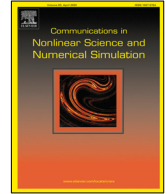


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# Communications in Nonlinear Science and Numerical Simulation

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## Highlights

### On the economic growth equilibria during the Covid-19 pandemic

Gian Italo Bischi, Francesca Grassetti\*, Edgar J. Sanchez Carrera

- We propose a fusion between economics (Solow's type model) and epidemiology (SIS model)
- Economies can fall into poverty trap if savings are low and the economic policies that reduce the spread of infection are severe
- If the suppression policies allow low infection levels and high recovery rates, the economies converge towards the equilibrium of high economic growth.

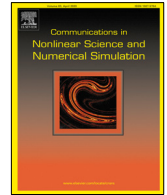
*Communications in Nonlinear Science and Numerical Simulation xxx (xxxx) xxx*

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## On the economic growth equilibria during the Covid-19 pandemic

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### ABSTRACT

The aim of this paper is to study the effects of the Covid-19 pandemic suppression policies (i.e. containment measures or lockdowns) on labor supply, capital accumulation, and so the economic growth. We merge an epidemic SIS population model and a Solow's type growth model, i.e. we propose a fusion between economics and epidemiology. We show the creation and the destruction of economic growth equilibria driven by the suppression policies and by the severity of the disease. The dynamic stability properties of the equilibria are mainly determined by (i) the stringency of the suppression policies, (ii) the proportion of infected workers, (iii) the recovery rate of workers, and (iv) the economy's saving rate. Thus, economies can fall into the stable equilibrium of the poverty trap if the propensity to save is low and the economic policies that reduce the spread of infection are severe enough with high levels of infection and low rates of illness recovery. Otherwise, with high savings rates and if the suppression policies perform in such a way that infection levels are low and recovery rates are high, then the economies converge towards the equilibrium of high economic growth with capital accumulation. The scenario is rather complex since there is a multiplicity of equilibria such that economies can be in one scenario or another, characterized by stability or (structural) instability, i.e. bifurcation paths. Numerical simulations corroborate our results.

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## 1. Introduction

Economics has rather neglected epidemiology in the past, but recently we have dramatically experienced how they are closely related, in both directions: on the one hand, sustained economic development is essential to face pandemics, on the other hand, the spread of disease can trigger serious crises and/or economic shocks [1]. It is well known that economic crises plunge economies into recession in the short, medium and long term. For example, the last major global financial crisis of 2008–09 was responsible for lower growth in total factor productivity, labor force, and corporate fixed investment (that is, determinants of potential Gross Domestic Product (GDP)). Without a doubt, the COVID-19 pandemic brings with it negative economic effects in the short, medium and long term on the main determinants of economic growth. The magnitude of this short- and long-term impact is highly uncertain. In the short term, the amplitude of the fluctuation depends significantly on how the containment measures are supposed to affect potential production. In the long term, it depends on the duration of the pandemic and the extent to which policy measures can protect the economy from excessive scarring, among other factors. Therefore, the current crisis (health and economic) is likely to bring about notable structural changes and, therefore, economic policies will play a key role in adopting these changes.

When the COVID-19 pandemic struck at the beginning of 2020, countries adopted very different strategies – containment measures namely policies to suppress socio-economic activities, like quarantines and business closures, i.e. policy makers aiming at taming the spread of the disease while minimizing the associated socioeconomic costs (see, for instance, [2]). Recall, for example, that China implemented an almost complete lockdown. European countries also introduced lockdowns of various degrees of restrictiveness. Italy, which was hit very hard by the COVID-19 infection early on, implemented a very restrictive lockdown.<sup>1</sup>

The empirical evidence on the socioeconomic effects of Covid-19 is quite extensive. The OECD estimates that repression policies (lockdowns) resulted in a 20%–25% reduction in GDP in several OECD countries in 2020, and furthermore that one out of five OECD/EU regions has at least the 30% of its employment levels potentially at risk in the short term as a result of containment measures or suppression policies (see [3]). Empirical evidence also suggests that the global economic impacts of the Covid-19 pandemic crisis have so far been very uneven across countries, with disproportionate negative effects. For example, evidence indicates that global economic growth (GDP) is estimated to have contracted between 6 and 7 percent in 2020, representing the largest economic crisis in a generation.

