Stand activities at the Big Bang Fair and my full involvement to all organizational stages of Bath Taps into Science on the side of Chris Budd enabled me to examine more closely the dynamics behind effective public engagement approaches and outreach activities, to understand better the problems hidden in maths communication to the public, and to evaluate what works and what does not work depending on actors and factors at play.

The fact that everything happens in front of the visitor and everything can be explained is the first thing I clearly noticed. There are no filters between the communicator and the public. Our first goal is to hook the public, then to entertain them. This is the first step towards being in a position to achieve success. Once we do that, we can start explaining what is the mathematics behind activities they’re doing. Public engagement is different from written communication in a number of ways. The most important is that there is a much larger role for the initial engagement of individuals. When communicating directly to the public, we must convince a grandparent, a child and her/his parents, a group of teenagers, in a very short amount of time, to pay attention to us versus whatever else is going on around them. Thus, in my opinion, grouping the mathematics exhibits and activities together to explain simple principles in an engaging way encourages and challenges intuition. In such a way, the messages don’t suffer from distraction: engagement and piqued curiosity succeed in challenging preconceived ideas. It is also undoubtedly true that seeing mathematics in action helps keeping it alive in the minds of people.

Another important matter in public engagement is that we have to meet people where they are. During the festivals, members of the public with whom we engaged in conversations, or with math puzzles, games and activities, arrive with many different levels of scientific knowledge. It was important to realize that and begin the conversation at their level. Most people find mathematics impossible as a topic for conversation and some will even resent it strongly. On the other hand, there are also several people who are prepared to engage in mathematical conversations if we can make them short and interesting. That renews my belief that a substantial part of the public in most countries would be interested under the right circumstances. Many kinds of toys, games and puzzles touch the same mental processes as mathematics, like imagination, intuition, systemization of experiences, combination of information, etc. Moreover, rational decisions in daily life and society at large are also indirectly using mental processes related to mathematics, like analyzing arguments, making hypotheses, testing their validity, reaching conclusions and finally making forecasts. If we can convince people that abilities they do not think of as related specifically to mathematics are among the most basic ideas pursued in mathematics, they will not immediately be frightened away from having a mathematical conversation. Indeed, the goal of public engagement is not to teach terminology, but to create a better-educated public that can understand the importance of mathematics. Then, if they also learn something from what we've explained to them, all the better.

A further very important point in drawing up outreach activities is to take into account the different learning styles people have. Studies of how people learn show that active-learning environments involving problem-solving discussions with peers are more effective than traditional lectures. Learning styles have been extended in the context of interactive science centers and festivals to encompass the theory of multiple intelligences[88]. According to this theory, there are seven basic intelligences which have been detected and which need to be addressed by, say, an interactive exhibit if it is so appeal to as wide an audience as possible. These are: linguistic intelligence, logical-mathematical intelligence (this incorporates scientific ability), spatial intelligence, musical intelligence, bodily-kinestetic intelligence (this is the ability to use the body to solve problems or to fashion products), interpersonal intelligence (the ability to understand and work with others), intrapersonal intelligence (the ability to form an accurate model of oneself and use it to operate effectively). These "intellingences", according to Gardner, are distinct. For any individual, one type will have more appeal than another depending on the degree to which that particular intelligence is developed. For effective learning to occur, relevant experiences embodying that intelligence must be provided for. Clearly, interactive exhibits offer a special opportunity to cater for multiple intelligences. The theory of constructivism embodies the essence of all successful communication – listening to and understanding the audience. Whether that audience be physically present or at a distance, ignoring the nature of the audience and how they think will result in failure to communicate. Everything about the ways in which science concepts are presented taps into constructivist theory: problems with relevance, with style, with language.

4.2 How I put into practice

4.2.1 FermHAmente Science Festival

In February 2017, I organized for the city of Fermo (Italy) the "FermHAmente" Science Festival. The project started essentially with two complementary goals: to enhance the interaction with schools and their teachers through the development of scientific material about mathematics and

[88] Gardner H. (1983), *Frames of Mind: The Theory of Multiple Intelligences*. Basic Books Inc. Publishers.

science in general through an innovative approach and the promotion of public awareness activities related to science and mathematics. In this public engagement effort, I collaborated with a couple of professionals in communication like Marco Brusati[89] and Mauro Labellarte[90] that helped me to support and train scientists and all the staff members for their engagement activities. As scientific director, with the support of Prof. Mara Felici[91], I focused my attention on scientific content and the quality of submitted public engagement activities. This required a careful work and a direct and effective interaction with the Universities of Urbino, Ancona, Macerata and Camerino, Fermo's high schools and all the other partners among which the "Pergolesi" Conservatory of Music of Fermo. FermHAmente has held exhibitions (6), educational programs (18) and special events (36), including scientific conferences (8), theatrical performances (2), concerts (2), workshops (24) among which many mathematical activities. The activities carried out was addressed to children and young people, schools (on Friday and Saturday morning) and families (on Saturday afternoon and Sunday). The total number of visitors was about ten thousands. During the festival, every post in our FermHAmente Facebook page reached 1.200/1300 people a day and, at the moment, has 1350 active followers.

In implementing the festival, we tried to mix a curated programme of thought-provoking and attractive events with activities of researchers from University, public and private societies, schools and other partners, always looking for high quality in communication and in underlying research, innovation and openness to discussion with the public. Fermo is a compact city with a beautiful historic centre, so we decided to develop five thematic itineraries (Science & Technology, Science & Cooking, Science & Travel, Science & Art, Science & Invisible) throughout the city, placed in 21 different locations. Within these itineraries, we preferred "staffed exhibits" where researchers together with students manage and explain interactive and handling objects for people to stop and explore a topic. These kinds of stands can engage children at table-top height while adults can discuss with researchers and perhaps look at adjacent visual materials. Laptops, games or equipment can be on the stand for visitors to try out a task, and many other activities constrained only by public's creativity, resource and, of course, the obligatory risk assessment. Many adults enjoy learning in a more playful style too. Furthermore, we included interdisciplinary panel discussions where audience was able to work through several views related to grand global challenges where science and technology have a role to play through researchers' perspectives from natural science to politics, economics, philosophy, mathematics, etc. Finally, we also offered informal formats in cafes and unexpected places like gardens or churches.

A fundamental aspect I personally took care of was the evaluation of the activities. The most difficult part in devising a public engagement activity is making a plan and manage time without forgetting economic aspects. Evaluation sounds less interesting than devising the activity, also because self-evaluation can be hard. But it's the only way that enables us to see whether we've achieved what we set out to do. With regards to festival contributors, there are many factors, like

[89] Since 2004, Marco Brusati is lecturer on Master's Degree Course in Communication at the University "La Sapienza" of Rome first, and now at the University of Florence. He is also designer and director of national and international events.

[90] Mauro Labellarte is producer of conferences, live shows and television events. He is also an instructor in the music business.

[91] Mara Felici is a teacher at Ipsia "Ricci" in Fermo.

ambition, aspiration, fear, ego, expectation, experience, inexperience and pressure, that can be good or bad things for good science communication and public engagement. Motivation is main key: if someone wants to do something, has energy, a spark of creativity and a willingness to share ideas on equal terms, we're on to a winner. It is very important to become engagement professionals to explore the "why" questions: for example, "Why am I doing the engagement?", "How does it link to my research?". This works best when the time and the willingness to share are there and when people are open to collaborating with us as a "critical friend". Then, to have a complete evaluation, it is essential public's feedbacks. We got the information we need through two or three minutes long interviews made by our school students staff during or at the end of a conference or workshop or at the location entrances (see Appendix B).

In the weeks before the festival, I and Marco Brusati gave lectures to the participating school students on the key elements necessary for planning and presenting a public engagement activity:

- to ensure that public engagement activities have a strong link to current and recent research or an overriding theme of STEM disciplines;
- to always consider two-way engagement and interaction with the public;
- to provide a chance for the organizers and the participants to explore scientific and mathematical issues via multiple lenses and perspectives;
- to have a great knowledge of the argument;
- to study all the related topics to the activity that we're going to present;
- to understand the relation to school science and mathematics;
- to figure out what the interplay is with the wider culture;
- to be clear why I am engaging with the public, both in my mind and in my communication with potential participants (do not raise unrealistic expectations amongst partecipants);
- to allow enough time to plan public engagement thoroughly, whether it is a small, one-off, event or a sustained programme;
- to carefully consider my engagement strategy: who I wish to engage, why and how, their interests and why they might be interested in, what the best approach is related to the context;
- to use my preferred communication styles and skills in the area of public engagement (my activities will be far successful if I am comfortable delivering them);
- to identify potential pitfalls;
- to think about my public engagement role as one that is ongoing (this will allow me to connect my activities, build my expertise and develop a rapport with the groups I'm engaging with;
- to schedule an evaluation at the start of the public engagement shared with other colleagues (if possible, make my evaluation reports publicly available so that others can benefit from my experience);
- to define and assess the impact of activities;
- to prepare a list of materials and the costs of my public engagement activity (insufficient funds could jeopardize my success on that activity).

Even in presenting a public lecture, one of the most familiar forms of science communication, these key elements are useful. The traditional format is very similar to a normal taught class or lecture, with a single speaker presenting on a specific topic, and the audience listening. Use of question and answer sessions at the end of the presentation can allow for some interaction between the speaker and the participants, however generally public lectures are considered a ‘transmission’ form of communication. We can overcome this traditional format by means of more importance being attached to the inclusion of ‘interactive’ elements within public lectures, in order to ensure greater involvement of the audience. For example, a more recent innovation along these lines uses electronic voting handsets to allow the audience to “choose their own lecture” – i.e. select from a range of options offered by the presenters, in order to allow the audience to take more control over the direction of the lecture[92].

4.2.2 A proposal for a Maths/Science Communication Course

Scientific advances are coming at an unprecedented pace, and they hold great promise for further improving the human condition. At the same time, however, society is exhibiting increased disaffection, fostered by instances of scientific fraud and by scientists charged with financial conflicts of interest. As remarked above, scientists and mathematicians must engage more fully with the public about scientific issues and the concerns that society has about them. Efforts that focus simply on increasing public understanding of science are not enough, because the problem is not merely a lack of scientific comprehension. In some cases, the public generally does understand scientific content in a fundamental way but still doesn't like it. Thus, the notion of public engagement goes beyond public education.

Effective public engagement requires long-term commitment, because many issues are complex. It would be convenient to leave this task in the hands of a few representatives selected especially for their communication skills, but that wouldn't work. Engaging the public effectively is an acquired skill, and preparation for outreach strategies has seldom been part of scientific training programs. Moreover, many scientists and mathematicians even feel that the culture of science actively discourages them from becoming involved in public outreach, because it would somehow be bad for their careers. But fortunately, something is changing.

First, the scientific reward system needs to support our colleagues' efforts to interact with the general public concerning their work and its implications. Funding agencies in the US and Europe have begun encouraging the scientists they support to include outreach efforts in their proposals. Academic institutions need to join in this chorus by rewarding faculty members who fulfill commitments to such work. That will entail putting public outreach efforts among the metrics used to decide promotion and tenure.

Second, university science departments should design specific programs to train graduate students and postdoctoral fellows in public communication. Unfortunately, this means adding yet another element to already overtaxed research training programs. Many students acquire teaching experience through assistantships, but public engagement activities are different and require other

[92] Bultitude K., Grant L. (2006), *2005 Institute of Physics Schools and Colleges Lecture Tour*. Graphic Science Unit.

strategies. We need to add media and communications training to the scientific training agenda[93]. This will doubtless be an additional burden on existing systems. If science and mathematics is going to fully serve its societal mission in the future, we need to both encourage and equip the next generation of scientists to effectively engage with the broader society in which we work and live.

Within this landscape, Silvia Benvenuti[94] (University of Camerino) and I presented a workshop entitled *Resource collection for a curriculum of a University maths/science communication course* at *Imaginary International Conference* held in Berlin in July 2016. The purpose of the workshop was to collect materials and ideas suitable to provide any student from a "scientific" degree course with the basic tools to communicate in matters related to his discipline to an audience of non-experts. The workshop was mainly targeted at an audience of researchers, teachers and science communicators. Our work team (Fig. 4.3) was composed by Gang Liu[95], Andreas Daniel Matt[96], Janine McIntosh[97], Hyungju Park[98], Jean-Paul Truc[99], Costanza Rojas-Molina[100], Agata A. Timón Garcia-Longoria[101] and, just for one day, Cedric Villani[102] (Fig. 4.4). Together we developed an initial draft of a syllabus by integrating our different points of views, expertise, cultural and mathematical backgrounds.

Fig. 4.3 – From left to right: Andrea Capozucca, Silvia Benvenuti, Agata A. Timón Garcia-Longoria, Janine McIntosh, Costanza Rojas-Molina, Hyungju Park, Andreas Daniel Matt, Jean-Paul Truc and Gang Liu

Fig. 4.4 – Cedric Villani and me

[93] Leshner A.I. (2007), *Outreach Training Needed*. Science, Vol. 315, Issue 5809, p. 161.

[94] Reseracher in Geometry at the University of Camerino (Italy), expert science communicator and member of the Raising Public Awareness Committee of the European Mathematical Society. She boasts a number of collaborations with television science show "Geo&Geo" and is also author of the book "Insalate di matematica" (Sironi, 2010).

[95] Associate professor, lecturer and Acting/Deputy Head for Mathematical Science at Xi'an Jiaotong Liverpool University in Suzhou (China).

[96] Mathematics communicator at the Mathematisches Forschunginstitut Oberwolfach (Germany), coordinator of Imaginary project and member of the Raising Public Awareness Commitee of the European Mathematical Society.

[97] Program Manager (Schools) at Australian Mathematical Science Institute in Melbourne (Australia).

[98] Professor at the Pohang University of Science and Technology (POSTECH), director of Center for Applications of Mathematical Principles (CAMP) and director of South Korea National Institute for Mathematical Sciences (NIMS). He is also a member of IMU Executive Commitee.

[99] Editor-in-chief of *Quadrature* mathematics magazine.

[100] Post-doctoral researcher at Institute for Applied Mathematics in Bonn (Germany).

[101] Coordinator of Communication and Popularization at Instituto de Ciencias Matemáticas in Madrid (Spain).

[102] 2010 Field Medal, professor at the University of Lyon and director of Institut Henri Poincaré in Paris (France).

The program of the workshop was divided into three stages : meet and greet (each of us introduced herself/himself and the field she/he was involved in to the others), learn and share (after an historical overview of science communication, we examined and analyzed different science communication programs taken from all over the world to draw up an assessment grid), collaborate and create (after a deep discussion and comparison of the day before assessments, we proposed a first syllabus for a maths/science communication course). The results are summarized in Table 4.1.

WHY	Students of science are very often called to explain the issues they care about to people who do not have specific knowledge of science: family, friends, colleagues who have chosen other paths. On the other hand, the training of science graduates usually not provide any course of communication techniques.
WHO	Academics supported by a team of experts
TO WHOM	Science and mathematics students from third year and above.
HOW	A semester long course with skill based modules in related topics. A face-to-face interactive course which combines theoretical and practical aspects.
WHERE	Some classroom lessons. Students will also be given the opportunity to work in groups on real tasks outside of the classroom.

WHAT	**Core modules**
	• A short history of science communication
	• Key elements of science communication plan
	  o Know and engage the audience
	• Start up a project
	  o Working with stakeholders
	  o Writing proposals / fundraising
	  o Project management
	• Journalism (basic writing skills)
	• Oral presentation skills
	Elective modules
	• Social media
	• Television and other visual media
	• Science museums and exhibitions
	• Communicating maths
	• ...

Table 4.1 – First syllabus draft

The work of the group will progress so that the fully revised proposal will be presented during the next Imaginary Conference which will take place in Uruguay in December 2018. All the documents produced and a summary of the conference discussions are open-source and available in the HackPad at https://imaginary.org. Anyone who wants to contribute to the debate, share ideas, propose amendments or extensions, implement our proposal, can do this directly from the website.

4.2.3 Unicam Science Outreach

Since July 2015, I am included in *Unicam Science Outreach* (USO) research project on science and mathematics communication hosted by School of Science and Technology of Camerino (Italy). The project group aims to reflect on the most effective ways of communicating science, starting from the territory and then, transcending local geographical boundaries, introducing the University of Camerino and the Marche region in a more global context. The relative abundance of mathematicians in this research group combined with the remarkable interdisciplinary approach guaranteed by the variety of backgrounds represented in it, are likely to provide optimal conditions for promoting an efficient cross-talk between the various scientific disciplines. This in turn will facilitate the presentation of a connected and systemic vision of science as a whole to the intended beneficiaries of the communication process to be planned. The complementarity of our skills, ranging from the purely scientific to the philosophical/historical, including the technological ones that allow us to easily handle the new tools for communication, constitute a very strong potential for innovation and research in the topics we address. We have planned to check the effectiveness of the resulting communication strategies using the citizens of Camerino and other nearby cities as a collective test-subject. This would probably be the first attempt since the university foundation[103] at improving or even inaugurating a process of communication such that the university activities might be no longer perceived by citizens as a black box but rather as a glass-box.

Among the goals reached during this two-year project, we may include:

- setting up of a task force aimed at responding to specific calls of Horizon 2020 (especially those whose specific impact is expected in the organization of exhibitions and science cafés engaging citizens in science and innovative ways to make science education and scientific careers attractive to young people);
- creation of a communication format for dissemination;
- creation of a network website;
- implementation of new technologies such as MOOCs, streaming network, digital platforms, digital interactive playground;
- extensive application of new information and communication technologies (ICT) for the dissemination and communication of science, possibly including social networks, as Facebook and Twitter, together with blog platforms;
- adoption and critically testing of the following frontiers technologies: the Massive Open On-line Courses (MOOCs) declined for dissemination, the periodic webinars, and the on-line cooperative work and networking by means of Moodle-based platforms;
- improvement of on-line collaborative works, networking and interactions with the researchers of the other partner Institutions;
- experimentation of non conventional means of communication to arise public debate on issues concerning science, the politics and sociology of science as well as ethical and epistemological aspects;
- report summarizing the extensive scrutiny of the literature (on the present status of science communication and popularization and, in prospect, the study of alternative strategies);
- theatrical events to be used to trigger series of thematic events on specific issues;
- scientific cafes and other public engagement activities.