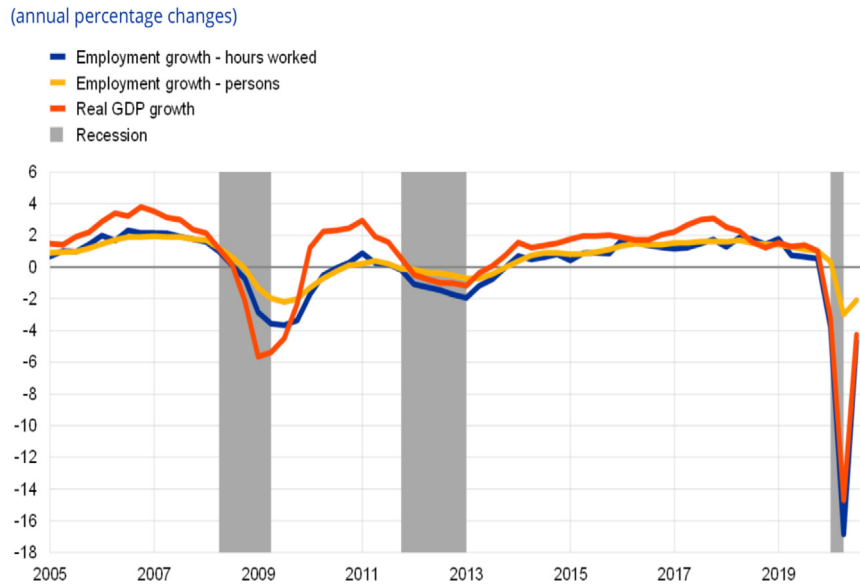
Unfortunately, the pandemic has affected employment levels. Looking at the stylized facts in European countries, we can see that the COVID-19 pandemic has caused a sharp contraction in employment and total hours worked. The data indicates that there has been a decrease in the number of people employed of approximately 44% [see 4]. In addition, despite the dramatically low employment growth observed in the first half of 2020, quarter-on-quarter employment adjustments remained relatively moderate compared to changes in GDP. Total hours worked changed substantially more than employment and also more than GDP. In the second quarter of 2020, the quarter most affected by containment measures (suppression policies), the total hours worked decreased by 16.8% and the average hours worked decreased by 14.3% in annual terms (see Fig. 1).

However, what may surprise us is that the economic recovery is already present, although not in a homogeneous way, since, for example, the levels of GDP growth and employment at the level of OECD countries are taking place in a very heterogeneous way. In other words, at the National level there are marked differences in terms of economic recovery and employment, and this empirical evidence is the main motivation for this research work: the existence of multiple possible paths for economies.<sup>2</sup>

This economic health crisis has dramatically profound effects on the job market. Unemployment has risen steadily and workers who have suffered job losses during the COVID-19 pandemic are not actively seeking to find new jobs and are therefore classified as “inactive or out of the workforce” in the official statistics which is simply a labor supply contraction ([6,7]; <https://ilostat.ilo.org/topics/covid-19/>). Apart from the reduction of the workforce due to the direct effects of the disease on the health of the population, the labor supply has been driven by behavioral channels; for example, Yu et al. [8] estimate that people’s behavioral response has a strong adverse impact on labor force participation, i.e. increasing inactive workforce during the pandemic, and this effect appears to be stronger in countries with high avoidance of uncertainty about pandemic control. Notably, putting temporary unemployment alongside permanent unemployment distinguishing between short-term and long-term unemployment is crucial to understanding the dynamics of the COVID-19 recession. Ignoring this heterogeneity will lead to a substantial overestimation of unemployment [9]. Thus the suppression policies to face the COVID-19 pandemic caused the sharpest contraction on record in employment and total hours worked in 2020 (see Fig. 2). This empirical evidence is another important motivation for our research paper.

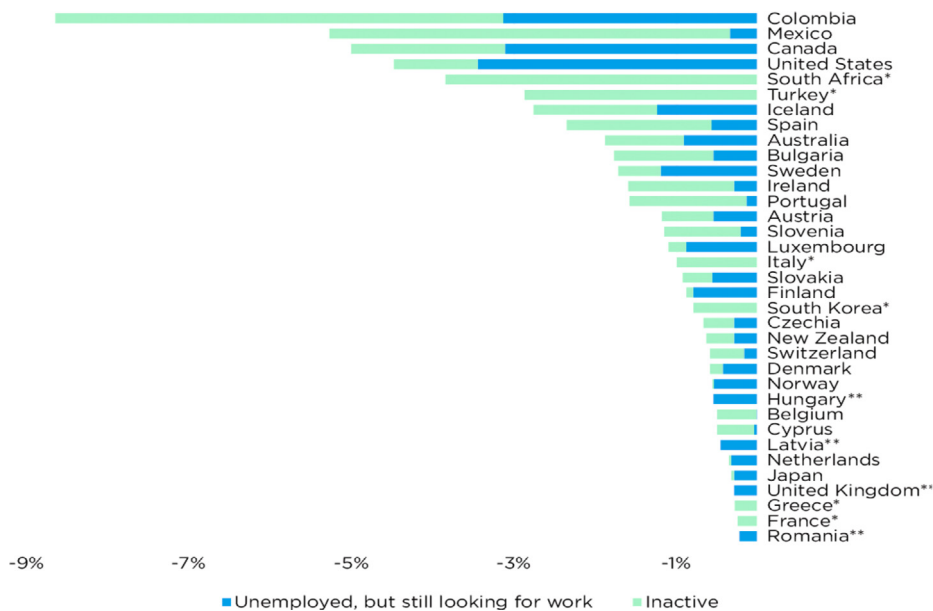
<sup>1</sup> See the Oxford Coronavirus Government Response Tracker (OxCGRT) Project measuring a Stringency Index which is a composite measure of: school closures; workplace closures; cancellation of public events; restrictions on public gatherings; closures of public transport; stay-at-home requirements; public information campaigns; restrictions on internal movements; and international travel controls. The index higher score indicates a stricter response (i.e. 100 = strictest response). If policies vary at the subnational level, the index is shown as the response level of the strictest sub-region. Notice, however, that this index does not imply the appropriateness or effectiveness of a country’s response. <https://ourworldindata.org/grapher/covid-stringency-index>.