[103] Founded in 1336, University of Camerino is one of the oldest universities in Europe.

In particular, I'd like to present three meaningful public engagement activities in which I was directly involved together with other group members as designer, promoter, speaker and even scientific entertainer.

4.2.4 VereMath Street

VereMath Street is a public engagement and outreach project held in June 2015 and June 2016 during the Veregra Street Festival[104]. I and Alessandra Renieri (University of Camerino) co-curated and designed the program in both years under the aegis of the City of Montegranaro.

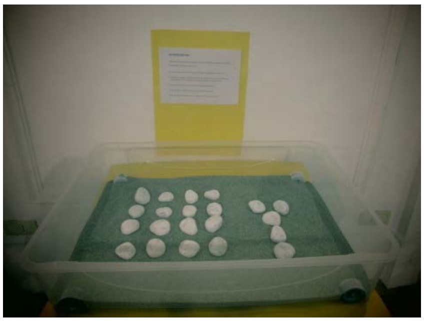
4.2.4.1 The 2015 edition

VereMath Exhibit

The exhibit illustrated the laboratorial-path occurred in March with 6-to-10 years students of the ISC School of Montegranaro.

It is made of four main parts:

- 5 panels on the mathematical basics of the laboratories;
- 5 hands-on areas for experiment those notions;
- 5 areas with photos and the works made by students;
- 1 panel with a story written by some students about this experience.



[104] Veregra Street Festival is a street art festival which takes place in Montegranaro (Italy) every year in the last week of June, since 1998. Theatricals, troubadours, jugglers and many others street performers, coming from all over the world, "invade" Montegranaro historic center for a whole week with wonderful and amazing shows.

Fig. 4.5 – Photos from VereMath Exhibit

VereMath Walk

It was a walk through Montegranaro historic center to discover mathematics around us. The math trail map pointed to places where walkers formulate, discuss and solve interesting mathematical problems linked to details hidden in a monument, in a building or in the surrounding environment.

The walk consisted of 6 different stages.

There was no competition or grading. Just the pleasure of having walked the trail and of having done some interesting mathematics.

Fig. 4.6 – Photos from VereMath Walk

Math Busking Show

The show tried to engage audiences that would not normally seek mathematical knowledge where mathematics is unexpected. It aimed to surprise and amuse street passers by with entertaining routines deep rooted in maths. It is like doing mathematics for public amusement by using the techniques of street entertainers.

The program involved live physics experiments, mathemagic tricks, theatrical engagement with a stand-up comedian, a live interactive band and scientific entertainers to tie it all together.

Fig. 4.7 – Photos from VereMath Busking Show

4.2.4.2 The 2016 edition

VereMath Lab
Alongside the exhibit (illustrating the new laboratorial-path occurred in April with 6-to-10 years students of the ISC School of Montegranaro), we planned 8 laboratories for children based on geometry with paper and geometry with the body. The laboratories changed almost every day on these two main streams.

AperiMathStreet
The 5 evening happenings were about raising public awareness of Mathematics through the voice of 5 expert maths communicator. The mission was to dispel the fear of maths that people pick up at school and to make maths as much a part of culture as art, music or literature. They are not lectures, but places where the expert guest and the audience were able to discuss about the idea that Mathematics is almost everywhere.

These have been experiments about new form of communication where we're able to approach people from scratch, attempts of communicating the wonder of mathematics to people in streets. Most mathematicians find mathematics incredible, but have put in years of study. Thus, the emphasis should be on entertainment and audience participation. I think that in such cases the important thing is to engage everybody, even those who don't want to know more. There's a risk, but it makes the effort worthwhile.

Of course, there could be problems linked to these kind of proposals. Sometimes tricks that seem thrilling in development stage do not work so well in the street, and making the links between the performances and the mathematics behind them can be a challenge. I already know that these things often remain a work in progress. I agree with Chris Budd when he claims that *"math busking should be seen as one of a variety of approaches for engaging people with maths, along with more formal methods such as popular lectures or science fairs"*. It's obviouos that in a performance we never explain mathematics because it's not an outdoor lesson. But in doing the performances, lots of people start shouting out answers or telling the performer how to play and that's when people are doing the math. And while they're doing it, they're actually learning the math. They are working it out themselves. Moreover, the combination of science with humour is beneficial, because enhances learning as well as connection between participants. More recently, innovative initiatives have been developed which are specifically aiming to engage adult audiences through the combination of science with humour.

One of the most significant achievements of VereMath Street format is its success in accessing young adult audiences outside of formal education (in this case aged between 20 and 40), who are often considered to be hard-to-reach through traditional means. The combination with scientific topics provides a unique opportunity to engage such audiences with science and mathematics. There is no overt expectation that audiences will learn anything from the event, although audience members reported being pleasantly surprised that learning does occur. This is clear by some comments of the participants:

"I thought it is really good because you can use your imagination and make up whatever you want. So, you are almost making your own maths tasks"

"I finally understand the meaning of all those strange things I studied at school"

"Me and my friend have a great time. But I can't still believe that we've had fun with mathematics".

"Well, you come to a street art festival and you expect to be entertained and amused, but you don't expect to learn anything! I really liked the mix of science and comedy ... It is a fantastic concept"

An idea on which we are still working together with USO group members is to create and train a communication group with participants taken from undergraduate and graduate students that in the future could become a network of buskers/science communicators who will be able to run science communication shows and events.

4.2.5 Scienza in Vacanza

Scienza in vacanza is a public engagement and outreach project held at Girasole Camping Village in Marina Palmense (Italy) on 24-26 June 2016. Its aim was to promote scientific culture and mathematical thinking through games, labs and evening scientific shows. A project of “smart entertainment” which was a mix of science and fun, talent and creativity, relax and engaging situations. A plot of interactive engagement full of trivia and insights on mathematics, physics, chemistry and robotics suitable for all ages was within the village. A series of workshops run throughout the three day: village guests were free to decide which to take part in according to their own interests and curiosity. A team of scientific entertainers were available for the public to discuss, answer questions or simply play together. All content have been designed to combine dissemination and recreation, thereby facilitating engagement among the partecipants in order to create moments of social gathering.

Data of the event are as follows:

- 5 engagement areas (average attendance: 50 participants)
  - Sunflower Casino – probability in gambling, analysis of certain types of gambling like roulette, high/low game, craps, win for life and Monty Hall problem
  - Winning strategy – free area with games like Pick up sticks, Nim, Bedlam Cube, Guess who, Four-in-a-row
  - Numbers out of a top hat – a real magician performing “mathemagical” shows, one in the morning and one in the afternoon
  - Issues of friction – hands-on activities to discover more closely the laws of friction and two experiments on air resistence and viscosity
  - Robotalent – how to make your own robot
- 8 laboratories (average attendance: 30 participants)
  - Carribean’s robo-pirates – robotics
  - Travelling mathematics – mathematics
  - Braids – mathematics and dance
  - Recyclemath – mathematics
  - Mind’s eyes – mathematics and perception
  - Physics on vacation – physics
  - Does Totti know that? – mathematics
  - Jugglers...you become – mathematics, physics and juggling
- 2 evening shows (average attendance: 150 participants)
  - Girasole under the stars – astronomy
  - Dreamy Physics – physics and juggling
- 1 exhibit “Fool your brain” on perceptual illusions

Total number of people meet during the three days: 750

Partecipation age: from 6 to 65 years old

Fig. 4.8 – Photos during the laboratories

Holiday is always a period of time away from work, study and normal lifestyle. Science and mathematics are words that call to mind sweat, anxiety and hard work. Thus, the great challenge was to bring science into a village where guests were there to relax. The result was astonishing. People sudden realized that one can have fun playing with mathematics, looking to the sky to discover its secrets or simply doing science experiments. At the end of the three days, children were enthusiastic as well as the adults. So excited that I'm working on a 2018 edition of Scienza in Vacanza under pressure of resort manager that will make available for guests a package devoted to the event.

4.2.6 Science Cafes

Two of the milestones inside the Unicam Science Outreach project are the identification of a network of science café where performing and the implementation of a scheme for different types of science café. Our research group set up 15 science cafes to date, many of which took place in Camerino. Something so common on an international scale such as a science café represents a never-seen-before innovation in a small town like Camerino. Moreover, though in its university all scientific disciplines are represented, its academic environment is small enough that is reasonable to conceive that under appropriate promotional actions specialists from various fields could easily be engaged in an open and constructive interdisciplinary cross-dialogue. We strongly believe that the combination of these premises constitutes a strength for our group, which aims to reflect on the most effective ways of communicating science, starting from the territory and then overcoming local geographical boundaries, introducing Camerino and the Marche region in a more global context.

Our science cafes classically start with a short introductory talk by an "expert" to introduce the central issue. On average, this introduction lasts for no more than 20 minutes, and in some cases as few as 10. Some cafes eschew technology that means no slides, microphones or dimmed lights to emphasize a hierarchical divide between the speaker and the audience. After the break comes an hour or so of discussion, questions, comments, thoughts and opinions between the speaker and the audience, the audience and the speaker and the audience and the audience. We don't have to forget that the most important ingredients for a successful café are audience's questions and reflections. Unlike traditional public lectures, the emphasis within science cafes is very much on a symmetrical relationship between the speaker and the audience. This does not necessarily mean that all events must involve high degrees of interactivity, but that it is important to consider whether greater audience involvement may result in higher levels of engagement and learning – both for them and for the presenter. Thus, I can define our science cafés as casual forums that host conversations between scientists and the public about current science topics. They have been generally small in size, taking place in cafés, restaurants, bookstores, or similar venues, always outside a traditional academic context. They have been open to everyone regardless of scientific knowledge or training. Just one science café has not complied with these parameters, but for a very good reason. On April 4 2017, we set out a big science café in Camerino inside a tensioned structure for 180 people. Special guest was Paolo Migone from TV program "Zelig", that together with Linda Pagli (University of Pisa), Pierluigi Crescenzi (University of Perugia), Silvia Benvenuti (University of Camerino), me and my music band spawned a special science café about artificial intelligence and Big Data (Fig. 4.9) the proceeds of which would have go to "Io non crollo", a local volunteer association that is still helping people in need after the last terrible earthquake which hit the area around Camerino.

Fig. 4.9 – The advertise of the event

We realized pretty soon that our science cafes have been and will able to be powerful approaches to science communication and popularization. To improve our work, we are member of the Italian Science Café Network which aims at:

- making available as soon as possible others science cafes experiences in the cities that are part of the network by facilitating the access to programs;
- connecting a range of local matters and their partners through exchange of connections, ideas and tips;
- creating a database of themes, experts and actions that can be easily available both to the partners and interested users to create new science cafes and be updated about proposed topics.

Throughout these science cafes, I realize that there are some important factors that should necessarily be taken into account to avoid the risk of failure, even if there is no universal formula for a science café. First, science café is not a lecture. There must always be a moderator capable of dampening experts' desire to be in the limelight and fostering the audience to express their opinion. Second, it should turn into a fight on a ring. Third, to discuss topics which are of close interest to particular communities. Fourth, say to the experts don't be too technical and use high quality images and short video only when necessary. Moreover, science café should be carried out in a nice place and there should always be a musical, literary, magic or theatrical interlude. It helps to relax the audience and predispose it to the listening. Finally, we have to take care of the quality of every component, food and drink included.

Fig. 4.10 – Photos from two different Science Cafes held in Sant'Elpidio a Mare, Montegranaro and Fermo

Another important thing I noticed is that science cafes engage the sciences with local cultures. Whereas the sciences are pretty much the same the whole world over, cultures are enormously varied. That's why science cafes can take many different forms. Each café has to engage with local behaviors, manners, beliefs and ways of thinking. Thus, science café must reflect local cultures and often that is a considerable challenge. But if we'd like to engage with more socially and economically deprived cultures problems, then problems start to emerge. Some of these are educational (resistence to science, lack of questioning, etc.), some social (fear of discussion, apathy about involvement, etc.), some financial and some others could be geographical, religious or personal. For example, sometimes people still consider ridiculous to go into a café and talk about science rather than gossip or sport with friends.

I think that we may try to change this attitude. It is really a great challenge to overcome some stereotypes, but science cafes could help to bring science back into culture and evenings are spent in a cultural examination of science, from which each member of the audience draws his or her own conclusions. Public discussion is a goal in itself, one interesting and enjoyable aspect of cultural exchange. Public engagement will take many forms, but it is important that many of them start from society, with social innovation, rather than from outreach by traditional institutions. Twenty-five years ago ethics and science were considered two completely different disciplines, but now ethics is crucial to any assessment of a science project and what it will accomplish.

But many people think that talking is not taking action. So, why have we to promote discussion through science cafes? I strongly believe that the networks of people engaged in interactive artistic and cultural pursuits create bonds of “civilty without civil society”. The respect shown for other individuals who have similar interests creates a common civilty. There is no higher purpose in science cafes other than being stimulated to consider and talk about subjects relevant and important to peoples’ lives. But that produces a new community, based on a common interest. Also the breadth of subjects discussed, from genetics to cosmology, from mathematics to biotechnology, and the quality of information and discussion, often undermines what the audience are told to believe by the media, government, advertisers and other powerful institutions. This provides people with an independent base for thought and action. Informal discussion of science is popular because there is a social and cultural gap between science and popular culture. Informal dialogue is likely to expand in many different directions, but its popularity and independence are important in this rapidly changing culture.

Chapter 5

Storytelling and Laboratorial Approach

The main concern in teaching a subject is to raise the level of knowledge and to develop the skills of individuals. On the other hand, outreach and public engagement with mathematics have more to do with raising public awareness and enthusiasm. Thus, teaching mathematics is not the same as raising public awareness of the subject, but the two tasks are mutually beneficial. Although not everyone agrees, there is no doubt that the best way of keeping mathematical thoughts and knowledge alive is to make sure that mathematics teaching works well since the very first day of school for every child. Therefore, mathematics teachers should be comfortable with a substantial catalogue of fundamental mathematical ideas and results in order to raise the awareness of the importance of mathematics and to create enthusiasm in the classroom. School is the home ground where we can work to make mathematics more attractive, interesting and useful for students of all ages that will be future citizens, but we have to identify obstacles and problems of mathematical education (quality of teaching, policies for reforming mathematics curricula, methods and assessments and so on) that many European countries have in common on the one hand and effective approaches to improve the situation on the other[105]. Good teachers may inspire students. Correspondingly, bad teachers can put students off a subject for life. Unfortunately, it is much easier to be a bad teacher than a good one. Moreover, we need to improve the public image of mathematics and create a suitable and proper language to put young people in touch with scientific expertise for this purpose. In short, mathematics education and communication should complement and not remain worlds apart. For example, if a means of communication is successful, I should consider translating it into regular education. In the same way, teaching techniques could be employed to enhance the quality of maths communication activities. Communicating mathematics is a complex process as well as teaching. At any stage of this process things may go wrong, making the communication less effective. For instance, the sender may not express what she/he wants to say clearly; or the room may be noisy; or the receiver may not understand the words the sender is using or more simply she/he is not attracted to the message. To be effective, teachers have to try to minimise these barriers to communication and the most important way to overcome the barriers is two-way communication.

[105] In 2011, European Commission began this process of analysis of mathematics education with his first Eurydice report entitled *Mathematics Education in Europe: Common Challenges and National Policies*. It aimed to contribute to European and national debate on how to improve the teaching and learning of mathematics and provide support to European cooperation in the field.

5.1 How communication can improve teaching

5.1.1 Popularization

"If we consider popularization in its widest sense – any effort to bridge the gap between science and its public understanding – it should include education. Until now, we have considered popularization as complementary to, indeed in some senses a correction for, the educational system, and we succeeded in expressing specific aims and features."[106] As Howson and Kahane, I strongly believe that part of it should be done in school. Popularization has a wider range (audience and topics), many more ways and more freedom compared to teaching. We have already seen that popularization is a one-way communication, but it should be a practice which every teacher should be familiar with. Its main purpose is to have students gain self-confidence in their mathematical capacities and to help them develop a positive attitude to mathematics. School mathematics needs not necessarily be boring, nor school out of touch with real life. Exhibit and projects can be realized in the schools. First, it is good to introduce open activities and non classical themes, and this could be true also for other subjects. Secondly, and this most important for the teaching of mathematics, it would contribute to make mathematics appear as a living science and not merely a collection of techniques or a universal language. For example, the Japanese approach to mathematical literacy is to build a whole curriculum where students have to learn something about mathematics without learning mathematics in the usual sense; it is part of the spirit of popularization in the curriculum itself.

The purpose of mathematics popularization is to put forward abstract concepts with different languages without prejudice to authenticity and quality of the content. However far off that might appear to the public, mathematics is an actual place and we can take them precisely there whenever we want to. On the contrary, maths problems from which we should start cannot be recognisable as such, and are often hard to see and explain. Our society and our own lives would be so much better if we had greater knowledge and understanding of what mathematics really is, what mathematicians do and what is the relationship between mathematics and "all the rest". Popularizing mathematics and getting young people excited about it could mean for them a successful life choice, both professionally and culturally. On one side, we should try to motivate and catch people's attention, or rather amuse and interest who we're up against at an appropriate level, even though it depends very much on the audience. On the other side, once they are willing to listen to us, we should try to inform them effectively. But we must be careful because there are some harmful popularization features that are ubiquitous. For example, someone tries to draw public attention telling stories or making examples that only marginally concern the subject or the mathematical object. That's why we witness to a flourishing of narratives and books focused on mathematicians' psychological and sexual lives or paradoxes and oddities. Moreover, scientific reality is often being used for ratings purposes. We need only think of Numb3rs TV series, although very useful, in which true amazing mathematical applications are presented, but sometimes in an unrealistic way[107]. But actually, this is

[106] Howson A.G., Kahane J.P. (1990), *The Popularization of Mathematics.* ICMI Study Series, Cambridge University Press.
[107] Capozucca A. (2017), *Di crimini, di numeri e di TV*. In: Bischi G.I., Klaver J.M.I., *Noir 2.0 – Il lato oscuro di Internet*. Collana Urbi*noir*-studi, Aras Edizioni.

good to a certain extent, but we're risking to push through wrong messages like "just weird people do maths" or "maths is just a game" or even "maths is funny". Bear in mind: mathematics isn't funny, at least not in the usual or recreational sense of the word. Mathematics can be "fun" in terms of excitement, fascination, motivation and so on.