<sup>2</sup> OECD [5], OECD Economic Outlook, Interim Report September 2021: Keeping the Recovery on Track, OECD Publishing, <https://doi.org/10.1787/490d4832-en>. Countries with higher levels of recovery, together with countries with still negative rates considering the data published by Eurostat: <https://ec.europa.eu/eurostat/web/products-euro-indicators/-/2-07092021-ap>.



**Fig. 1.** Employment developments in the euro area. Notes: Annual changes are based on seasonally and working-day adjusted data. Shaded bars indicate euro area recessions, defined as two consecutive quarters of negative GDP growth. Latest observation: third quarter of 2020.

Source: Anderton et al. [4], Eurostat [https://www.ecb.europa.eu/pub/economic-bulletin/articles/2021/html/ecb.ebart202008\\_02-bc749d90e7.en.html](https://www.ecb.europa.eu/pub/economic-bulletin/articles/2021/html/ecb.ebart202008_02-bc749d90e7.en.html).



**Fig. 2.** Change in employment from 2019 to 2020 (%). It shows the overall decrease in employment accounted for by increases in unemployment and inactivity from Q1–Q3 2019 to Q1–Q3 2020. \* Denotes countries where unemployment decreased, but was overcompensated by the rise in inactivity. \*\* Denotes countries where inactivity decreased, but was overcompensated by the rise in unemployment.

Source: Cotofan et al. [7], International labor Organization (ILOSTAT), <https://worldhappiness.report/ed/2021/work-and-well-being-during-covid-19-impact-inequalities-resilience-and-the-future-of-work/>.

So we wonder how the labor market, particularly the labor supply, and economic growth have been affected by the policies of suppression–containment of socioeconomic activities that have aimed at facing the Covid-19 pandemic. What will the economic growth scenario be, that is, would economic recovery take place in a single or multiple economic growth equilibrium? It seems that the current empirical evidence shows us that there are various economic growth scenarios (multiple equilibria) triggered by the way economies responded to the Covid-19 pandemic. Therefore, we can affirm that the COVID-19 pandemic and related containment measures affect economies to an extent that is likely to modify

potential economic growth and generate various growth scenarios. Here we get another empirical motivation for what we theoretically develop in this research work.

Hence, the aim of this paper is to analyze the economic effects of suppression policies (containment measures, lockdowns, reductions in economic activities) that are imposed to face contagious diseases in times of pandemic. In this paper we merge an epidemic SIS population model and a Solow's type growth model, i.e. we propose a fusion between economics and epidemiology. We use a basic epidemiological model, a SIR model, which is parametrized in discrete time considering the labor force and capital accumulation as engines of economic growth.<sup>3</sup> To this aim, we blend a SIS (Susceptible, Infectious, Susceptible) epidemic population model à-la Hethcote and van den Driessche [11] and Hethcote [12] with a Solow-type economic growth model [13] based on labor and physical capital dynamics in order to study the effects of Covid-19 pandemic suppression policies (i.e., containment measures or lockdowns) on labor supply, capital accumulation, and thus on economic growth in general. The purpose of our analysis is to shed light on some of the underlying trade-offs between economic performance and the labor market when suppression policies of economic activities in pandemic times are implemented.

There are too many research articles that analyze the implications of the policy response in relation to the COVID-19 pandemic. To point out some important ones, for example, Hall et al. [14] analyze the optimal trade-off between consumption losses and pandemic deaths. Jones et al. [15] study the interaction of public and private mitigation efforts. Other policy options are discussed in [16,17]. More closely related to us, several recent papers [18–20] specifically analyze the consequences of the pandemic on economic growth and the labor market. Some other papers, such as Anderton et al. [4] discuss how mitigation policies will affect the COVID-19 pandemic. There are different approaches such as those of Acemoglu et al. [21], Alvarez et al. [22] and Gonzalez-Eiras and Niepelt [23] applying the theory of optimal control to determine the optimal route of a quarantine that can be varied continuously considering the socioeconomic costs. We do not calibrate our model for any particular country and we do not use control theory to specify an optimal isolation path.

Instead, this paper blends epidemic population models with Solow-style models. In principle, our model mixture can generate multiple equilibria with have both bifurcations and traps. With a Solow growth model, and with a particular non-concave, piecewise-defined production function, our aim is to shed light on some of the underlying trade-offs between economic performance and the labor market when suppression policies of economic activities in pandemic times are implemented. In this vein, we show that multiple equilibria arise, and that the poverty trap equilibrium exists in pandemic times. This trap is a stable situation, and any economy (advanced, developing or poor countries) can fall into such a steady state.