When I use popularization approach within the classroom, I have to accept two rules. The first is: say one thing less, but say it better. And the second is: use metaphors, examples, similarities and parabolas to put into practice abstract and theoretical concepts without using any formula, at least at the beginning. Sometimes they could seem unnatural, complex or difficult to understand or design, but they are all powerful rules which are at popularizers and teachers disposal. It's a useful approach for most teachers. Over the last few years, I trained myself to follow these popularization rules and I first-hand experienced how important is communication in teaching/learning process and how much it was useful to improve my own teaching skills.

5.1.2 Storytelling

Robert W. Frucht (1906-1997), a German/Chilean mathematician known for his studies in graph theory, stated that *"using mathematics to tell stories and using stories to explain mathematics are two sides of the same coin. They join what never should have been separated: the scientist's and the artist's ways of uncovering truths about the world"*[108]. Communicating mathematics to the public should never be just an information list like an instruction manual, albeit translated in simple language. Faced with new information, our mind always creates a common thread to hang on to. That's why stories serve as trails to lead the public through the deep forest of proposed themes encouraging them to keep going without making them feel lost[109]. Stories are experiences that carry people in situations in which they can identify with the characters, understand what drives a character to act in a certain way, find an immersive scenario and a precise timing in which stories take place. A great story should have the power to pull us out of the world for a while and make us feel as if the clocks have stopped, time's authority over us suspended. Along these lines, Lewis Carroll's "Alice in Wonderland" (1865) represents a real masterpiece.

In recent years, there has been a renewed interest in the role of stories in mathematics education, both as a means of disseminating knowledge and as a cognitive tool for understanding mathematics. This renewal coincided with Andrew Wiles' solution to Fermat's Last Theorem in 1995. His prolific proof received international public attention and was followed by the best-selling book, "Fermat's Enigma" by Simon Singh. With mathematics suddenly becoming fashionable, several works, both fiction and non-fiction have appeared that draw their subject from the field of mathematics[110]. The number zero, the prime numbers, as well as the number pi and other irrational numbers became the characters of many popular books, while films such as "Proof", "A Beautiful Mind", and "Good Will Hunting" deal with such advanced topics as number theory, combinatorics, and graph theory. This leads us to the idea that stories can also be used in the classroom in very

[108] Lipsey S.I., Pasternack B.S. (2002), *Mathematics in Literature*. The Mathematics Education into the 21st Century Project: The Humanistic Renaissence in Mathematics Education, Terrasini, Palermo, p. 1.
[109] See Chapter 2: Capozucca A. (2017), *Communicating Mathematics in Europe. Episode 2: Alex Bellos*. Lettera Matematica International Edition, Springer-Verlag.
[110] Tomlin S. (2005), *What's the Plot?* Nature, 436 (051), p. 622-623.

practical ways to serve different educational goals. In mathematics education, storytelling is a powerful cognitive tool that can assist in providing meaning to abstract ideas and seemingly irrelevant facts, algorithms and procedures students learn in class. In some cases, a story may be loosely tied to the mathematical content, while in others, the story and the mathematics are one in the same. There many ways stories can be used to engage students' imaginations and in the following I briefly present some of these along the path indicated by the classification laid out by Zazkis and Liljedahl[111] (but not literally).

One type is a story that accompanies a subject. Let us look at the ancient Greek, Archimedes of Syracuse. There are two familiar stories about his life. The first account is given by the Roman architect, Vitruvius, and describes how he discovered the principle of buoyancy; the other story is an account of the dramatic way Archimedes met his demise. Both of these colourful tales will engage the imaginations of the listeners. What student is not fascinated with people running around naked in public or an execution by sword? In terms of mathematics, the first story illustrates the sheer ecstasy of discovering a mathematical truth, so much so, that any contest around is ignored. The second story shows the degree to which someone might be engaged with mathematics – even to the point of ignoring the dangers around them. However, both stories are what Mazur calls, *"raisins in the pudding"*[112]. These "anecdotal digressions" neither help bring students closer to understanding the mathematical content, nor do they fit in as part of the structure of the argument presented. There are several familiar stories that are ornamental in nature, many of which are derived from the history of mathematics, and that often focus on the lives of mathematicians. For example, Galois killed in a duel, young Gauss' boredom in math class, or Hippasus drowned at sea. The greater risk is that these "raisins" often outweigh our primary purpose – to help students understand and find meaning in the mathematical content.

Stories can also be used to introduce a mathematical idea or concept. "Origin stories" are commonly used to introduce new topics and answer the question of how the mathematician came to work on the material and how the ideas were originated. When students see a concept through the emotions that were involved in its discovery, they grasp its deeper human meaning. The facts and algorithms we teach our students will no longer be viewed as meaningless symbols and abstract ideas, but as the product of human passions, hopes, and fears. Researches made by VanSledright and Brophy reveal that primary school students are particularly interested in the history, and are concerned with human motives and cause-effect relationships[113]. This propensity for historic understanding does not go away as students grow up. Instead, it becomes a search for patterns and overarching schemas in order to make sense of the present and how they fit into the world around them. Examples of origin-stories are "Descartes' fly", "The bridges of Konigsberg", "The tower of Hanoi" and many others.

Another type of story is one that intertwines with a mathematical topic. Consider how area and perimeter concepts, algebraic equation manipulation, and proofs can be intertwined with "The Story

[111] Zazkis R., Liljedahl P. (2009), *Teaching Mathematics as Storytelling*. Sense Publishers, Rotterdam/Taipei.
[112] Mazur B. (2007), *"Eureka" and other stories*. Retrieved on July 12, 2006, from http://www.thalesandfriends.org/en/papers/pdf/mazur_paper.pdf.
[113] VanSledright B., Brophy J. (1992), *Storytelling, Imagination, and Fanciful Elaboration in Children's Historical Reconstructions*. American Educational Research Journal, 29 (4), p. 837-859.

of Princess Dido", for instance. Before telling students how Dido cut the hide, we might ask them to come up with their own ideas for how to do this. Remember the trick where you tell someone you can make one cut in a letter size paper so that you can walk through it? We might show this trick to students after they have considered the problem for a while. The real task, though, is how to maximize the area of the shape given a fixed length (the long thin strand of bull's hide). We could also tell students that the chieftain gave Dido the restriction that she must only use rectangular shapes or the deal is off. In this way, we are introducing constraints into the mathematical problem through the events of the story. Students will soon discover there are many different areas that can be contained by this amount of rope under this constraint. Lastly, we might want students to generalize their findings using a simple proof. Perhaps the highest degree to which a story can intertwine with the mathematics is in the form of what Mazur calls "pure mathematical narrative". In this type of story, the mathematics itself is the story, and the drama lies wholly in the way mathematical ideas unfold. If a story is to keep our attention, there must be some type of conflict or tension present that gives rise to a pattern of suspense, and it should force the listener to ask: "What happens next?" In this type of story, however, the conflict is not between two mathematicians, or two schools of thought, but it occurs within the realm of ideas. Some might call this "intellectual curiosity". While this form of mathematical narrative is engaging to the mathematician for its conundrum and its purity, it leaves most non-mathematicians somewhat less enthused. It appears that a necessary prerequisite for enjoying this kind of narrative is a high level of mathematical sophistication. It could seem obvious that we cannot engage students in a drama of abstraction, but, contrary to many of the current theories, children have a certain proficiency at abstract thinking. Combined with the fact that children have a proclivity for narrative, stories can be used to frame abstract mathematical ideas in a way that engages students' imaginations. In order to access these higher levels of thought, we often need some other more indirect instrument. Zazkis and Liljedahl suggest that one way to bring abstract mathematical ideas to life is to cast mathematical objects as characters in our stories. Giving inanimate objects, animals, or concepts human qualities is a familiar technique used in a wide variety of children's literature. "Thomas the Tugboat", "The Little Engine that Could", and "Peter Rabbit" are just a few of the many examples. This approach takes into account the way young children perceive the world around them, and their affinity for fantasy play. Although the tendency to humanize inanimate objects diminishes as children approach secondary school age, in later adolescence and adulthood, it regains its fervour—a way to sort of recapture their childhood. Nevertheless, this literary device has already worked its way into a wide variety of mathematics literature. "The Greedy Triangle", "The Silly Story of Goldie Locks" and "The Three Squares, and The Dot and the Line" all have familiar geometric shapes as their main characters[114]. Stories at the secondary level include: "Flatland: A Romance of Many Dimensions" by Edwin Abbott Abbott, "The Parrot's Theorem" and "Zero" by Denis Guedj, and "Class Reunion" by Colin Adams.

A further type of story is a story that explains a rule or algorithm. Mathematical content is mostly perceived as a set of logical facts, algorithms, and skills that students need to learn. Many of these facts, algorithms, and rules have been constructed through centuries of mathematical endeavors and carry with them a part of the rich culture of mathematics we want students to embrace.

[114] Delano Moore S. (2002), *Teaching Geometry and Measurement Through Literature*. Mathematics Teacher in the Middle School, 8 (2), p. 78-84.

Disappointingly, direct instruction often promotes rote mechanistic memorization of those facts, algorithms and skills (i.e. invert and multiply, do no divide by zero, negative times a negative equals a positive). This seems counterintuitive to establishing mathematical meaning for students, but the teaching of facts, algorithms and rules can be a powerful learning activity that connects students' skills and understandings to real mathematical meaning. Story structures can provide the framework for giving meaning to these facts, algorithms, and rules. Even facts, algorithms and rules need the development of narrative tools in order to be used effectively. We incorporate new skills and facts into our conceptual frameworks more readily according to their significance and emotional meaning rather than chronologically or sequentially. We organize and remember facts and rules more easily in narratives, than from logically ordered lists. Few people will remember the precipitation on a particular day five years ago—unless perhaps, it was the day their house flooded, or the day they were splashed by a car while walking along the sidewalk. Likewise, few adults will remember the relationship between the perimeter and area of a rectangle, until it becomes part of a plot involving treachery and seduction, and few students will care if a hexagon has 5 sides or 6, unless it means the difference between being a member of the 49 middle class or the upper classes of society. Stories can better capture the complexity of experience than a list of facts, rules and procedures. More importantly, they provide a familiar way for students to connect the seemingly disjointed bits of information into a meaningful network of concepts.

One last type of story is a story that introduces an activity. Activities can play a vital role in helping a story establish meaning. For example, children often understand difficult mathematical concepts presented within the context of a story only after they engage in activities that support those ideas. There are three different levels at which students are asked to engage with the mathematics they encounter in stories[115]. At the most basic level, students observe what characters in the story do with the mathematics they encounter, but are not given any explanation as to how the mathematics is done. At the next level, students listen in to a character or narrator's thoughts while mathematics is being done, so that they can see how to do the mathematics. However, students do not do any mathematics themselves. At the highest and most effective level of Schiro's hierarchy, students are required to do the mathematics in the story in order to fully understand both the story and the mathematics within. Stories often prompt activities that emerge from within the story and that support connections to students' own lives. Activities often lead to discussions about mathematical concepts. When children share their work with each other and help each other, they become involved, not only in the individual construction of meaning, but a social construction of meaning as well. This connects them to a larger and more varied knowledge base, while at the same time adds connectedness and comradeship to the ongoing web of intrigue that forms about their lives. Moreover, when problems are situated in a lengthier story, students will be more likely to dream about, discuss, imagine, share ideas about, and re-play in their minds the events of the story, what will happen next, and how the activities and related problems emerging from within the story can be solved.

Thus, stories can be used in the classroom in various ways to highlight, introduce, or even explain the mathematical content we wish our students to understand. The degree to which students actually understand the mathematics embedded within a story largely depends on the type of story chosen,

[115] Schiro M. (2004), *Oral Storytelling and Teaching Mathematics: Pedagogical and Multicultural Perspectives*. Thousands Oaks, Sage Publications.

as well as the level at which students engage with the mathematical content they encounter. Presenting a problem using familiar natural language and recognizable contexts may not be enough. Students must be given a chance to, not only observe what characters in the story do with the mathematics, but also interact with the mathematics themselves.

5.1.3 Laboratory as learning environment

In schools, it is increasingly necessary to communicate mathematics not only in the two traditional directions, teacher who explains to student and student who answer to teacher, but especially between equals. Students must also communicate with each other by having mathematics as subject of communication. Laboratory approach is a powerful tool for this purpose. It promotes students getting involved and has the following main objectives:

- to improve reasoning skills, rational thinking and analytical capabilities;
- to develop ability to address and solve problems;
- to encourage operational dimensions, personal autonomy and creativity, and accountability.

To achieve long-term results, teachers have to make meaningful choices as the body of the discipline and the area of interpersonal relations are concerned. They need to tune in with students' motivations, interests and pre-existing knowledge in order to be consistent on the front of mathematics communication and classroom management. For example, the selection of a topic can't be set only by thinking of a single education segment , but it must take into account the global students' schooling, from the first cycle classes up to the end of secondary school. Students, for their part, try to solve problems – taken from everyday life or within the math-world – by using mathematical knowledge and beyond, connecting foreground in different areas, and putting their personal assets on the line. This research for solutions naturally allows to recreate the stages of the mathematical research activity: begin with a problem whose we know nothing about possible solutions, analyze the problem to find a path to follow, trial and error, interpret the obtained results and generalize them, communicate the results to the others. Thus, teaching mathematics in this way allows students to work together with a common goal to be achieved by enabling them to express high efficiency and accuracy levels, enhance usually underestimated, or not even recognized, abilities and be more involved in learning process.

As there isn't a single definition of "laboratory", I think it's useful to give my own definition of laboratory to which I shall refer from here on out. I define laboratory an educational activity in which students play an active role by working in small groups, teachers play the role of expert guides (who observe, listen to students' needs and answer to their possible questions in order to steer what is put in place or divert from insignificant paths) and a two-way communication and a focused debate between different parts is always guaranteed. The proposal of such laboratorial activities is a working method for an effective mathematical learning at any school level. Subject to appropriate adjustments, it can even work as outreach activity for general public. The important thing in both cases is a careful choice of what to convey, arrangements, schedule, communication styles and group management. This is the main way to ensure the coexistence of high aspirations and educational efficacy.

With this in mind, communication becomes a fundamental aspect in designing a laboratory where students are able to explore the opportunities and challenges connected with the language. As stated in the previous chapters, the technical language of mathematics may be an obstacle, often overwhelming, for most students. Teachers cannot neglect the difficulties faced by students in effort to help them. In laboratorial activities, students start using a familiar language with specific terms borrowed from the common language that seem to clearly describe the mathematical situation in which they're in. Only after, students feel the need of a more rigorous language to achieve mutual understanding, communicate more effectively and establish the meaning of the terms in that specific context. Outside the laboratory, it's uncommon that students have the opportunity to talk to each other about maths. Mathematics communication is rarely *inter pares*, but generally between who knows and who doesn't know. Against this backdrop, rigour, language and errors hold a different significance compared to traditional teaching/learning process. In a peer-to-peer communication, the first thing to do is to understand each other. It is precisely this need that bring students to emphasize rigour and the search for an unambiguous language. As a consequence, errors are valuable: they can be useful to better understand a problem and move ahead towards its solution, if they are appropriately discussed. Thus, laboratory consists of two stages both strictly related to communication: the informal part in which students experiment, make mistakes, discuss using common language, adjust their assumptions and try again, and the formal part in which fix things, discuss using a more rigorous language, formalize and communicate what they figure out. It often happens that an interesting cause/effect circularity arises between these two stages leading students to a deeper and more meaningful learning and a strong metacognitive reinforcement. This is essential because there exists a transition between the moment of true insight of a given concept and the moment in which the concept is to be considered acquired. And this happen when a students are able to communicate it to themselves and to the others, teachers included.

5.2 How I put into practice

During my PhD years, I examined in detail and tested these methods through a deep literature study, meetings, interviews and direct cooperation with experts (Alex Bellos, Simon Singh and Erik Stern), and implementation of two different research projects (a children's book and a math-dance laboratory for teachers and students).

5.2.1 The book "Il Tranello e la soluzione matematica" by Capozucca A. and Cerquetti B.

5.2.1.1 Synopsis

Silvia loves the peaceable life of the college. It is the ideal atmosphere to soothe the nightmares and memories of the war bombings. But the calm ends when Andrea, the young servant of her family castle, reveals that her father, the Earl of Duino, has mysteriously disappeared. Then, to prevent the sale of the manor, the story of Silvia and Andrea takes place among plots and mistaken identities. By day, they pretend that nothing has changed; by night, they investigate what might have really happened to the Earl of Duino. To get back her father, Silvia will be able to learn to see what is seemingly invisible. In a whirl of plot twists, lifelike statues, talking dragons and great personalities of the past, Silvia will face an extraordinary journey in the Magic Dimension of mathematics. Perhaps she will understand that it's useless to remove the past without groped before understanding.

Fig. 5.1 – The cover of the book

Fig. 5.2 – Barbara and I before the book launch at the University of Camerino

5.2.1.2 About the book

In February 2016, the book entitled *Il tranello e la soluzione matematica* (Fig. 5.1) came out after a year's work together with Barbara Cerquetti[116] (Fig. 5.2). As clearly stated by the title, the book is about mathematics, but it's not a textbook. It's a coming-of-age story aimed at children aged from 9 and up. Writing books for young readers is a serious and delicate matter. The interests of ten-years-old kids are very different from those of a forty-years-old man or woman. Preadolescent language and sense of humour don't match up with those of adults. Since the beginning of the writing process, we shared the idea that to engage young people we needed to be very clear, wipe out innuendo, choose the appropriate terms. And above all, never underestimate young readers. They really know what they like and are open to new experiences. They have a highly developed critical sense, but at the same time are more tolerant and open-minded than most of the adults. Thus, mixing the ingredients in writing a book that bears the indication "from 9 to up" has been challenging for us also because the lack of the age's upper limit requires that it should be readable by all. However, our biggest challenge was to talk about mathematics, teach mathematics, without writing a book that was conspicuously about mathematics. So the idea of a story, a children's story, which calls into question the mathematics almost by accident, because mathematics is everywhere and it's not impossible to make it filter through the plot.

[116] She is a philosopher and a well-known children's author. She wrote other books for Giacone Editore: *Il bosco delle lucciole. Una storia di amicizia e avventura nei Sibillini* (2011), *Il giardino delle farfalle e altre storie farfallose* (2015) and *L'isola perduta. Viaggio tra i ricordi del Conero* (2015).

We decided to carry out the narrative dimension and the mathematical dimension at once. The book starts with Silvia's story, a young girl who lives with her hunt and is happy to end up in a boarding school, which took place in the fifties during the rainy days after the end of the World War II.

> *"Durante la Seconda Guerra Mondiale le città ritenute obiettivo militare vennero ripetutamente bombardate. Quando l'unico rumore che si sente attorno è il rombo degli aerei, il fischio degli ordigni e le esplosioni che sbriciolano i palazzi, quando il cielo non è più un luogo verso cui rivolgersi, perché proprio dal cielo arriva il pericolo mortale, può accadere che si desideri diventare invisibili, sparire nel nulla e ritrovarsi per magia lontano e al sicuro.*
>
> *La nostra storia inizia cinque anni dopo la fine di quel conflitto. Gli aerei avevano smesso da un pezzo di essere strumenti di morte e in ogni paese venivano aperti grandi cantieri per costruire edifici vistosi e luccicanti che sostituissero quelli disintegrati dall'esplosivo. Il mondo voleva spazzare via le macerie del passato e ricominciare a vivere. Tutti tenevano lo sguardo rivolto al futuro e alla ricostruzione che sarebbe stata rapida, grandiosa e vorace. O forse sarebbe meglio dire quasi tutti.*
>
> *Da qualche parte infatti c'era una bambina che poco ricordava del periodo dei bombardamenti ma che con tutta se stessa desiderava ancora rannicchiarsi in un angolo, farsi piccola e diventare invisibile."*

We didn't use any narrative sweetener because young readers are able to read and imagine what's going on. After this historic introduction and more information about Silvia, we included a dash of adventure to introduce the other protagonist Andrea, a young boy with completely different interests and habits compared to her.

> *"Fu in quell'istante che qualcosa, fuori dalla finestra, frusciò. Una mano inguantata di nero fece toc toc sul vetro, seguita da una faccia magra, scura, con gli occhi spiritati.*
>
> *Silvia balzò dal letto e spalancò la bocca per strillare di terrore, ma le gambe le si fecero di gelatina, mille puntini scuri le oscurarono la vista e tutto quanto all'improvviso divenne nero.*
>
> *La prima cosa che Silvia avvertì riprendendo i sensi fu la voce di qualcuno che parlava senza riprendere fiato. «Speriamo che non sia morta, porca miseria. Che faccio se è morta? Magari non lo è... ma non lo so... Come si capisce? Forse posso provare a tapparle il naso e vedere se apre la bocca. Sì, faccio così: se apre la bocca allora sto tranquillo che non è morta...» Neanche due secondi e Silvia si sentì pizzicare forte il naso, tanto che spalancò gli occhi e strillò di indignazione: «Lasciami subito!»*
>
> *«E' viva!» gracchiò di rimando l'altro indietreggiando spaventato. Si trattava di un ragazzino, suppergiù della sua stessa età."*

From here on out, we started mixing variouos story elements (historic, adventurous, magic, humorous, etc.) that are prelude for mathematics. Silvia and Andrea must deal with trials and puzzles to overcome the obstacles to their goal and mathematics is the only thing that can help them. That's why we included mathematical insights and breaks throughout the story, so that young readers can investigate mathematical topics such as cryptography, prime numbers, computation, tesselletions, polyminos and lot of recreational mathematics, or meet Euclid, Polybius, Fibonacci, Piet Hein, Loyd, Gauss, Ada Lovelace and Maurits Cornelius Escher, who have the privilege of being a real character inside the story.

> *"Il locomotore si fermò. Le porte si aprirono con uno scatto secco. Ne scese un passeggero, uno solo. Le porte si richiusero e il treno ripartì. L'uomo era alto, magro, aveva i capelli e una barbetta aguzza, portava con sé una piccola valigia e chiaramente non era Enrico Beton. Però si guardava intorno da tutte le parti come se stesse cercando qualcuno. I ragazzi gli si avvicinarono perplessi. «Scusi» fece Andrea, «è lei che oggi ha mandato un telegramma al castello Visconti?»*
>
> *L'uomo si rilassò visibilmente: «Ja. Il duca Duino ha mandato voi a prendermi?» Andrea rimase sul vago. «Diciamo di sì, vossignoria. Io sono Andrea, il garzone del castello.»*
>
> *«Ed io sono Silvia Visconti, la figlia del duca.» Lo sconosciuto afferrò la mano di Silvia e la strinse cordialmente. «Heel blij, mislopen.» «Ahem...» «Perdono. È olandese, la mia lingua madre. Vuol dire: molto lieto, signorina.» Andrea lanciò un'occhiataccia allo straniero. Non gli era affatto simpatico, questo tizio. Silvia invece sembrava colpita dai suoi modi formali e distinti.*
>
> *«Mi scusi, signore. Ma lei chi è?» chiese timidamente intanto che si avviavano fuori dalla stazione, verso il castello. «Mi presento, mislopen: mi chiamo Maurits Cornelis Escher, per servirla.»"*

While reading the book one wonders if we used the story to talk about mathematics or we used the mathematics to enrich the story. What we wanted to demonstrate is that there's no incompatibility between literature and science. In my opinion, they can coexist without difficulty because both literary magic and mathematical magic have the ability to amaze. Besides, book's ultimate goal, at least as far as young readers are concerned, is neither to push them towards mathematics nor towards narrative, but to convince them of something more important that is hidden in the title itself. The story is full of puzzles, questions, problems to solve, mysteries, but no "traps" in the true sense of the term. What this means is enclosed in the phrase

> *"Non cadere mai nel tranello della tua insicurezza. Il pregiudizio verso te stessa e verso il mondo sarà sempre il tuo peggiore nemico."*

that is crucial for every young reader's life, but above all for every teacher's mission.

5.2.1.3 Aims of the book

The first aim is to present an active and joyful engagement with mathematical ideas and present mathematics as a meaningful human experience. In the story, mathematics becomes the focus of the narrative and it is presented with historical perspective and thematic coherence with the historical period in which the story takes place. The novel is interspersed with insights of the mathematical topics that the protagonists are faced with, and with biographies and curiosities about historical characters and their work related to mathematics that appear throughout the story. At the end of the book there also are worksheets to be cut, related to the topics, that allow young readers to join in the story in an interactive way.

Another aim is to return to mathematics those moments and situations from which it is artificially separated when it is codified in an article or in a textbook, and highlight the creative and dynamic aspect that characterized it by trying to reveal how it is the very essence of the inherent reasoning in each of us and in the understanding of the world around us.

A further aim is to make maths available without debasing by doing save the crucial and fundamental articulations of the discipline through the dramatization of concepts and characters. The book is not intended to be a guide, but it wants to be a shared experience which puts young readers in a role in which are the readers themselves to be the manual to draw. A sort of "socratic method" where the book doesn't guide the reader to discover mathematical truths, but it tries to stimulate reflection and personal research, by highlighting, where possible, connections and similarities between the various parts of the narrative.

Our final aim is to make available to mathematics teachers an innovative tool they can use within their educational programs. Telling a story is a way of establishing meaning. And establishing meaning should be central thread in teaching mathematics – a subject that is too often perceived as the manipulation of symbols, the meaning of which is often far from clear to students.

> *Tutta quella fatica ti porterà ad un momento in cui i tuoi occhi, all'improvviso, la vedranno e allora dirai: oh eccola, come ho fatto ad essere così cieco, ce l'avevo proprio davanti agli occhi! E da quel giorno in poi, per sempre, la vedrai immediatamente, senza neanche doverci pensare.*

Creating interest with a story is an important initial step. Describing a chain of events may engage students, create excitement, mystery or suspense, and motivate thinking about a particular problem presented throughout the story. A story may convey passion and enthusiasm. It may also introduce ways of thinking and acting like their heroes, create empathy, and make the material more accessible and memorable.

> *«Noi non dobbiamo mai arrenderci all'impossibile, o temerlo. Dobbiamo anzi accettarlo, imparare a vederlo e a dargli un nome. Solo così potremo aprirci verso le meraviglie dell'universo.»*

Stories in which students identify with the heroes may also make the reading, as well as the lesson, more relevant and more vivid. A story that involves specific examples may help readers and students relax as it provides something to hold to when moving to general theory or technical detail.

> *Un bravo matematico cerca sempre di trovare delle analogie tra le cose. Questo gli permette, a volte, di elaborare un metodo generale che poi può applicare in contesti diversi anche apparentemente distanti.*

5.2.1.4 Why a story and not a handbook?

One of the aspects that makes hostile learning mathematics is the language; the difficulty lies largely in the fact that it does not admit ambiguities, appearing hermetic and enigmatic. Rather than having to deal with such maths, it could be more pleasant the reading of a book because the language used in a novel is closer to the common sense, despite the complexity of the plot and the difficulty to grasp all the links of reasoning. The novel is written in a language that is able to emotionally engage because populated by situations and objects with which we are more familiar and comfortable. The attempt is to transfer knowledge and ideas about mathematics to all readers without frightening or making them feel dumb, instilling interest and curiosity. Each reader can choose what to read: only the story, only the insights or both for a deep and more comprehensive view of the story as a whole. Indeed, the insights and the related worksheets are designed to be an integral part of the story and not mere compendium to the reading.

5.2.1.5 Interdisciplinary approach

I decided to write the book with Barbara Cerquetti because I strongly think that knowledge transfer is more effective when different skills are able to interact in a cross-disciplinary approach. In this case, the different communication styles become binding devices of a shared quality and, at the same time, reasoning ideas on diversity in practice and in communication formats. The basic idea is to contaminate the languages typical of the “two cultures”, which means the ability to have new different tools to understand better the mathematical world. This fruitful “four-hands” writing was possible because both Barbara and I were willing to contaminate our own specialization through the learning of the language and communication techniques of the other. This contamination is not synonymous with superficiality, dispersion and amateurism, but an opportunity to create special connections and synergies that can lead to deeper and original visions of those results that could be obtained in an internal logic to the individual disciplines. Anyway, the quality of the knowledge requires a careful and patient study, a deep debate and intellectual honesty. We worked together sharing feelings about the style of better writing, selecting coherently the right topics and trying to put ourselves in readers’ shoes. That was a hard task.

5.2.1.6 What the story wants to convey and how

Through the story of Silvia and Andrea we tried to present the way in which mathematicians think and deal with problems. First of all, make young readers understand that mathematics is just not a genius legacy, but everyone can approach the mathematical world at different level. Examples are the two protagonists of the story. We give them difficult problems, let them struggle and get frustrated. Then, see what they came up with. Wait until they are dying for an idea, then it’s time for some technique. This means that technique in mathematics can be effectively learned in context.

Then, as mentioned above, we wanted to present mathematics as a fascinating, funny and free-mind experience and as a fundamental part of our personal growth as human beings. For example, the inner evolution of the protagonists throughout the story is linked to their mathematical awareness. The secret is to create an healthy and vibrant atmosphere and get used to being flexible and open to sudden changes in direction to which our curiosity may lead. A mathematical mind is a creative mind that represents this synergy of productive action. A complex thought that gives space to reality and imagination can generate a person motivated to do, motivated to understand, motivated to grope around and go beyond what apperas to its curious eyes.

Before writing the book, I and Barbara met every week to choose together the mathematical arguments to be included in the story. Time after time, Barbara wanted to understand step by step solutions to the puzzles and problems. She always told me that she loved to see my delight in doing maths. Thus, she wondered how to put this feeling within the story. Mathematicians enjoy thinking about the simplest possible things, and the simplest possible things are “imaginary”. In mathematics things are what you want them to be. You have endless choices. The only way to get at the truth about our imaginations is to use our imagination. So, there must not necessarily be a practical purpose to do maths. Sometimes it could be just a game. When a mathematician wants to prove a theorem, it does not list the hypotheses and then ponders on them. What he does is a set of experiments and attempts (trial and error), strategies and forecasts, following an inspiration and sometimes hoping in a dumb luck. Is not this what we do when we try to solve a puzzle or a math puzzle? Mathematics is deeply linked to the game, especially in fielded procedures and attitudes. This could help students to understand why, in the history of mathematics, it often happened that a game or a fun problem ignited a whole line of research or theories that then pledged mathematicians for years or even centuries (e.g. The Fermat’s Last Theorem). Problem solving and play are approaches that can reveal a mathematical face which remains hidden behind the outward appearance of the discipline as presented at school or in textbooks. “The dark side of the moon” is able to exert a fascination that can excite the study of a discipline generally considered boring. That’s why we chose to include in the story many math puzzles linked to particular mathematical topics.

Furthermore, we used many mathematical ideas in the construction of the plot (prime numbers, non-linear dynamical systems, fourth dimension, ecc.). For example, the behaviour of the two protagonists, Silvia and Andrea, is similar to that of a non-linear dynamical system. Two simple agents that interact with other characters and the external environment in which the story takes place. They show an adaptive evolution throughout the story assessing whether the adaptation is useful or not for their final goal. Moreover, they also experience a circular relationship between causes and effects that allow them to understand the complex situations which they are involved in. Emergent behaviours arise every time the story presents an unexpected “bifurcation” situation until the protagonists are able to seize “order from chaos”. Thus, the evolution of the individual characters (Silvia and Andrea) is not given by the sum of the their behaviours, but it is greater than their sum and it’s closely connected with global evolution of the whole story.

5.2.1.7 The book as a teaching tool

Since February 2016, *Il tranello e la soluzione matematica* has been chosen as textbook by 11 primary schools and 16 middle school teachers. More than 700 students in the Marche region have read the book, 275 primary students aged from 9 to 10 and 723 middle school students from 11 to 13, without taking into account other 948 copies sold throughout Italy. Teachers are showing a great interest and appreciation for the book and all the possible educational activities connected with it. It may be useful to read some of their comments:

"We gave attendance to the reading of IL TRANELLO and to the resolution of the puzzles and some proposed games with special attention to tessellation in 2D space. Students were really impressed to discover that often looking at the problem from a different angle is all you need is to get creative solutions, and surprised by how much mathematics is hidden all around us. It was enlightening to interpret a reality inherent in us through mathematical concepts as in the fourth dimension explanation. We learned that thinking out of the box all the pieces of the puzzle fit perfectly together and everything is amazingly clear! An intuitive concept like the point linked with the present time, here and now, further explain that reality is mathematics."

Michela Morresi, fifth year of Primary School, ISC "Rodari-Marconi" Porto Sant'Elpidio (FM)

"My students' curiosity and interest increase every time we read the book in the classroom. That makes my work a lot easier!"

Stefania D'Emidio, fourth year of Primary School, ISC "De Carolis" Acquaviva Picena (AP)

"After reading the book, students were excited and ready to put themselves out there to such an extent that some of them wrote a story about numbers and others invented a board game similar to Hex. Not in nine and half years have I come across such an enthusiasm."

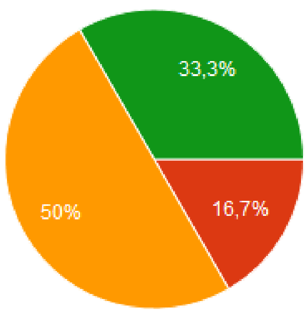
Chiara Vitali, second year of Middle School, ISC "Rodari-Marconi" Porto Sant'Elpidio (FM)

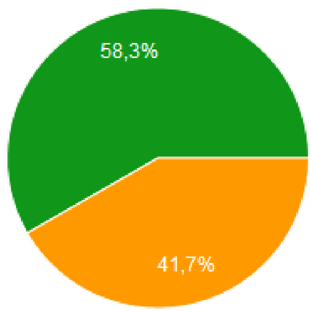
At the beginning of 2017, I conducted a small survey among students of the schools that adopted our book to observe, before and after reading:

- students perception of mathematics;
- students involvement in classroom activities;
- students awareness of the presence of mathematics in the world around us.

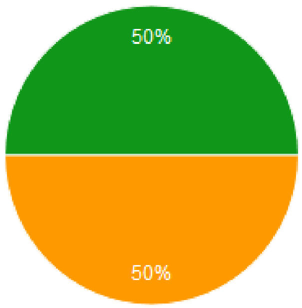
These are the results up-to-date as of August 31st, 2017 (Table 5.1). See Appendix C for templates of BEFORE and AFTER questionnaires.

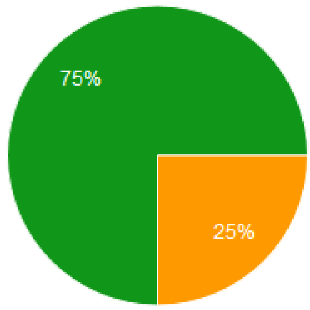
Statistical sample : 411 students aged from 9 to 14 Meaning of colours : Good (green), Normal (yellow), Bad (red)	
BEFORE	**AFTER**



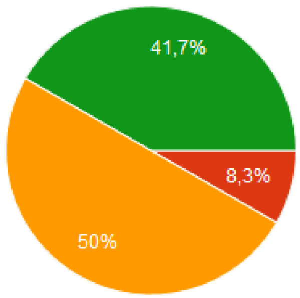


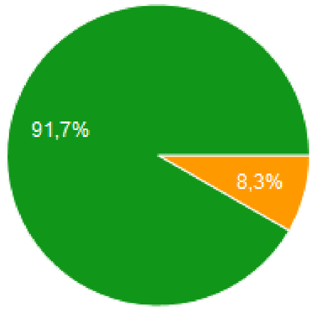
Students perception of mathematics





Students involvement in classroom activities





Students awareness of the presence of mathematics in everyday life

Table 5.1 – Results of the survey

5.2.2 Math-Dance laboratorial activities

Since August 2016, I entered into a collaboration with Erik Stern[117] on two interdisciplinary projects involving mathematics and dance. The first of these is a mathematics communication project which aim is to present science of complexity to the general public through a show/exhibit involving music, dance and visual graphics, and I will present it in detail later in Chapter 6. The other is an educational project which provides ideas and materials for teachers to teach mathematical concepts through body movements and choreographic exercises.