The remainder of the paper is organized as follows. Section 2 proposes the basic dynamic model. Section 3 presents the main results, propositions and corollaries, on the existence and stability of the equilibria of the developed dynamic model. Section 4 presents numerical simulations and interpretations of the results. Section 5 concludes.

## 2. Model setup

In this section we propose a growth model driven by labor force and physical capital (i.e. capital accumulation à la [13]), in a discrete time framework  $t \in \mathbb{N}$ , incorporating a disease transmission model (somewhat related to the approaches developed by Goenka and Liu [24]). Let us start from the SIS (Susceptible, Infectious, Susceptible) continuous time model (for example the baseline model developed by Hethcote and van den Driessche [11] and Hethcote [12])<sup>4</sup>:

$$\dot{S} = bN + \gamma I - \alpha \left( \frac{I}{N} \right) S - dS \quad (1)$$

$$I = N - S \quad (2)$$

where  $S \geq 0$  are susceptible,  $I \geq 0$  are infected,  $N \geq 0$  is the total population,  $\gamma \in [0, 1]$  is the disease recovery rate,  $\alpha \in [0, 1]$  is the fraction of susceptible workers that become infected (infection rate).<sup>5</sup> Notice that this transmission rate parameter  $\alpha$  could be interpreted in terms of number of contacts among individuals and probability of infections, and so to have other leverages to analyze the effects of the policy instruments. Parameters  $b$  and  $d$  are respectively birth and death rate. We rewrite model (1) in discrete time ( $S = S_{t+1} - S_t$ ), and we identify the susceptible population with the

<sup>3</sup> An initial modeling framework coupling epidemiological modeling with economic growth modeling is presented in Eichenbaum et al. [10]. They construct what they call a SIR-macro model with endogenous consumption and labor supply. These authors show that the competitive equilibrium is suboptimal due to fact that agents do not fully internalize the externality of their economic interactions. Instead, we model a macroeconomy that responds to repressive policies by controlling the dynamics of the epidemic, with the aim of limiting infections but at the same time reducing labor supply and economic performance.

<sup>4</sup> The SIS model can be interpreted as an extension of the classical SIR (Susceptible, Infectious, Recovered) model, since for some infections, for example, those from the common cold and influenza or thought the Covid-19 variants, do not confer any long-lasting immunity. Such infections do not give immunity upon recovery from infection, and individuals become susceptible again.

<sup>5</sup> Notice that this SIS model structure (Eqs. (1)–(2)) does not allow for any dynamics of infectious, although either analytically and numerically we may perform our next results for any given infection rate. In any case, we are very aware that the dynamics of infection levels could be represented as a dynamic equation in time. Which we claim it as future research in our concluding remarks.

susceptible worker population  $S_t = L_t$ , hence  $N = \bar{L}_t$  is the total population of workers, while  $I_t = \bar{L}_t - L_t$  is the number of infected workers. Therefore:

$$L_{t+1} = L_t + b\bar{L}_t + \gamma(\bar{L}_t - L_t) - \alpha \left( \frac{\bar{L}_t - L_t}{\bar{L}_t} \right) L_t - dL_t$$

Let us now introduce a simplified measure for the Covid-19 pandemic suppression policies.

**Definition 1.** A parameter  $\rho \in [0, 1]$  measures the degree of the COVID-19 suppression policies, i.e. the lockdown measures, that is  $\rho$  represents the set of norms in order to decrease contacts among workers. Such norms decrease economic activities as well, while decreasing contagion among workers. This latter effect can be represented by replacing the contagion rate  $\alpha$  by  $\alpha(1 - \rho)$ .

Thus labor supply dynamics becomes:

$$L_{t+1} = b\bar{L}_t + \gamma(\bar{L}_t - L_t) - \alpha(1 - \rho) \left( 1 - \frac{L_t}{\bar{L}_t} \right) L_t + (1 - d)L_t$$

As we have mentioned, the parameters  $\alpha$  and  $\gamma$  respectively represent the fraction of newly infected and recovered in each period. We assume the following.

**Assumption 1.** A disease spreads when the number of newly infected is greater than the number recovered, so we assume:  $0 < \gamma < \alpha < 1$ .

Regarding the pandemic suppression policy rate, we keep in mind that a full lockdown measure, that is  $\rho = 1$ , is not somewhat realistic since the policy measures applied by governments in case of pandemics tend to cause the least possible damage to economic growth. However, aggressive measures are needed when a strongly contagious disease exists.

**Assumption 2.** In accordance with these considerations we assume that  $0 < \rho < 1 - \frac{\gamma}{\alpha}$ . This means that the suppression measures can assume different values considering the wide scenarios and diversity of the world economies, despite the fact that there is a maximum limit that depends on the severity of the disease, represented by  $\gamma$  and  $\alpha$ . The worse situation  $\rho \rightarrow 1$  can arise when  $\gamma \rightarrow 0$ .