Our monthly Skype meetings have been inspiring and invaluable moments of discussion and reflection on the projects. We shared ideas and concerns on what could have been the best way to teach and communicate mathematics using the whole body, how to better engage teachers and students in a teaching/learning process starting from informal situations to arrive at a formal knowledge, and how much making room for creativity is essential in these kind of laboratorial activities. Then, I visited Erik for a week on August 2017 to deepen and first-hand experience his methods and approaches, and meet his collaborators (Rachel Bachman, Adam Johnston and Julian Chan) to develop my projects from their embryonic stage. To speak with them, see how they work and implement my projects with their precious help and suggestions, have been positively mind

[117] He has toured with Tandy Beal & Company, acted with Shakespeare Santa Cruz, composed numerous scores for dances, and collaborated with numerous other artists. He is Associate Professor of Dance at Weber State University in Ogden, Utah, and received his B.A. in Biology from UC Santa Cruz, and Master of Fine Arts Degree from California Institute of Arts. He is Co-Artistic Director of Dr. Schaffer and Mr. Stern Dance Ensamble.

enrichment and concrete situations to improve my mathematical and artistic skills. Below, I present a first outcome of our educational project.

5.2.2.1 Physical problem solving: Math+Dance

The purpose of this workshop is to introduce teachers to the use of physical problem solving to teach mathematical concepts. The workshop alternates between creative problem solving and reflective discussion. These activities are for everyone and do not require special training in dance or mathematics (although training or interest in these subjects can be helpful). The activities begin by addressing the most basic universal elements of dance and mathematics: recognizing and remembering patterns, walking, counting, moving, making shapes. The activities are designed to be flexible and can be extended to suit various levels of students. During the workshop, participants will:

1. solve problems physically individually and in groups of two to four;
2. discuss problems in smaller groups and as a class;
3. work to formalize the creative problem solving strategies developed.

We'd like to give particular attention to time for discussing how these activities provide meaningful entrance points to the mathematics curriculum (Clear Content Connections), and how the practices integral to this method provide transferrable pedagogical practices for all mathematics classrooms (Teaching Nuggets).

5.2.2.2 Background information on Pattern Play: movement and mathematics

In 2013, Weber State University Mathematics faculty members Julian Chan and Erik Stern began adapting Schaffer-Stern's math dance work to suit an interdisciplinary college course. Drawing on Schaffer-Stern's master classes for college level mathematics and dance courses, Julian and Erik devised Pattern Play: Movement and Mathematics, which paired Creative Arts general education requirements with a Contemporary Mathematics quantitative literacy course. Julian and Erik also devised several new lessons. Rachel Bachman joined the team to qualitatively and quantitatively assess Pattern Play students and a comparable math course taught in a conventional manner. The team has published several papers on the resulting assessment.

5.2.2.3 Why Math and Dance?

The use of the art form of dance in the classroom has been shown to develop students' creativity, critical thinking, symbol and vocabulary use, memory, neural plasticity, visual-spatial skills, self-confidence, persistence, and social skills[118]. Moreover, the use of dance to study mathematics has been documented to significantly increase student attitudes toward mathematics[119]. A well-

[118] Hanna J.L. (2015), *Dancing to learn: The Brain's Cognition, Emotion and Movement*. Lanham, MD: Rowman & Littlefield Education.

[119] Brillon R. (2014), *Choreograph Math: Teaching Geometric Concepts Through Dance Movement and Design*. Doctoral dissertation. Retrieved from ProQuest Dissertation & Theses database (UMI n.1495482).

constructed dance activity gets students experiencing fundamental mathematics concepts in time, space and feeling, forming a basis to understand the symbolic approach to that mathematical concept. Thurston states, *"People have very powerful facilities for taking in information visually or kinesthetically, and thinking with their spatial sense"*[120]. Thurston goes on to explain that when reasoning spatially, *"we tend to think more effectively with spatial imagery on a larger scale: it's as if our brains take larger things more seriously and can devote more resources to them"*. Dance provides this opportunity to make "small" internal concepts visible and large by experiencing them kinesthetically and interactively. This process of externalizing mathematical ideas creates an environment conducive to experimentation and frees students from traditional confines and expectations of mathematics learning.

Arts integration methods foster access to the human creative process with collaboration, team building, and interpersonal work, as a result linking the affective and cognitive domains. A highly participatory arts integration activity serves as a bridge between the palpable to the abstract. This progression aligns with the principles of cultural-historical activity theory that emphasizes educational tasks that has students first investigating a concept through the study of real objects and activities in a lab setting. These guided activities are followed by carefully scaffolding inquiry that leads to the more abstracted, symbolic representation of a given concept. The performative aspect of arts integration activities cements their understanding gained through this progression from real, creative, social and physical activities to symbolic representations and gives learners personal connections to the curriculum being studied.

In the Fall 2014 semester, Bachman researched the effectiveness of Pattern Play by contrasting measures taken from that class to measures taken from a traditional mathematics class covering the same content. Students in Pattern Play outperformed the traditional control group students in the areas of mathematical content knowledge, attitudes toward mathematics, and persistence in problem solving[121]. Furthermore, the students in Pattern Play exhibited a greater use of the Standards of Mathematical Practice when asked to solve nonstandard tasks related to the content of the course. They persisted longer, recognized more patterns, spent more time understanding the problems, drew more pictures, described their solutions more fully, and arrived at correction solutions more frequently.

5.2.2.4 Math-Dance activities: an example

Title	**EXIT MOVE GAME**
Devised by	Chan and Stern
Revised by	Capozucca
Mathematical Concepts	Probability, central distribution, spread, sampling distributions, normal curve, data displays
Materials	Dice
Space	Large open room

[120] Thurston W. (2006), *On Proofs and Progress in Mathematics*. In: Hersh R. (ed.), *18 Unconventional Essays on the Nature of Mathematics*. New York, Springer, p. 41.

[121] Bachman R.M., Stern E., Chan J., Berezay K., Tripp L. (2016), *Math Dance: A Study of Effectiveness*. In: Adolphson K.A., Olson T. (eds.), *Proceedings of the 43rd Annual Meeting of the Research Council on Mathematics Learning*. Orlando, Florida, p. 132-139.

STAGES OF THE ACTIVITY	
Stage 1 WARM UP	Use the "What can it do?" (page 33) warm up to get students exploring what different parts of their bodies do. Challenge them to really explore (e.g., how do your legs bend, how else, what can you spine do, and what are multiple ways the arms can move?)
Stage 2 TRIO SHAPE GAME	• Dancers in groups of trios (pairs or quartets work too). Decide on an order. Dancer 1 makes a shape. Dancer 2 makes a shape that somehow relates to Dancer 1's shape. Dancer 3 relates to the two-person shape made by Dancers 1 & 2. And so on. The goal is to make conscious shape choices. Examples: do a similar shape, do the opposite shape, connect shapes, separate them, change one aspect such as the right arm, or facings, etc. • While circulating, prompt students to draw from a wide range of movement possibilities. Challenge them to vary their use of levels, spatial orientation, and dynamics. • As a class, discuss ways groups consciously chose to alter or relate to others' shapes. Create a list. Example: try a different level, mirror image, other side of the room, make a name for what you see and do the opposite of that name, do the same and change one detail. • Return to groups to experiment more. • The groups think about one shape they created that was particularly memorable or unusual or intriguing.
Stage 3 THE EXIT GAME	• Instruct each group to make four additional shapes by manipulating their first group shape using parameters the class listed in Stage 2. When done, each group has five shapes: the original one and the four new ones. NOTE: it's okay if the new shapes are very different from the original, or if they're similar. • Have each group come up with a dynamic way to exit the dance space. Discuss options as a class, but encourage groups to differ from each other. • Each group now has six dance "actions": an original group shape, four other shapes, and an Exit Move. Groups practice these actions. • Each group gets a die and assigns an action to each face of the die. For example, the Exit Move could be the face with one dot, and so on. • Each group rolls their die <u>until they get the number that corresponds to their Exit Move</u>. Someone in the group records the order. (For example, if the Exit Move is five, and they roll 3, 4, 3, 1, 3, 5, they stop at the five). Repeat this three times, recording the order of the rolls until the Exit Move number. Each group selects the roll length that was the middle amount of rolles. For example, if a group rolled 3 times until the Exit Move, 17 times until the Exit Move, and 6 times until the Exit Move, that group chooses the 6 roll sequence, since it was the middle amount of rolls. • Each group learns the dance of the roll sequence by putting the group shapes in that order, and inventing transitions between the group shapes. The transitions can be simple or elaborate. Rehearse. The key is that the group is able to perform without verbal cues. • Assign an order for the group performs. Remind the student to hold applause until the entire class performance is finished.

Stage 4
 	
THE FOLLOW-UP
	• Ask groups what they noticed about the dances and what was intriguing.
	• Eventually guide the discussion to the varying lengths of the dances.
	    o What do you think was the typical number of moves in each dance?
	    o How likely (or What's the probability) do you think it is to arrive at the dance length your group did?
	    o Make a scatterplot at the board.
	    o Draw a number line at the board from 0 to 20.
	    o Give each group a sticker and have them place the sticker on the scatterplot.

	- Continue the discussion about what values were typical or likely. Depending on what topic you link into, the discussions may take many turns at this point. - For central tendency - Have students think about what the best measure of "typical" would be and why. - Explicitly ask students to evaluate the mean, median, and mode of the data set they produced in the scatterplot. - How would the "typical" value be affected if many more trials were performed? - How would the typical value change if we weren't picking the middle length each time? - For inference - Describe the difference between conducting this experiment once and conducting it 1000 times. How would our scatter plot change? - If this experiment was repeated many more times, what range of lengths do you expect most of the data to fall within? Half of the data? All the data? - These questions open up discussion for confidence intervals and hypothesis testing, normal distribution, and the law of large numbers. - For probability - What is the probability a dance sequence has only one move? Two moves? Six moves? Twenty moves? - What is the probability has more than one dance move? More than two?

I think that the "Exit Game" activity helps prepare students to answer the questions of what is typical, what is likely, and what do you expect. First interacting with the problem through estimation and intuition gives students a basis of experiential understand to help them successfully navigating calculations and evaluating the reasonableness of their answers. I also enjoy how this activity serves as a gateway to discussions about a variety of probability and statistics topics such as central tendency, the difference between a sample/trial and a sampling distribution, and the likelihood of various events. Moreover, choosing how to make a shape that relates to someone else's shape is a rich physical and intellectual challenge. It's almost inexhaustible, and introduces pre-dance and pre-mathematical thinking skills by presenting students with essential questions, such as, "Why is my shape different (or similar)?" or "How can I make it more different?".

Another important aspect is that I have to carefully observe what groups are doing. If students are ready for a challenge, ask them how they can better fulfill what they're trying to do. I can push them a bit. For example, if a student wants to make a low shape to contrast a high one, ask if they are bending their legs as much as they can.

Chapter 6

Communicating Complexity

Could the experience of beauty arising from sensorial sources such as hearing and sight be used to improve both our own cognitive and emotional experience with mathematics? According to reports from last neural-scientific studies, the answer is yes[122]. But the thing is to embrace the beauty of mathematics, a non-trivial knowledge of the subject is required: a knowledge to which not everybody has access until after years of study. However, it is true that we can tell the story of mathematics and its beauty carrying the audience in a world where the boundaries among disciplines are fluid. The beauty of mathematics is different from the sensorial beauty: it represents the most extreme example of beauty because it depends on culture and learning. Only the most skilled among mathematicians can feel and look the beauty of a formula, or a geometric shape represented by equations, or the logical structure of a theorem statement or of a proof. On the contrary, regardless of Country and culture of origin, any individual may consider a visual scene, an art work or a piece of music beautiful or not, and she/he can appreciate them according to the intensity of the feelings arousing by the beauty when she/he looks at or listen to those stimuli.

I always thought that a mathematical theorem is not so different from a musical score. Both are the synthesis of a creative process written using a symbolism obscure to most of the people. If someone read a theorem with its proof or a score without having any knowledge of mathematics or music, she/he probably would be able to figure out nothing, least of all appreciate the beauty. Only an expert mathematician or composer could grasp the nuances at a glance, because their mind read and play every line by default. All those symbols have a profound meaning and value in their eyes. But talking about music, any individual can appreciate a piece of music without being necessarily a composer. The score can be played by performers (a musician, a singer, a band or even an orchestra) and who's listening can go with the flow and enjoy that piece of music. Thus, performers play a central role in communicating to the audience the "message" written in the score. They have to know better composers' background and what they want to convey through the composition and lyrics, master the musical instrument they're going to play, and breathe life into the score to make the performance available to the public. And the more passion they will put in the performance, the more who's listening will enjoy it. Then, maybe from there, someone in the audience would like to learn more about music composition and arrangement, or find out whether there are other versions of that piece of music, or even know what those black dots and symbols on that page full of parallel lines are. What is essential is that all can appreciate the beauty of the music at their own level of understanding and engagement. Why can't it be the same for mathematics?

---

[122] Zeki S., *Sesso, bellezza ed equazioni*. Scienza e filosofia, Il Sole 24 Ore domenica (23 aprile 2017).

My PhD research project lies within Science of Complexity curriculum. One of the doctorate key words is "better society". It becomes increasingly clear that in 21st century Europe, the growth, essential element for the future, can't just be economic; it should above all be in quality of life and knowledge. In this vision, science communication becomes a duty to raise public awareness of what's happening in the world around us in scientific terms.

Science of Complexity is a term used to describe a set of concepts, principles, propositions and ideas that have emerged and clustered together over the course of the 20th century. It encompasses more than one theoretical framework and is highly interdisciplinary. The concepts of complexity science present a way of better describing and understanding dynamics and processes of change found in a range of physical and biological phenomena. As a result, complexity science has been described as being *"at the forefront of science and mathematics"*[123], and is the focus of an impressive literature and an increasingly specialised vocabulary. The scholars in complexity science have identified underlying parallels in phenomena as diverse as the rise and fall of civilisations, the human immune system, the origins of life, the evolution of species, the workings of the human brain, the onset of psychiatric illnesses, ecological systems, genetic selection, flocking birds, the stock market, the world economy, the Internet and the Social Network. This breadth of coverage means that few fields of scientific endeavour have not been examined, in one way or another, by use of the concepts of complexity science. Scientists from a range of disciplines collaborate with computing experts to conduct interdisciplinary work on the application of complexity science to new fields and questions. Increasing attention is being paid to how the key ideas and concepts of complexity science can help researchers and practitioners to understand and influence social, economic and political phenomena.

In a work widely acknowledged as helping to popularise an understanding of perhaps the most famous element of complexity – namely, chaos – science journalist James Gleick describes how, in the 1960s and 1970s, a scattered group of scientists in the US and Europe started to investigate ways to understand and explain disorder and change. The most famous of these was Edward Lorenz, whose "butterfly effect" is still the strongest metaphor for chaos in the public imagination. The work of Lorenz was crucial because it was an early – and highly influential – step in illuminating a world of nonlinear dynamics. Deterministic modes of thinking, in which the future is seen as a straightforward extrapolation of past trends, were reassessed in light of this discovery. What happens after this breaking point is a story worth telling. Thus, I started wondering: *"What could be an effective way to communicate this story to the public?"*, *"How could I present the science of complexity and its main features in an engaging way?"*, *"Which languages and tools could be more suitable for this purpose?"*. After a period of reflection, I came to the conclusion that a show/exhibition would have been the answer. This led to the FOTCAB Project.

[123] Berreby D. (1998), *Complexity Theory: Fact Free Science or Business Tool?* Strategies & Business, 10, p. 40-50.

6.1 FOTCAB Project: Communicating Complexity

The acronym FOTCAB stands for "From Order To Chaos And Back" and is closely related to the idea underlying this project. FOTCAB aims to be a voyage of discovery and reflection on the concepts of order and chaos that at first sight seem to be worlds apart. But actually, the two aspects are not mutually exclusive because we can find disorder in order and order in chaos. The project is not intended to be a guided tour, but a completely immersive experience for the public, where they really feel like they are part of the experience: a multilayer path in which the public can choose what to see, what to hear and what to do. Therefore, the whole journey would like to stimulate reflections and personal research, and highlight, where possible, connections and similarities between the mathematics behind the scenes and its useful applications in everyday life. Presenting mathematics and science according to the culture and the interests of different audiences could become mine of ideas for new thoughts and new insights. Obviously, the goal is not to transform the whole public in mathematicians or scientists, but strive showing mathematics and science as meaningful human experiences that everyone is able to joyfully and surprisingly appreciate and engage with.

FOTCAB project is made up of two interlinked parts: a music/dance performance and an interactive exhibition. The first part foresees a 45 minutes play where dance, music, projections and live visual art performances on stage will take the audience on a journey "from order to chaos and back" going through the concepts of symmetry in time and space, linear and non-linear dynamical systems, fractal geometry, chaos theory, entropy, complexity, strange attractors, emergent behaviors and tendency to spontaneous order (synchronization). Every action performed on stage is inspired by mathematical or physical ideas, but no one will figure out what's going on. The audience will attend to a veritable and inspiring show during which will not fail interactive situations between the public and the performers. The second part, instead, starts right after the end of the first. Performers will get off the stage and invite the public to move to another place. There, three main areas (Order, Chaos, Synch) within which they will find interactive exhibits, hands-on locations, interactive experiments and experts at their disposal to go into rather more detail regarding mathematics and science behind the scenes. For example, there will be cross-sectional exhibits about pendulums and mirrors which are objects widely used during the performance both choreographically and visually (Fig. 6.1 and Fig. 6.2) to express order and chaos.

There will also be three dedicated areas where people will be able to interact directly with the dancers, the musicians and the computer graphic experts that have made the show and understand deeply how much and what kind of mathematics or science are behind that particular choreography, piece of music or projection; or "real world" zones in which will be presented some of the latest applications of science of complexity to real world problems such as weather forecasting, turbulence, population growth and others.

These two part of the project are strictly connected, but at the same time modular. To mean that, the public could decide freely which one to join in by taking into consideration their interests and curiosity, but only through partecipation in the whole "journey" they can have a deeper understanding and awareness of the mathematical and scientific message carried by the project.

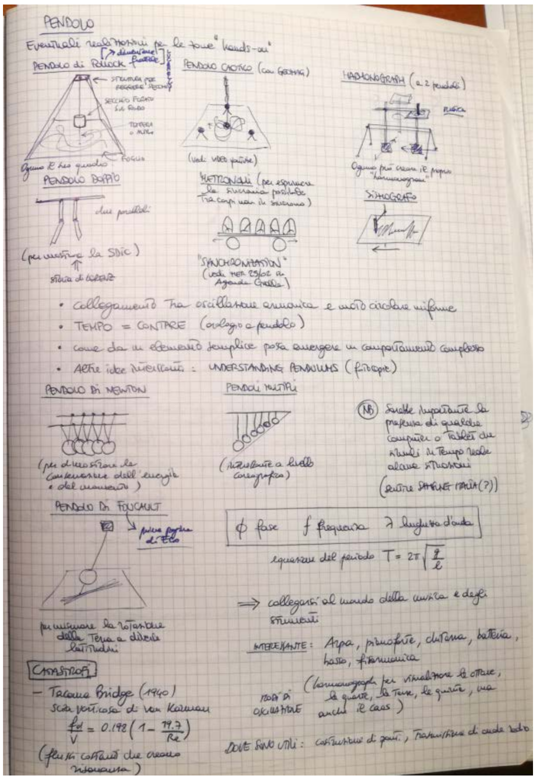


Fig. 6.1 – Sketches for pendulum cross-sectional exhibit

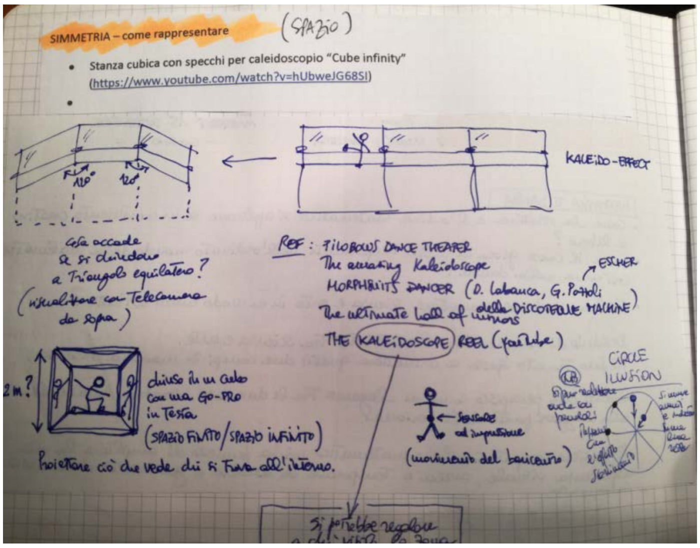


Fig. 6.2 – Sketches for choreography with mirrors

Thus, the main aims of the FOTCAB project are:

- to communicate mathematical and scientific ideas to general public using the languages of music, dance and visual art design;
- to promote interest in mathematics and science, and raise awareness of a variety of recent discoveries: the mathematics seen in schools is only a fraction of the mathematics out there;
- to experiment unconventional means of communication to arise public engagement with mathematics and science through an interactive and immersive experience;
- to conduct and consolidate academic research into the topic "mathematics, art and performing arts";
- to test new interdisciplinary approaches to maths communication working in an cross-disciplinary group to share skills and expertise;
- to produce high quality original art and scientific works;
- to create effective and powerful hands-on exhibits.

6.1.1 Implementing and developing the project

Since the very beginning of the project, I was able to count on the excellent cooperation of the Italian physicist and musician Marco Fermani to start and to develop the idea. Along the way, others have joined us: choreographers and dancers (Valentina Di Sante and Silvia Gambini), a musician (Simone Giorgini), videomakers (Alessandro Bravetti and his company Red Spot), a software engineer, a scenographer and even an architect. Furthermore, after having taken part in

Bridges Conference 2016 in Jyvaskyla (Finland), I started a remarkable collaboration with Kristof Fenyvesi[124] (University of Jyvaskyla) and Erik Stern[125] (Weber State University) which provided me with competent technical support, helpful insights and fruitful exchanges of views on the project. This interdisciplinary group has led to different but complementary points of view, improvement of our creative thinking and greater number of tools available. I strongly believe that knowledge transfer is more effective when different skills are able to interact in a cross-disciplinary approach. The different communication styles become binding devices of a shared quality and, at the same time, reasoning ideas on diversity in practice and in communication formats. We could contaminate each one's languages with the others. This has meant to have different and innovative tools to represent mathematical and scientific concepts that are inside the science of complexity. Each of us has put at the service of the others her/his own specialization and expertise, and learnt from the others how to have an effective integrated approach to the project. This contamination could be a risk, but it's not synonymous with superficiality, dispersion and amateurism. It is an opportunity to create special connections and synergies that can lead to deeper and original visions of those things which could be obtained in an internal logic to the individual disciplines. We are working hard sharing feelings about the best communication style and techniques to express complexity, order and chaos in a sound way, to make mathematics attractive and clear without lowering its level and avoiding too spectacular tones, and to create a "work of art". Chaos theory taught us to expect the unexpected and our hope is that the result of this project will be greater than the sum of its parts.

6.1.2 Project outline

I began working on the project shortly after the beginning of my first PhD year in December 2014. The first months were spent mainly sourcing links between mathematics and performing arts, and gathering resources. I used many books and websites in my research, but also had to equip myself with the programming skills needed for later in the project. Meanwhile, I started studying the mathematics behind the science of complexity and chaos theory more thoroughly; it is impossible to communicate mathematics without having a full understanding of it, so it was important to spend time doing this. For this purpose, I attended with success two online courses about complexity and chaos theory on Complexity Explorer project of the Santa Fe Institute. Around this time, I also began thinking the space and equipment needed to hold the show/exhibition.

By the end of October 2015, I had completed my necessary background research. Then,in the middle of March 2016, I spent a two weeks visiting period in UK at the University of Bath. During this period, I had the unique opportunity to visit the London Science Museum, the Royal Institution and some scientific exhibitions in Birmingham and Bristol, searching for scientific and artistic inspiration on how to make interactive hands-on areas and exhibits. At the same time, it was useful to gain valuable historical insight into my subject. In August 2016, I took part in Bridges

[124] He is a researcher of contemporary culture at University of Jyvaskyla's Department of Art and Culture Studies, member of the Board of Directors and Director of Community Events at the world's largest art and mathematics community, the Bridges Organization (www.bridgesmathart.org), chief executive officer of International Symmetry Association (www.symmetry.hu) and director of Experience Workshop – International Math-Art Movement for Experience-centered Education of Mathematics (www.experienceworkshop.hu).

[125] See Chapter 4, p. 16.

Conference at University of Jyvaskyla during which I presented a workshop about how to translate in music and dance ideas coming from complexity. It was a great opportunity to share my ideas with other expert colleagues on how to implement the show by balancing the mathematical and artistic dimensions. As stated above, it was here that I met Kristof Fenyvesi and Erik Stern. Bridges Conference was a moment of revelation which taught me a lot about how to communicate mathematics through performing arts and art in general.

On returning to Italy, I began to design and develop the show/exhibition more in detail, although I was simultaneously conducting further research into both mathematics and art connections to create hands-on exhibits suitable for the project. The first step was to choose what mathematical topics deal with that I believed had potential and scope for representation, and developed a couple of artistic ideas for each. From these I chose, aided by the interdisciplinary work group advices, what part of the show begin developing more accurately. We decided to focus our attention on the transition from order to chaos, and chose the logistic map as inspiration for creating a piece of music and a choreography. On July 2017, I took part in Bridges Conference 2017 in Waterloo (Canada) by giving a talk in which I presented the piece of music we composed starting from the logistic map as well as the related dance performance during the Bridges Family Day[126]. Then at the beginning of August 2016, I spent a week period in Ogden (Utah) at the Weber State University with Erik Stern to work more specifically on some concrete ideas together with his group.

By now, I'm continuing to work on the other parts of the show and to design and build the exhibits. FOTCAB project is still "work in progress" and probably its final version will be on stage not before 2019.

6.1.3 Listening to the Logistic Map

A part of my talk at Bridges Conference 2017 has become a short paper published in the annual Bridges Conference Proceedings on art and mathematics, reported here below. See Appendix D for Logistic Map Song score.

Listening to the Logistic Map

By Andrea Capozucca, Marco Fermani, Simone Giorgini

Abstract

A logistic map is one of the simplest nonlinear dynamical systems that clearly exhibits a route to chaos. In this paper, we present the creation of a musical piece based on a logistic map. We explored the evolution of the logistic map using a free sonification program called "Music2Chaos". We divided the one-dimensional interval [0,1] into eleven equal parts, and associated a note of a pentatonic scale to each segment. Every time an iteration took place a corresponding orbit was generated and translated into a sequence of notes. Then, we built up the main theme of the musical piece by choosing the sequences that best express the surprising array of dynamical behaviors of the logistic map, and assembled them according to harmonic rules of music. Time and musical arrangements were chosen to best fit and define the complete song structure.

[126] The Bridges Family Day is a free public event that offer a range of hands-on activities, workshops, games, exhibitions, and performances that connect mathematics with art. It usually takes place on the last day of the Bridges Conference.

Introduction

Difference equations arise in many contexts in the biological, economic and social sciences. Such equations, even though simple and deterministic, can exhibit a surprising array of dynamical behavior, from stable equilibrium points to cyclic patterns and even "chaotic" behavior, a regime which, although fully deterministic, is in many respect indistinguishable from a random process. The behavior of such equations can be studied by manipulating and processing large amounts of data via a computer. The results of such calculations often have to be transformed into visual information to be easily accessible to the human analyzer, and provide a clear vision of the dynamical system's global evolution (e.g., bifurcation diagrams or strange attractors). If we are able to "see" the behavior of such equations, why could we not "hear" them?

Converting the logistic map to music

The logistic map is, perhaps, the simplest example of how a nonlinear dynamic equation can give rise to very complex behavior. Initially introduced as a mathematical model of population growth, it rapidly found applications in diverse areas like mathematical biology, biometry, demography, condensed matter, physics, econophysics, and computation [1]. The logistic map function is defined as

$$x_{n+1} = kx_n(1 - x_n) \equiv f_k(x) \,, \qquad (1)$$

where k is a model-dependent exogenous parameter and x_n is the population in the *n*th-period, scaled so that its value fits the interval [0,1].

We used the outputs of the logistic map to generate music. The form of the piece is always subject to our artistic aims as composers taking care not to manipulate the raw material in an "extensive and idiosyncratic" way [2]. It is important that the music remains compelling to an audience and, unlike Babbitt [3], we must care if they listen.

Choosing a musical scale. We chose the pentatonic scale, first of all, for its universality. Humans show a natural predisposition towards the pentatonic scale. Pentatonic scales are found all over the world, from Pre-Columbian music to African music, Irish folk, Chinese music and the 35,000 year old bone flute from Hohle Fels. Furthermore, music performed using pentatonic scales does not have musical dissonances and always sounds pleasant. Several anthropologists have reported that we are innately prone to appreciate these melodic sounds and our ears prefer the simple mathematical ratios between frequencies [4]. We decided to use the A minor pentatonic scale (A-C-D-E-G), which is considered a gapped blues scale and comprises the same tones as the C major pentatonic, because the arrangement of notes in minor scales plays a large role in determining the mood of music.

Music2Chaos (M2C). This is a free sonification program[127] that allowed us to transform a sequence of numbers (the orbit generated from the iteration of the logistic map function f_k starting from an initial seed) into a sequence of notes visible within a red frame, easy to listen to and

[127] "Music2Chaos" has been created for the purpouse by Andrea Capozucca and Diego Zallocco.

exportable via a midi file. As shown in Fig.4, in the upper part of the M2C program window, we can choose the Midi Device, tempo in beats per minute (bpm), number of tracks to play simultaneously, number of iterations to calculate, the parameter k value to start the simulation, to play the sequence and to save it in a specific format. On the right, we have the red frame inside which the sequences are stored according to the number of iterations (we can choose the value of the initial seed $x_0 \in [0,1]$). On the left, the array of notes chosen for the simulation are shown: two A minor pentatonic scales starting with A3[128] and ending with A5. In this case, we divided the [0,1] interval into eleven equal parts and mapped one-to-one to a single note in the array. When the logistic map produces certain values, the program sets the corresponding note to the numeric output according to the interval partition (e.g, $x_2 = 0.70875$ corresponds to D5). After several iterations, it is possible to visualize the orbit obtained and listen to the musical sequence (Fig. 6.3).

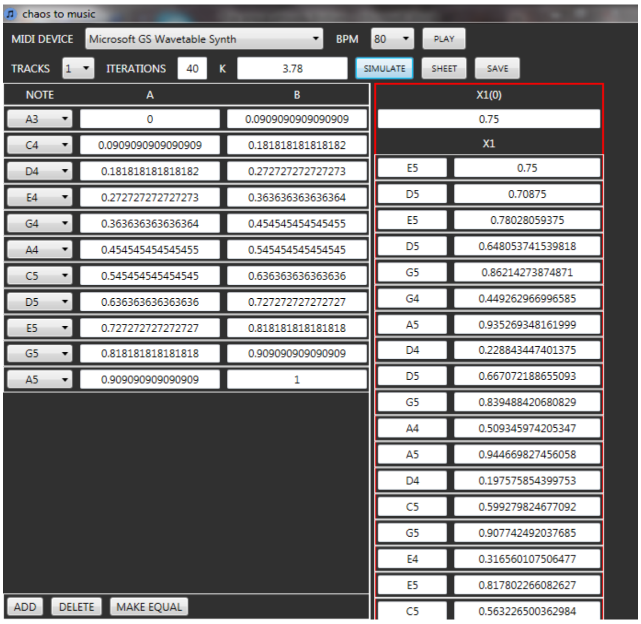


Fig. 6.3 – A snapshot of “Music2Chaos” interface window

Choosing a starting note. We wanted to give a sad and weird feeling to the musical piece. So we chose A3 as the starting note for every simulation regardless of parameter k value. For that to happen, we chose as the initial seed for every simulation a value within the range [0,0.090909] corresponding to the A3 note. $x_0 = 0.078$ was the choice that leads to music the authors liked. In addition, starting from A3 allowed us to get ever-increasing note sequences concerning stable fixed-point behavior of the logistic map ($1 < k < 3$).

Instruments. We decided to use a contrabass, an accordion and a synthesizer because their tones best fit the desired mood of the musical piece. Along the track, a musical interplay developed that created movement and flow within the song and musically described different feelings and settings for every system state. Moreover, we used 8 different beat-box types for the rhythm section, to increasingly emphasize the sound picture we had in mind. Beat-box is present for the duration of the song, except at some crucial steps.

[128] A3 is the 3rd octave regular A in a standard 8-octave keyboard.

K-sequence generation and final assembling

We divided the musical piece into three main sections: a stable fixed-point (SFP) section, a periodic even cycle (PEC) section and a chaotic (CPW) section with periodic (odd cycle) windows inside. Using M2C, we generated several orbits choosing appropriate values for k. The tempo was set at 120 bpm.

k	Note sequences	Fixed point x
0.8573	A3 A3 A3 A3 A3 **A3** ...	0
1.19	A3 A3 C4 C4 C4 C4 **C4** ...	0.15966386...
1.3687	A3 C4 C4 C4 C4 D4 **D4** ...	0.26937970...
1.5224	A3 C4 C4 D4 D4 E4 **E4** ...	0.34314241...
1.7957	A3 C4 D4 E4 G4 G4 **G4** ...	0.44311410...
1.9042	A3 C4 D4 E4 G4 A4 **A4** ...	0.47484507...
2.2271	A3 C4 E4 A4 C5 C5 **C5** ...	0.54954954...
2.7804	A3 D4 G4 D5 C5 D5 C5 D5 C5 **D5** ...	0.64028776...

SFP section ($0 \leq k < 3$)

We built this section assembling the note sequences shown in the table on the left, comprised of five beats each. The duration of every single note was established following two simple rules:
1) the first part of the sequence was emphasized to highlight the speed of the convergence to stability;
2) a standard duration (2/4) was assigned to notes corresponding to a fixed-point value.

In this section, all the note sequences in the table above are played by the contrabass, whereas the accordion just plays a musical accompaniment, in addition performing the breath beat-box. Note duration can be freely chosen to improve the musicality.

k	Note sequences	2^n-cycles
3.0214	A3 ... **D5 C5** ...	2-cycle
3.089	A3 ... **E5 C5** ...	
3.1987	A3 ... **E5 A4** ...	
3.2845	A3 ... **G5 A4** ...	
3.4452	A3 ... **G5 G4** ...	
3.4972	A3 ... **G5 G4 G5 A4**...	4-cycle
3.5523	A3 ... **G5 G4 G5 A4 G5 E4 E5 A4** ...	8-cycle

PEC section ($3 < k < 3.569946$)

To better express the periodic behavior, we purposely chose to cut the first part of the sequences and take into account only the notes contained in the cycles[129] (see the bold notes on the left table). We also decided to match each cycle with a musical mood (2-cycle/waltz, 4-cycle/funk, 8-cycle/reggae) to convey different feelings to each system state.

[129] We generate more 2-cycle sequences than 4-cycles or 8-cycles to better show what happen after the first bifurcation point for $k = 3$. After this point, the two branches of the bifurcation diagram tends to broaden due to different stable point values.

Unfortunately, we cannot “observe” higher periodicity (like a 16-period or higher) with this setup because of the discretization used for the interval [0,1]. In order to increase the periodicity, we would have to divide the [0,1] interval into more than eleven parts, which would create too many notes to handle and, therefore, less musical enjoyment.

In this section, we disregarded the beginning notes of every sequence taking into account only the bold notes in the table above, corresponding to that stable cycle. Furthermore, we introduced an interesting role reversal between instruments to better express these periodic behaviors. In 2-cycle/waltz and 8-cycle/reggae the contrabass plays the notes, whereas the accordion is part of the arrangement. In the 4-cycle/funk, the role of the instruments was interchanged.

k	**Note sequences**	**Behavior**
3.65764	A3 D4 A4 E5 D5 E5 D5 E5 C5 G5 E4 G5 A4 A5 E4 E5 D5 G5 ...	Chaotic
3.70164	A3 D4 D5 E5 D5 C5 A5 E4 E5 D5 **G5 A4 A5 D4 D5 E5 D5**...	7-cycle
3.72048	A3 D4 E5 E5 D5 E5 D5 E5 C5 A5 E4 E5 D5 E5 C5 G5 G4...	Chaotic
3.7383	A3 D4 E5 E5 E5 D5 E5 C5 G5 G4 G5 E4 **G5 A4 A5 D4 D5** ...	5-cycle
3.7964	A3 E4 E5 D5 E5 C5 G5 G4 G5 E4 G5 A4 A5 D4 C5 A5 D4...	Chaotic
3.82843	A3 E4 E5 D5 E5 C5 A5 D4 D5 G5 A4 A5 **C4 A4 A5** ...	3-cycle
3.9375	A3 E4 E5 C5 A5 D4 D5 G5 G4 A5 C4 E4 G5 G4 A5 A3 A3...	Chaotic

CPW section $(3.569946 < k \leq 4)$

We assigned a standard duration (1/4) to chaotic notes and a blues/swing mood to the section with the initial setup. A synthesizer played the chaotic sequence and instruments played a musical improvisation over the track. In this section, the beat-box is 1/4 ahead of the beat to avoid overlapping contrabass. Moreover, a periodic window appears every 12 beats: first 7-cycle, then 5-cycle and finally 3-cycle (see left table). The musical piece ends by speeding up the beat until it becomes white noise.