Let us also consider that outbreaks caused by infectious disease typically last for few years (Spanish flu 1918–19, Asian flu 1957–60, Hong Kong flu 1968–69, H1N1 pandemic 2009–10), therefore we can assume in our model that the total labor force is fixed, while labor supply depends on disease spread. We include these considerations by setting  $\bar{L}_t = \bar{L} \geq 0$  and  $b = d = 0$ . Therefore the dynamic equation governing labor supply reduces to:

$$L_{t+1} = \mathcal{L}(L_t) := \frac{\alpha(1 - \rho)}{\bar{L}} L_t^2 + (1 - \gamma - \alpha(1 - \rho))L_t + \bar{L}\gamma \quad (3)$$

On the other hand, to describe the production technology, economic growth theories typically adopt the Cobb–Douglas function to model the impact of capital investments and labor on economic growth.<sup>6</sup> That is,  $f(K, L) = L^{1-\phi}K^\phi$ ,  $\phi \in (0, 1)$ , where  $K \geq 0$  denotes capital. Such production function fulfills conditions  $f(0, L) = 0$  and  $\lim_{K \rightarrow 0} f_K = +\infty$ , where  $f_K$  represents the partial derivative. From an economic point of view, these conditions imply that an economy almost without capital gains infinitely high returns investing a small amount in capital. Instead, we argue that a critical level of capital should be reached before getting economic output, we hence propose a production function shifting the Cobb–Douglas function by such a threshold level (see, for example, Buera and Kaboski [29]; Capasso et al. [30]; Dong and Xu [31]).<sup>7</sup> Notice that a more accurate solution for avoiding the unrealistic Inada condition  $\lim_{K \rightarrow 0} f_K = +\infty$  without imposing a kink in the production function (i.e. the threshold level for positive output) would be to select an S-shaped (sigmoidal) production function as in Grasseti et al. [32]. Despite so, in order to avoid excessive complexity that would prevent explicit results when an additional parameter (related to economic policy) is embedded in the production technology, in the following we adopt the simplified function proposed in Capasso et al. [30] that sets equal to zero the output generated when capital is lower than a given threshold.<sup>8</sup>

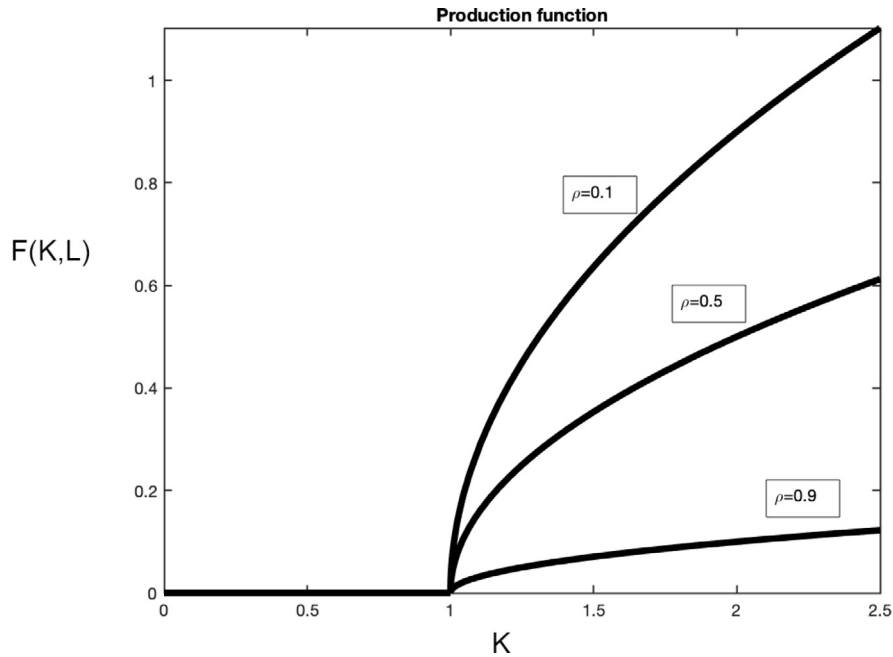
<sup>6</sup> Seminal contributes made by Romer [25], Lucas [26], Mankiw et al. [27] and Benhabib and Spiegel [28].

<sup>7</sup> In particular, the applications of such a production function by Buera and Kaboski [29] describing a multi-sector environment in which some sectors may remain idle (output = 0) while others produce something, or the applications by Dong and Xu [31] which consider a frictional banking sector that may lead to endogenous boom–bust cycles.

<sup>8</sup> Specifically by characterizing the Cobb–Douglas production function and with the aim of justifying the adoption of an alternative production function that does not satisfy the Inada condition, Capasso et al. [30] on p. 3860 write:

We consider a region with almost no physical capital, that is there are no machines to produce goods, no infrastructure, etc. This condition says that it is possible to gain infinitely high returns by investing only a small amount of money. This obviously cannot be realistic.