In this section, chaotic sequences played by the synthesizer exhibit endless variations within the limits of the scales utilized. Instrumental improvisation and composer add-ons enhance this underlying structure. Moreover, the piece presents three periodic windows at regular intervals representing the most significant periodic behaviors in the chaotic region.

Conclusion

Turning the logistic map into a musical piece that makes sense is no trivial task, especially regarding its chaotic behavior. Finding a correct subdivision of [0,1] intervals and, consequently, an appropriate array of notes is not difficult, but is also not guaranteed. Furthermore, in SFP and PEC sections, we have to face a problem of orbit quick convergence that forces us to carefully choose specific k values to generate meaningful sequences. In CPW section, the use of the A minor pentatonic scale proved to be useful to blend a compelling chaotic background structure with the whole arrangement. We agree with Truax’s statement that “the musicality may reside in the musical knowledge of the mapper than in the source function"[5]. But there is an inevitable level of error in our attempt at describing a logistic map through music, either in terms of rigor or in terms of communicability. Finally, to better understand the musical piece as a description of a logistic map, behaviors could be displayed with a bifurcation diagram with appropriate text and graphic representations to complement its listening. It could also be a different way to present, in a classroom, the practical and creative applications of non-linear dynamic systems.

References

[1] M. Ausloos, M. Dirickx (eds.). *The Logistic Map and the Route to Chaos: From the Beginnings to Modern Applications*. Springer, New York (2006)

[2] R. A. Bidlack. *Chaotic systems as simple (but complex) compositional algorithms.* Computer Music Journal (1992), p. 33-47.

[3] M. Babbitt. *Who Cares if You Listen?*. High Fidelity (Feb. 1958)

[4] F. J. Ballesteros. *E.T. Talk: How We Will Communicate With Intelligent Life on Other Worlds*. Springer (2010), p. 160.

[5] B. Truax. *Chaotic non-linear systems and digital synthesis: An explanatory study.* Proceedings of the International Computer Music Conference (ICMC 1990), p. 100-103.

Chapter 7

Conclusions

The foreseeable future is going to be one of unprecedented pervasiveness of mathematical thought throughout the sciences. In a data driven world, mathematical concepts and algorithmic processes will be the primary navigational tools. This makes mathematics increasingly important for many of the science and engineering advances to come, but also for ensuring an active and aware citizenship to the society at large. Along these lines, mathematics communication is increasingly becoming a field of advanced research, which will require strong innovation and development. If I compare the need for mathematical skills of future generations to the current need for reading skills, it will take a long time to achieve widespread mathematical literacy as well as it took a long time to achieve widespread reading literacy. As Hilbert declared in his famous 1900 address to the International Congress of Mathematicians in Paris, *"a mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street."*. Thus, we must make mathematics visible and appealing. There are many ways of doing this and I've shown some of them thoroughly in Chapters 4, 5 and 6.

The main aims of my research project have been to find some effective ways to communicate complexity theory to the general public and use ideas taken from complexity theory to improve science and mathematics communication. That required an intensive study of science communication framework with its methods and approaches, a deep understanding of the language of complexity, emergent phenomena and, more generally, non linearity as key tools for innovative proposals in the world of science communication, and a thoughtful investigation of appropriate interdisciplinary ways to communicate mathematics. Without a huge amount of literature on this, I've made it a point to get to know the current state of the art of mathematics communication in Europe and I've done it through a series of interviews with the greatest European experts in that field. I've collected three of these interviews in Chapter 2, each one dedicated to a specific aspect of maths communication: the first with Chris Budd on engagement and outreach activities, the second with Alex Bellos on storytelling, and the third with Andrew Jeffrey on magic and teaching. At the same time, I've studied in detail science and mathematics communication actors, methods, techniques and approaches through available literature, as shown in Chapter 3. This initial exercise has been intense but necessary, and has laid the groundwork for carrying on my research and dealing with the complexity embodied in communicating mathematics.

With this in mind, I've studied specific aspects of maths communication in-depth like engagement and outreach, storytelling, laboratorial approach to learning and interdisciplinarity. I've set up and first-hand experienced projects and activities strictly related to them. Some engagement and outreach activities, as presented in Chapter 4, have been valuable opportunities to see their impact on the public and understand how essential both mathematical and communication skills are to maintain an active participation when presenting a certain kind of activities rather than others. I've understood that each of them has its pros and cons. Doing math busking, for example, has proven to be more difficult than carrying out a science cafe because you have to approach people from scratch. Set up a science festival, instead, requires a multi-faceted scientific knowledge because of the wide variety of the proposals. In order to reach a successful performance, appeal must be differentiated when I'm outreaching to non-expert people depending on the targeted audience. Regardless of the specific activity, I've also realized that it's crucial to share my enthusiasm with the audience and be clear about the message I want to convey. Maths communication is not something I can just jump right into, but it requires constant training, proper preparation and excellent grasp of the issues and methods.

Then, through the book "Il tranello e la soluzione matematica" and math-dance laboratories, I've directly tested how important the use of storytelling and laboratorial approach are within the teaching/learning process. These approaches don't deny the traditional approach to mathematics, but would like to be other doors to open to observe and discover the mathematical world from another point of view. Even popularization techniques can come in handy to teachers and help students to achieve meaningful results and self-confidence with mathematics. That's why in Chapter 5 I've argued that they could be innovative, although there have always been in human path to self-knowledge, and interactive approaches to improve maths teaching and students awareness and interest in mathematics. We still have a long way to go, but we've set the right course.

In the light of experience gathered during the implementations of projects and activities described above, I've really understood how complex communicating mathematics is and how many obstacles we have to overcome before reaching a successful communication. Thus, I've poured all my acquired knowledge and expertise in communicating complexity science to the public through the FOTCAB project. As shown in Chapter 6, it is a huge and challeneging project that involves music, dance, visual graphics, hands-on exhibits and different communication styles: a high interdisciplinary context in which different skills and visions creatively interact to give form and coherence to ideas taken from complexity and chaos. There is clear evidence from what I've already made that the project has managed to successfully communicate mathematics behind complexity and chaos through art, at least to the people who attended the workshops and the performance "From order to chaos". Undoubtedly, to determine exactly how effective performing arts are at communicating mathematics would require much further study, but it seems as though the performing arts, as well as art in general, function perfectly for catching people's eyes and that a well-crafted hands-on exhibit with well-written explanation and a high engagement factor holds people's attention for long enough for them to absorb the information. Furthermore, this math-art approach has proved to be an extremely valuable tool in teaching mathematics, as shown in Chapter 5 explaining math-dance activities. By providing people with a novel settings for mathematics and showing them a variety of topics coming from complexity and chaos, the project allowed them to develop their interest in, and knowledge of, mathematics in accordance with their interests, attitudes and expectations, sometimes without even realising it. Although communicating mathematics through performing arts would be of limited use in preparation for exam-based assessment, it provides people with a wider mathematical background and meaningful emotional live experience encouraging general learning and awareness.

In carrying out my engagement and outreach activities, I've encountered various limitations. The first has been with regards to funding. Implementing, developing and setting up relevant and effective communication activities requires medium-high budget. There are a lot of cost items related to them such as video projection and sound systems, production of materials, equipments and so on without taking into account staff involved. For example, more than half of the budget at our disposal for FermHAmente Science Festival was used for operating costs. In order to guarantee the quality of the "products", a proper funding is necessary. Thus, each time I plan an engagement and outreach activity I have to include inside the planning a fund rising action to ensure activity's success.

Another limitation is the amount of time needed to implement and develop projects like these. In my case, this time has been largely due to the breadth of the subjects concerned, especially in FOTCAB project: although I've managed to satisfy all my aims in the time I had available, there is vastly more material that I could have covered in most stages of my project. This has forced me to make choices about what mathematical topics to propose, what communication style to use, what hands-on exhibits to build and how to get there. More time means room for improvement of the quality of my research and of what I'm going to make. But I've realized that the best way to overcome this limitation is teamwork and focused on one thing at a time. It's better less doing right than much doing wrong.

A further limitation is the space required to carry out the activities. Finding the appropriate location where to run a math busking or a science cafe or a festival carries more weight than one may realize. For example, a noisy cafeteria could jeopardize a science cafe success as well as run science festival events in locations too far apart could discourage people to take part in all. Every detail counts and helps the public to feel comfortable and safe by allowing them to concentrate on the activities or talks or whatever else is going on.

Finally, another limitation comes from the communication of mathematics itself. It has been much harder than I thought to put myself in the shoes of someone who hasn't a mathematical background. Meeting people, engaging them with mathematics, carrying out activities for students and teachers, performing music and dance require a wide range of skills and the ability to manage a dynamic situation which is constantly changing in mood, participation and perception. Often it's not enough to be good in mathematics. It seems paradoxical but this is what happens when it comes to communication.

This study presents some issues that deserve future investigations. In order to have a more comprehensive vision about the European state of the art, other maths communication aspects will be treated in the forthcoming interviews with Simon Singh and Michele Emmer on non-fiction and documentary films, Kristof Fenyvesi on maths through art, Andreas Matt on visual and interactive exhibitions, Rogerio Martins on TV programs, Eduardo Saenz De Cabezon on talks and math busking and others. It has been impossible to include these interviews within this thesis for a matter of timing. As mentioned earlier at the end of Introduction, each interview will become an article in the series "Communicating mathematics in Europe" in the quarterly magazine *Lettera Matematica*.

In my research project I've tried to create the conditions for possible new scenarios on communicating mathematics that can be adaptable and flexible to the public. For this reason, I will further develop engagement and outreach activities in collaboration with the U.S.O. communication group by starting with those already active such as FermHAmente Science Festival, Scienza in Vacanza and science cafes, as well as set up new ones. For example, I've started to work on a format called "Scienza in Contrada" in which there will be engaging and interactive talks and experiments on science and mathematics of the past in an historical context with a view to the future. The slogan will be: "understand the past to learn more about the future".

Then, I would like to take the book "Il tranello e la soluzione matematica" to a new level by turning it into a theatre play. It could be an interesting way to test if the approach to mathematics used in the book will still work with a live audience. Such an approach will need a progressively tightening scientific basis. That's why I'll do my best to create practical forms of synergy among the members of the interdisciplicary group for this project. The hope is that these forms will acquire theorical and methodological dignity both in mathematics communication activities and in teaching practice.

I will continue my collaboration with Erik Stern and Kristof Fenyvesi for what concern the math-art approach to learning and communication. What is important is to continue sharing ideas and seeking advice from them with regards to both the quality of the art and mathematics proposed and the success of our mathematical communication activities. I'm going to keep experimenting laboratorial approach with students through new math-dance activities and training courses for teachers organized in co-operation with Mauro Comoglio[130] for MathUp courses[131].

Finally, I will implement and develop FOTCAB project by following four main strands:

- Dance – At the beginning of 2018, it will begin a new partnership with the University of Malta in addition to that with Erik Stern and Weber State University. We will work in synergy to build the whole choreography in the first part of the project and to consolidate my research into mathematics and dance more formally.
- Music – After the "Logistic Map Song", we will go further with music composition aimed at the show. We are also developing three hands-on exhibits for every dedicated exhibition area (order, chaos and synch).
- Visual graphics – This group is in a brainstorming phase and is waiting for a complete vision of the whole performace structure. In the meantime, we are implementing computer simulations that will be useful for exhibits.
- Hands-on exhibits – We've almost finished the design phase. Next step will be to measure the construction costs and search for funding.

Throughout this project, I've had many opportunities to both contribute to society and make personal gain. Primarly, I got a unique opportunity to pass on my knowledge of something I'm passionate about to members of the general public, as well as to colleagues coming from arts and performing arts, and in a novel way. Looking at those first results, I believe that the final project will not only show what a varied and exciting subject mathematics is, but encourage people of all ages to take more interest in learning more about the subject, at least I hope. Complexity and chaos

[130] He teaches mathematics at "Liceo Massimo D'Azeglio" (Turin) and has a long collaboration with the Bocconi University Pristem Research Center. He is interested in popularization and history of mathematics.

[131] They are training courses for all educational levels maths teachers based on laboratory approach to teaching/learning process.

could give to the public a new and unexpexted view on mathematics and make them aware that any mathematics exists beyond what they did at school. Moreover, the interdisciplinary nature of this project has given me a chance to research and link the mathematical and artistic worlds of great interest to me. This has allowed me to develop a range of transferable skills over the course of the project so far and has enhanced my music performing skills giving me further inspirations for the continuation of the project itself.

I've developed a 15% of the whole project to this day. I am extremely pleased with the initial outcome of my project and the good reviews received from the audience after the talk and workshop held in Waterloo last summer during Bridges 2017. At the same time, I appreciate that there is a lot that could have done better and there is a vast scope to further the project.

Appendix A

Ten Laws of Human Communication

By Mackay H. (1994), *Why Don't People Listen? Solving the Communication Problem*
Pan Macmillan Australia

Australian psychologist and social researcher, Hugh Mackay's "10 Laws of Human Communication" has become the go to guide for the Public Relations Industry. It's not only for the PR pros though. It is a great set of rules that can help us all better improve communication with each other.

1. It's not what our message does to the listener, but what the listener does with our message, that determines our success as communicators.
2. Listeners generally interpret the message in ways which make them feel comfortable and secure.
3. When people's attitudes are attacked head-on, they are likely to defend those attitudes and, in the process, to reinforce them.
4. People pay most attention to messages which are relevant to their own circumstances and point of view.
5. People who feel insecure in a relationship are unlikely to be good listeners.
6. People are more likely to listen to us if we also listen to them.
7. People are more likely to change in response to a combination of new experiences and communication than in response to communication alone.
8. People are more likely to support a change which affects them if they are consulted before the change is made.
9. The message in what is said will be interpreted in the light of how, when, and where and by whom it is said.
10. Lack of self-knowledge and unwillingness to resolve our own internal conflicts makes it harder for us to communicate with other people.

Appendix B

Festival survey fac-simile

Small survey to be completed by each family or individual.

Number of adult(s)	Number of children (<18)
I/we enjoyed the talk[132]	☐ Strongly agree ☐ Agree ☐ Neither ☐ Disagree ☐ Strongly disagree
I/we understood the talk	☐ Strongly agree ☐ Agree ☐ Neither ☐ Disagree ☐ Strongly disagree
I/we will look into this area more at home	☐ Strongly agree ☐ Agree ☐ Neither ☐ Disagree ☐ Strongly disagree
At least one of us got a degree	☐ Agree ☐ Disagree
I/we have been to an event like this before	☐ Agree ☐ Disagree
I/we will return to the event	☐ Agree ☐ Disagree
I heard about the event via	....
Further thoughts	

[132] There were four more surveys in which the word “talk” was replaced by “workshop”, “live show”, “exhibit” and “festival” respectively.

Appendix C

Before/After questionnaires

Before starting and after finishing the laboratorial activities related to the book “Il tranello e la soluzione matematica”, I gave to school students that joined the project a questionnaire to fill out. These are the templates of those questionnaires that helped me with the survey.

BEFORE Questionnaire

Name:	Age:	Grade:
1. How is your relationship with mathematics?		☐ Good ☐ Normal ☐ Bad
2. Explain why.		
3. Do you carry out classroom activities with your teacher?		☐ Yes ☐ No
4. How many occasions per year?		
5. How would you define your engagement with mathematics during these activities?		☐ Good ☐ Normal ☐ Bad
6. Mathematics is all around us. What’s your level of awareness about this fact?		☐ Good ☐ Normal ☐ Bad

7. Look around and list three things or objects that, according to you, have to do with maths. …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………
8. Explain why. …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………

AFTER Questionnaire

Name:	Age:	Grade:
Now that you've read the book and taken part into laboratorial activities strictly linked to some mathematical topics that are presented inside the book, answer honestly to the questions:		
1. How is your relationship with mathematics?	☐ Good ☐ Normal ☐ Bad	
2. Explain why. …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………		
3. Which topics have been more interesting? ……………………………………………………………………………………………		
4. Have you found useful the classroom activities?	☐ Yes ☐ No	

5. Explain why. …………………………………………………………………………………………………… …………………………………………………………………………………………………… …………………………………………………………………………………………………… …………………………………………………………………………………………………… ……………………………………………………………………………………………………	
6. How would you define your engagement with mathematics during these activities?	☐ Good ☐ Normal ☐ Bad
7. What difficulties have you encountered? …………………………………………………………………………………………………… …………………………………………………………………………………………………… ……………………………………………………………………………………………………	
8. Mathematics is all around us. What's now your level of awareness about this fact?	☐ Good ☐ Normal ☐ Bad
9. Which activity has made you more aware? …………………………………………………………………………………………………… ……………………………………………………………………………………………………	
10. Explain why. …………………………………………………………………………………………………… …………………………………………………………………………………………………… …………………………………………………………………………………………………… …………………………………………………………………………………………………… ……………………………………………………………………………………………………	

Appendix D

Logistic Map Song Score

Logistic Map Song

Capozucca / Fermani / Giorgini

2


Am F Am G Dm G Dm G Dm G Dm G Dm Em F Em
Am F Em Am F Em Am F Em
Funky
x8
Reggae
x8
Blues Mood (12 + 12 Beat)
/E
x8
Blues Mood (12 Beat)
/E
x8
Blues Mood (12 Beat)
/E
x8
Blues Mood (12 + 12 Beat)

References

AAVV (2010), *Science and Technology Report*. Special Eurobarometer 340 / Wave 73.1 – TNS Opinion & Social, p. 8-24

Attard C. (2012), *Engagement with Mathematics: What Does It Mean and What Does It Look Like?* APMC, 17 (1), p. 9-13.