**Fig. 3.** Effects of the suppression policy on the production function (2). Note that output is positive from the minimum level of capital,  $K_c \geq 1$ , required to produce.

Furthermore, in response to the epidemic crisis, as we argued above, governments implement measures that suppress economic activities, in particular, the related containment and lockdown measures are likely to affect most components of potential production. Therefore, considering these facts, the resulting production function is given by:

$$F(K, L) := \begin{cases} 0 & K \leq K_c \\ (1 - \rho)L^{1-\beta}(K - K_c)^\beta & K > K_c \end{cases} \quad (4)$$

where  $K_c$  is the threshold level of capital necessary for production,  $\beta \in (0, 1)$  is the output elasticity of capital while  $\rho$  is the suppression-policy above discussed that causes loss of total economic output.<sup>9</sup> Note that we do not include technological progress in production function since we consider relatively short periods of time: the model aims at describing the dynamics during the pandemic regime and it is not suitable for the analysis of the long run dynamics in case of diseases with endemic regime. In other words, the understanding should be that after a while the economy could be out of the pandemic regime and, perhaps after a transitory period in an epidemic regime, it could end up in an endemic regime, for example when the Covid – 19 will become a common virus like the flu, so that suppression policies are no longer applied. Hence, we are proposing a model (the following system (7)) to describe a dynamic for an economy that enters the basin of attraction of the poverty trap during the pandemic, because from that moment on, production is zero and capital is doomed to annihilate as  $t \rightarrow \infty$  (see Remark 1 below), there is no hope of recovery, even after the pandemic has gone. But what we are proposing and affirming is that after the pandemic, the proposal dynamics of capital must change to another type of dynamic representation that we do not represent here. Because, our proposal model simply describes the “transition dynamics during a pandemic without attempting to explain what happens after the pandemic ends”.

Fig. 3 provides a graphical representation of the Cobb–Douglas shifted function affected by the suppression parameter. As observed, for higher levels of suppression, total production is reduced.

Now consider that capital evolves over time according to the classical Solow [13] model that describes the change in physical capital. Notice that Solow’s seminal model assumes a supply of goods based on a production function with constant returns to scale. Labor grows at a constant rate, the level of technology is constant over time, the saving rate is constant and capital depreciates at a positive constant rate, that is, at each point in time, a constant fraction of the capital stocks can no longer be used for production. Within the model, the capital stock is a key determinant of the economy’s output, and changes in the capital stock affect economic growth. The main findings can be summarized in two stationary states of the economy: one characterized by zero production and capital per capita; the other one achieved as a result of the way the production function is constructed. The crucial assumption is about the marginal productivity

<sup>9</sup> Research articles project that the GDP losses from COVID-19 would range from 7% to 30% [21,22,33–36]. See: [https://www.ecb.europa.eu/pub/economic-bulletin/articles/2020/html/ecb.ebart202007\\_01-ef0a77a516.en.html](https://www.ecb.europa.eu/pub/economic-bulletin/articles/2020/html/ecb.ebart202007_01-ef0a77a516.en.html).

of capital that would tend to zero when capital tends to infinity. Such an assumption assures global stability towards the output-per-person stationary state [37]. Hence in each new period, capital depends on the previous (depreciated) capital and on the part of production that is saved and invested:

$$K_{t+1} = (1 - \delta)K_t + sF(K_t, L_t) \quad (5)$$

where  $s \in (0, 1)$  is the saving rate and  $\delta \in [0, 1]$  is the depreciation rate of capital. Substituting (4) into (5), the evolution of capital over time is given by:

$$K_{t+1} = \mathcal{K}(K_t, L_t) := \begin{cases} (1 - \delta)K_t & K_t \leq K_c \\ (1 - \delta)K_t + s(1 - \rho)L_t^{1-\beta}(K_t - K_c)^\beta & K_t > K_c \end{cases} \quad (6)$$

Therefore, the two-dimensional map (Eqs. (3) and (6)) which describes the dynamic evolution of the economy that we have just described, is defined by:

$$T := \begin{cases} K_{t+1} = \mathcal{K}(K_t, L_t) \\ L_{t+1} = \mathcal{L}(L_t) \end{cases} \quad (7)$$

This is an unidirectionally coupled (or master-slave) nonlinear dynamical system, as the dynamic evolution of  $L_t$  only depends on  $L_t$ , whereas the dynamic evolution of  $K_t$  is conditioned by both  $L_t$  and  $K_t$ . This kind of dynamical systems has some particular dynamical properties related to the triangular structure of the Jacobian matrix of the map, see Cabral Balreira et al. [38], Dieci et al. [39], Kolyada and Sharkovski [40].

**Remark 1.** Because of the piecewise definition of  $\mathcal{K}(K_t, L_t)$  the two dynamic variables are decoupled whenever  $K_t \leq K_c$ , and in this case the map that governs the dynamics of  $K_t$  is a contraction, therefore  $K_t \rightarrow 0$  as  $t$  increases whenever  $K_t < K_c$  for a finite  $t$ . So, the strip  $K \leq K_c$  represents a poverty trap that leads to a long-term evolution along the invariant poverty set  $K = 0$  (since  $K_t = 0$  implies  $K_{t+1} = 0$  as well) along which the one-dimensional dynamics of  $L_t$  is governed by (3).

An economy falls into the poverty trap according to its initial conditions, that is, the initial levels of work and physical capital, but also according to the crucial parameters mentioned above that govern the dynamic model. For example, the suppression policy  $\rho$  to contain the spread of the virus while suppressing socioeconomic activities. Let us now analyze in detail the dynamic properties (existence and stability of equilibria) of this model (the two-dimensional map (7)), this with the aim of studying the interrelation between labor supply and economic growth in times of pandemics.

### 3. Existence and stability of the equilibria

The first step for the qualitative analysis of the dynamic properties of the two-dimensional map (7) consists in locating its equilibria, that is the steady states of the economic system described above. The following result (proved in Appendix) shows the existence and position of such equilibrium points, which mainly depend on the suppression policy  $\rho$  and threshold capital  $K_c$  as well as some aggregate parameters that will act as reference or bifurcation parameters.

**Proposition 1** (Existence of Equilibria). *Given the two solutions  $L_1^* = \frac{\gamma \bar{L}}{\alpha(1-\rho)}$  and  $L_2^* = \bar{L}$ ,  $L_1^* < L_2^*$  of the second degree equation  $\mathcal{L}(L) - L = 0$ . Consider the following threshold parameters:*

1. A threshold of segregation of capital accumulation with labor supply:

$$K_{M,i} = K_c + \left( \frac{\beta s(1 - \rho)}{\delta} \right)^{\frac{1}{1-\beta}} L_i^*, \quad i \in \{1, 2\};$$

2. A segregation threshold for the initial condition of capital such that positive economic growth is achieved (a condition to get positive production levels):

$$z = \left[ \frac{K_c}{\bar{L}(1 - \beta)} \right]^{1-\beta} \frac{\delta}{s\beta^\beta};$$

3. A threshold for segregation of suppression policies at high (very stringent) and low (not so stringent) levels:

$$\rho_1 = 1 - z^{\frac{1}{\beta}} \left( \frac{\alpha}{\gamma} \right)^{\frac{1-\beta}{\beta}}, \quad \rho_2 = 1 - z.$$

Then:

- the low-level (poverty trap) equilibria  $E_{0,1} = (0, L_1^*)$  and  $E_{0,2} = (0, L_2^*)$  always exist
- if the initial conditions are very high or difficult to reach,  $z \geq 1$  then no additional equilibria exist
- if the initial conditions are  $z \in \left[ \left( \frac{\gamma}{\alpha} \right)^{1-\beta}, 1 \right)$ , then  $\rho_1 < 0 < \rho_2$ , and



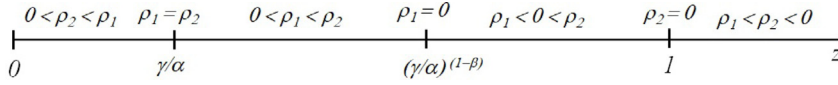


Fig. 4. Schematic representation of the results of Proposition 1.

- for a suppression policy  $\rho > \rho_2$  no additional equilibria exist
- for a suppression policy  $\rho = \rho_2$  one non hyperbolic equilibrium exists given by  $E_{M,2} = (K_{M,2}, L_2^*)$
- for a suppression policy  $\rho < \rho_2$  two positives equilibrium points exist, given by  $E_{1,2} = (K_{1,2}^*, L_2^*)$  and  $E_{2,2} = (K_{2,2}^*, L_2^*)$ , with  $K_{1,2}^* < K_{M,2} < K_{2,2}^*$
- if the initial conditions are  $z \in \left(\frac{\gamma}{\alpha}, \left(\frac{\gamma}{\alpha}\right)^{1-\beta}\right)$ , then  $0 < \rho_1 < \rho_2$ , and
  - for a suppression policy  $\rho > \rho_2$  no additional equilibria exist
  - for a suppression policy  $\rho = \rho_2$  one positive non hyperbolic equilibrium exists given by  $E_{M,2}$
  - for a suppression policy  $\rho_1 < \rho < \rho_2$  two positives equilibrium points exist, given by  $E_{1,2}$  and  $E_{2,2}$  (like in the statement above)
  - for a suppression policy  $\rho = \rho_1$  three positive fixed point exist given by the non hyperbolic  $E_{M,1} = (K_{M,1}, L_1^*)$ , together with  $E_{1,2}$  and  $E_{2,2}$ , with  $K_{1,2}^* < K_{M,1} < K_{M,2} < K_{2,2}^*$
  - for a suppression policy  $\rho < \rho_1$  four positive fixed point exist given by  $E_{1,1} = (K_{1,1}^*, L_1^*)$ ,  $E_{2,1} = (K_{2,1}^*, L_1^*)$ ,  $E_{1,2}$  and  $E_{2,2}$ , with  $K_{1,2}^* < K_{1,1}^* < K_{M,1} < K_{M,2} < K_{2,1}^* < K_{2,2}^*$
- in the case that the threshold  $z \leq \frac{\gamma}{\alpha}$  four positive fixed point exist given by  $E_{1,1}$ ,  $E_{2,1}$ ,  $E_{1,2}$  and  $E_{2,2}$ .