Attard C. (2015), *Engagement and Mathematics: What Does It Look Like in Your Classroom?* Journal of Professional Learning, Semester 2. Available at www.cpl.asn.au/journal.

Ausloos M., Dirickx M. (eds) (2006), *The Logistic Map and the Route to Chaos: From the Beginnings to Modern Applications*. New York, Springer.

Babbitt M. (1958), *Who Cares if You Listen?*. High Fidelity.

Bachma n R.M, Stern E., Chan J., Berezay K., Tripp L. (2016). *Math Dance: A study of Effectiveness*. In: Adolphson K.A., Olson T. (eds.), *Proceedings of the 43rd Annual Meeting of the Research Council on Mathematics Learning*. Orlando FL, p. 132-139.

Ballesteros F.J. (2010), *E.T. Talk: How We Will Communicate With Intelligent Life on Other Worlds*. Springer, p. 160.

Baram-Tsabari A., Lewenstein B.V. (2017), *Preparing Scientists to Be Science Communicators*. Preparing Informal Science Educators, Springer.

Behrends E., Crato N., Rodrigues J.F. (eds) (2012), *Raising Public Awareness of Mathematics*. New York Dordrecht London, Springer Heidelberg.

Bellos A. (2010), *Alex's Adventures in Numberland*. Bloomsbury, London.

Bellos A. (2014), *Alex Through the Looking Glass*. Bloomsbury, London.

Bellos A. (2016), *Can You Solve My Problems? A Casebook of Ingenious, Perplexing and Totally Satisfying Puzzles*. Guardian Faber, London.

Benneworth P. (2009), *The Challenges for 21st Century Science – A Review of the Evidence Base Surrounding the Value of Public Engagement by Scientists*. Univesiteit Twente.

Berreby D. (1998), *Complexity Theory: Fact Free Science or Business Tool?* Strategies & Business, 10, p. 40-50.

Bertuglia C.S., Vaio F. (2003), *Non linearità, caos, complessità*. Saggi Scienze, Bollati Boringhieri.

Bertuglia C.S., Vaio F. (2011), *Complessità e modelli*. Saggi Scienze, Bollati Boringhieri.

Bidlack R.A. (1992), *Chaotic systems as simple (but complex) compositional algorithms.* Computer Music Journal, p. 33-47.

Biehler R., Scholz R.W., Strässer R., Winkelmann B. (eds) (2002), *Didactics of Mathematics as a Scientific Discipline*. Dordrecht, Kluwer Academic Publishers.

Bischi G.I. (2015), *Matematica e Letteratura: dalla Divina Commedia al noir*. Alice & Bob 42, Egea.

Bradley M.J. (2006), *The Foundation of Mathematics*. Infobase, New York.

Brillon R. (2014), *Choreograph Math: Teaching Geometric Concepts Through Dance Movement and Design*. Doctoral dissertation. Retrieved from ProQuest Dissertation & Theses database, UMI n.1495482.

Bruner J. (1987), *Life as Narrative*. Soc. Res. 54, p. 11–32.

Bruno J. (2011). *Math Movement: The Integration of Geometry and Dance*. Doctoral dissertation retrieved from *ProQuest Dissertations & Theses database*. UMI No. 1495482.

Bucchi M. (2008), *Of Deficits, Deviations and Dialogues: Theories of Public Communication of Science*. In Bucchi M., Trench B. (eds), *Handbook of Public Communication of Science and Technology*. London and New York, Routledge.

Bucchi M., Trench B. (2016), *Science Communication and Science in Society: A Conceptual Review in Ten Words*. Tecnoscienza, Italian Journal of Science & Technology Studies, 7(2), p. 155-156.

Buescu J., Rodrigues J-F. (2012), *The Importance of Useful Mathematics: On Tools for its Popularization, from Industry to Art*. In: Beherends et al., *Raising Public Awareness of Mathematics*. Springer-Verlag Berlin Heidelberg, p. 313-330.

Bultitude K. (2011), *The Why and How of Science Communication*. In: Rosulek P. (ed), *Science Communication*. Pilsen, European Commission.

Bultitude K., Grant L. (2006), *2005 Institute of Physics Schools and Colleges Lecture Tour*. Graphic Science Unit.

Bush V. (1945), *Science: the Endless Frontier, a Report to the President*. Washington, United States Government Printing Office. Available at https://www.nsf.gov./od/lpa/nsf50/vbush1945.htm

Calvino I. (1968), Interview on “L’Approdo letterario” magazine. In: Calvino I., *Una pietra sopra*. Milano, Mondadori.

Capozucca A. (2017), *Di crimini, di numeri e di TV*. In: Bischi G.I., Klaver J.M.I., *Noir 2.0 – Il lato oscuro di Internet*. Collana Urbi*noir*-studi, Aras Edizioni.

Carrada G. (2005), *Comunicare la scienza. Kit di sopravvivenza per ricercatori*. I Quaderni del MdS.

Carroll L. (1998), *Una storia intricata – Racconti matematici*. Stampa Alternativa, Nuovi Equilibri.

Carroll L. (2007), *Alice nel paese delle meraviglie*. Classici Illustrati, Mondadori.

Catterall J. S. (2002). *The Arts and the Transfer of learning*. In: Deasy R.J. (ed.), *Critical links: Learning in the Arts and Student Academic and Social Development*. Washington, DC, Arts Education Partnership, p. 151-157.

Caulton T. (1998), *Hands-on Exhibitions: Managing Interactive Museums and Science Centres*. London and New York, Routledge.

Cerrato S. et al. (1996), *Caos e complessità*. Quaderni di divulgazione, Laboratorio interdisciplinare SISSA, Cuen srl.

D'Amore B., Pinilla M.I.F. (2007), *Modi di comunicare*. La vita scolastica, vol.61, n.16, p. 20-22.

Davies S. et al. (2008), *Discussing Dialogue: Perspectives on the Value of Science Dialogue Events That do not Inform Policy*. In: Public Understanding of Science, 18 (3), p. 338-353.

Davydov V. V. (2008). *Problems of Developmental Instruction: A Theoretical and Experimental Psychological Study*. New York, Nova Science Publishers, Inc.

Dedò M. (2012), *Rigour in Communicating Maths: a Mathematical Feature or an Unnecessary Pedantry?* In: Behrends E. et al. (eds), *Raising Public Awareness of Mathematics*. Springer-Verlag Berlin Heidelberg.

Delano Moore S. (2002), *Teaching Geometry and Measurement Through Literature*. Mathematics Teacher in the Middle School, 8 (2), p. 78-84.

Doxiadis A., Mazur B. (2012), *Circles Disturbed: The Interplay of Mathematics and Narrative*. Princeton University Press.

Eccles J.S., Wigfield A. (2002), *Motivational Beliefs, Values and Goals*. Annual Review of Psycology, 53 (1), p. 109-132.

Einsiedel E. (2014), *Publics and their Partecipation in Science and Technology: Changing Roles, Blurring Boundaries*. In: Bucchi M., Trench B. (eds) *Routeledge Handbook of Public Communication of Science and Technology*. London and New York, Routeledge, p. 125-139.

ESF Forward Look on "Mathematics and Industry" (2010), European Science Foundation, Strasburg. www.esf.org

Euler L. (1768), *Lettres a une Princesse d'Allemagne sur divers sujets de Physique et de Philosophie*. Petersburg. A third edition of the English translation was published in Edinburgh in 1823 and is available at http://www.euelrarchive.org.

Gardner H. (1983), *Frames of Mind: The Theory of Multiple Intelligences*. Basic Books Inc. Publishers.

Gleick J. (1988), *Chaos: Making a New Science*. Penguin Books.

Gouthier D. (2002), *Termini e linguaggio per comunicare la matematica*. JCOM 1 (2), June.

Gouthier D., Pitrelli N., Pupolizio I. (2001), *La lingua perfetta e i matematici: il caso di Giuseppe Peano*. Jekyll.comm, 1, SISSA.

Greco P. (2004), *Il modello Venezia. La comunicazione nell'era post accademica della scienza*. In: Pitrelli N., Sturloni G. (eds), *La comunicazione della scienza. Atti del I e II convegno nazionale*. Roma-Milano, Zadig, p. 11-38.

Greuel G-M. (2014), *Mathematics Communication for the Future. Mission and Implementation*. Berlin, Imaginary. Available at www.imaginary.org/background-material

Greuel G-M. (2015), *Mathematics between Research, Application and Communication*. EMS Newsletter 97, p. 44-51.

Gross A.G. (1994), *The Roles of Rhetoric in the Public Understanding of Science*, Public Understanding of Science 3(1), Sage Journal, p. 3-23.

Hanna J.L. (1999). *Partnering Dance and Education: Intelligent Moves for Changing Times*. Champaign, Human Kinetics.

Hanna J.L. (2000). *Learning Through Dance: Why Your Schools Should Teach Dance*. American School Board Journal, 187(6), p. 47-48.

Hanna, J. L. (2015). *Dancing to Learn: The Brain's Cognition, Emotion and Movement*. Lanham, Rowman & Littlefield Education.

Hansen V.L. (2002). *Popularizing Mathematics: From Eight to Infinity*. In: Tatsien L.I. (ed.), *Proceedings of the International Congress of Mathematicians*, Beijing, Higher Education Press, Vol. III, p. 885-895.

Hansen V.L. (2005). *Mathematics Alive and in* Action. In: Araki H. (ed.), *Mathematics: History, Concepts, and Foundations*. Encyclopedia of Life Support Systems (EOLSS), Oxford, Publication developed under the auspices of the UNESCO by Eolss Publishers. Available at http://www.eolss.net.

Hardy G.H. (1940), *A Mathematician's Apology*. Cambridge University Press.

Hawking S. (2011), *A Brief History of Time: From Big Bang to Black Holes*. Transworld Publishers Ltd.

Hilgartner S., *The Dominant View of Popularization: Conceptual Problems, Political Uses*. In: Social Studies of Science, 20 (3), (1990), p. 519-539.

Holt J. (1964), *How Children Fail*. Pitman, London.

Howson A.G., Kahane J.P. (1990), *The Popularization of Mathematics.* ICMI Study Series, Cambridge University Press.

Ipsos MORI (2011), *Public Attitudes to Science 2011*. Department for Business, Innovation and Skills, p. 71.

James F.A.J.L. (2007), *Christmas at the Royal Institution: an Anthology of Lectures*. Singapore, World Scientific Publishing Co. Pte Ltd.

Jasanoff S. (2005), *Designs on Nature: Science and Democracy in Europe and the United States*. Princeton, Princeton University Press.

Kennedy H. (1983), *Peano. Storia di un matematico*, Torino, Boringhieri.

Kline M. (1972). *Mathematical Thought from Ancient to Modern Times*. Oxford University Press.

Kline M. (1985). *Mathematics and the Search for Knowledge*. Oxford University Press.

Kozulin A. (1990). *Vygotsky's Psychology: A Biography of Ideas*. Cambridge, Harvard University Press.

Kreimer P. et al. (2010), *Popularization by Argentine Researchers: The Activities and Motivations of CONICET Scientists*. Public Understanding of Science, Sage Journal, p.42.

Latour B. (1987), *Science in Action: How to Follow Scientists and Engineers Through Society*. Harvard University Press.

Lesen A.E., Rogan A., Blum M.J. (2016), *Science Communication Through Art: Objectives, Challenges and Outcomes*. Trends in Ecology & Evolution, Elsevier.

Leshner A.I. (2007), *Outreach Training Needed*. Science, Vol. 315, Issue 5809, p. 161.

Lévy-Leblond J.M. (1998), *La pietra di paragone*. Napoli, Cuen.

Lévy-Leblond J-M. (1996), *La Pierre de Touche – la science a l'épreuve*. Paris, Gallimard.

Lewenstein B. (2008), *Popularisation*. In: Heilbron J.L. (ed), *The Oxford Companion to the History of Modern Science*. Oxford, Oxford University Press, p. 667-668.

Lightman B. (2007), *Victorian Popularizers of Science. Designing Nature for New Audiences*. Chicago, University of Chicago Press.

Lipsey S.I., Pasternack B.S. (2002), *Mathematics in Literature*. The Mathematics Education into the 21st Century Project: The Humanistic Renaissence in Mathematics Education, Palermo, Terrasini, p. 1.

Mackay H. (1994), *Why Don't People Listen? Solving the Communication Problem*. Pan Macmillan Australia.

Mazur B. (2007), *"Eureka" and Other Stories*. Available at http://www.thalesandfriends.org/en/papers/pdf/mazur_paper.pdf.

McCallie E. et al. (2009), *Many Experts, Many Audiences: Public Engagement with Science and Information Science Education*. Washington: Centre for Advancement of Informal Science Education.

McCrory P. (2010), *Developing Interest in Science Through Emotional Engagement*. In: Harlen W. (ed), *ASE Guide to Primary Science Education*. ASE, Hatfield, p. 94-101.

McGillion C., McKinnon M. (2014), *Participatory Theater as a Science Communication Tool*. Timor Leste Science Communication.

Merton R.K. (1981), *La sociologia della scienza*. Milano, Franco Angeli.

Mesure S. (2007), *The CreScENDO Project: Final Report for NESTA*. London, National Endowment for Science, Technology and the Arts.

Miller S. et al. (2009), *Can Science Communication Workshops Train Scientists for Reflexive Public Engagement?*. In: Science Communication, 31 (1), p. 116-126.

Montebelli V. (1998), *I giochi matematici nel "De Viribus Quantitatis"*. In: Giusti E. (ed), *Luca Pacioli e la Matematica del Rinascimento*. Petruzzi Editore, p. 313-330.

Natalini R. (2015), *To Infinity and Beyond*. EMS Newsletter, n.97, September, p. 60-61

Osborne J., Monk M. (2000), *Good Practice in Science Teaching: What Research Has to Say*. Open University Press.

Paulos J.A. (1990), *Innumeracy – Mathematical Illiteracy and Its Consequences*. New York, Vintage Books.

Peano G. (1959), *Opere scelte.* Vol.III, a cura di U.M.I., p. 273.

Peterson I. (1991), *Searching for New Mathematics*, SIAM Review 13, p. 37-42.

Pyne C., Bates V., Turner W. (1995), *Is it Possible to Change People's Negative Attitudes to Mathematics?* Mathematics Teaching, vol. 15, p. 8-10.

Radford L., Demers S. (2006), *Comunicazione e apprendimento. Riferimenti concettuali e pratici per le ore di matematica*. Bologna, Pitagora.

Rowe G., Frewer L. (2005), *A Typology of Public Engagement Mechanisms*. Science Technology Human Values 30.

Schaffer K., Stern E., Kim S. (2001). *Math Dance with Dr. Schaffer and Mr. Stern*. Santa Cruz, MoveSpeakSpin.

Schiro M. (2004), *Oral Storytelling and Teaching Mathematics: Pedagogical and Multicultural Perspectives*. Thousands Oaks, Sage Publications.

Science for All (2010a), *Report and Action Plan from Science for All Expert Group*. London: Department for Business, Innovation and Skills. Available from http://interactive.bis.gov.uk/scienceandsociety

Science for All (2010b), *Public Engagement with Science and Society – A Conversational Tool*. London, Department for Business, Innovation and Skills. Available from http://interactive.bis.gov.uk/scienceandsociety.

Seth Taylor R. (2007), *The Wonder of Magic: Eliciting Wonder and Analyzing its Expression*. Mphil thesis, Massachusetts Institute of Technology.

Snow C.P. (1959), *The Two Cultures*. Cambridge, Cambridge University Press.

Spinelli A. (2008/2009), *La partita aperta tra didattica e comunicazione*. Rivista Scuola IaD, p. 44-66.

Steen L. (1990), *Mathematical News That's Fit to Print*. In: Howson A.G., Kahane J.P. (eds), *The popularization of mathematics*. Cambridge University Press, ICMI Study #5, p. 194-206.

Stewart I. (2006), *Mathematics, the Media and the Public*. Proceedings of the International Congress of Mathematicians, Madrid, Spain, p. 1631-1644.

Thomson (Lord Kelvin) W. (1894), *Popular Lectures and Essays, vol.II*. London, Macmillan and Co., p. 571-592.

Thurston W. (2006). *On Proofs and Progress in Mathematics*. In: Hersh R. (ed.), *18 Unconventional Essays on the Nature of Mathematics*. New York, Springer, p. 37-55.

Tomlin S. (2005), *What's the Plot?* Nature, 436 (051), p. 622-623.

Truax B. (1990), *Chaotic non-linear systems and digital synthesis: An explanatory study*. Proceedings of the International Computer Music Conference (ICMC), p. 100-103.

Ulam S. (1991), *Adventures of a Mathematician*. University of California Press.

VanSledright B., Brophy J. (1992), *Storytelling, Imagination, and Fanciful Elaboration in Children's Historical Reconstructions*. American Educational Research Journal, 29 (4), p. 837-859.

Werner L. (2001). *Changing Student Attitudes Toward Math: Using Dance to Teach Math*. Prepared for The Minneapolis Public Schools. Available at http://www.cehd.umn.edu/carei/publications/documents/mathattitude.pdf

Wilkinson C., Weitkamp E. (2016), *Creative Research Communication: Theory and Practice*. Manchester, Manchester University Press.

Yorks L., Kasl E. (2002), *Towards a Theory and Practice for Whole-person Learning: Reconceptualizing Experience and the Role of Affect*. Adult Education Quarterly, vol. 52, n.3, p. 176-192.

Zazkis R., Liljedahl P. (2009), *Teaching Mathematics as Storytelling*. Rotterdam/Taipei, Sense Publishers.

Zdravkovska S. (1987), *Conversation with Vladimir Igorevich Arnol'd*. Math. Intell. 9, p. 28–32.

Zeki S. (2017), *Sesso, bellezza ed equazioni*. In: *Scienza e filosofia*, Il Sole 24 Ore domenica (23 aprile 2017).

Ziegler G.M. (2010), *Communicating Mathematics to Society at Large*. Proceedings of the International Congress of Mathematicians, Hyderabad, India, p. 721.

Ziman J. (1980), *Teaching and learning about science and society*. Cambridge University Press.

Ziman J. (1991), *Public Understanding of Science*. In: Science, Technology and Human Values, 16 (1), p. 99-105.