The different positions of  $\rho_1$  and  $\rho_2$  listed in the Proposition, are summarized in Fig. 4 as the aggregate parameter  $z$  varies.

It is worth to remark (see also the proof in Appendix) that the presence of the non hyperbolic equilibrium points mark fold (or saddle–node) bifurcations, leading to the creation (or destruction) of a pair of equilibrium points, node and saddle respectively. If the node is stable, then the stable set of the associated saddle constitutes the boundary of the basin of attraction of the stable node (as we shall confirm numerically in the next section).

It is also worth to notice that  $L_1^*$  and  $L_2^*$  are the two fixed points of the one-dimensional quadratic map (3), conjugate with the well-known logistic map, and under the constraints on the parameters considered in this model  $L_1^*$  is always stable whereas  $L_2^*$  is always unstable. In fact, it is immediate to show that the derivative  $\mathcal{L}'(L_1^*) = 1 + \gamma - \alpha(1 - \rho)$  is such that  $0 < \mathcal{L}'(L_1^*) < 1$  (stability without oscillations), and  $\mathcal{L}'(L_2^*) = 1 + \alpha(1 - \rho) - \gamma > 1$  for any set of feasible parameters.

Each equilibrium is characterized by a pair  $(K^*, L^*)$  and it is necessary to analyze its properties. With regard to the labor force,  $L_2^*$  represents the case in which there are no ill workers, i.e. the case without infections. Conversely, the equilibrium level  $L_1^*$  is characterized by a lower level of labor supply, that is  $L_1^* < L_2^*$ , and depends on factors related to the pandemic, where:

$$\frac{\partial L_1^*}{\partial \alpha} < 0, \quad \frac{\partial L_1^*}{\partial \gamma} > 0 \quad \text{and} \quad \frac{\partial L_1^*}{\partial \rho} > 0$$

It is easy to see that the greater the severity of the pandemic (high  $\alpha$ , low  $\gamma$ ) the lower the level of labor supply as the number of ill workers increases. Parameter  $\rho$  plays the expected role, that is, the decision of governments to suppress economic activities is intended to reduce socio-economic contacts and, consequently, the number of infected workers decreases thus giving more available equilibrium labor supply.

**Corollary 1.** Positive equilibria exist depending on the value of the threshold level for capital  $K_c$  (which influences  $z$ , with  $\frac{\partial z}{\partial K_c} > 0$ ). For  $K_c$  excessively high no positive equilibria exist since the economy would not be able to reach a level of capital for which economic production is possible.

**Corollary 2.** For sufficiently low levels of  $K_c$ , the existence of the equilibria depends on the severity of the pandemic. For  $\alpha \leq \frac{\gamma}{z}$ , there are four positive fixed points, and as the contagiousness of the illness increases the positive fixed points might be destroyed via saddle–node bifurcation. Concerning the suppression policy implemented by government, for an excessively high value of  $\rho$  there exists only those equilibria characterizing the poverty traps.

In summary, the model (7) states that poverty traps, characterized by very low (symbolically 0) equilibrium capital levels, always exist. As the equilibrium level of labor supply is concerned,  $L_2^*$  represents the case of usual economic times, while  $L_1^*$  represents the case of pandemic times and its level decreases when the severity of the illness increases and it increases when the government implement too severe policies to reduce economic activities. Indeed, the severity of the

illness (parametrized by  $\alpha$ ) and the severity of the implemented policies (parametrized by  $\rho$ ) might destroy the positive equilibria, so that only those labeled as poverty traps exist.

Note that, according to the inequalities represented by capital levels:

$$K_c < K_{1,2}^* < K_{1,1}^* < K_{2,1}^* < K_{2,2}^*, \quad (8)$$

the equilibria can characterize several economies, that is,

- $E_{0,1}$  and  $E_{0,2}$  represent poor economies in a poverty trap
- $E_{1,1}$  and  $E_{1,2}$  represent economies trying to overcome poverty, or emerging economies (for instance, the developing middle-income countries)
- $E_{2,1}$  and  $E_{2,2}$  are advanced economies (developed countries)

In pandemic times, a policy to suppress economic activities plays a crucial role. As discussed, increasing values of parameter  $\rho$  may lead to the disappearance of equilibrium points characterized by positive capital. In other words, suppression policies that reduce potential output could destroy the high-level equilibria. However, when there are equilibria with positive capital, their levels are affected by such policies, according to the following proposition (proof in [Appendix](#)):

**Proposition 2.** Consider that  $\rho < \rho_i$ ,  $i \in \{1, 2\}$ . As  $\rho$  increases,  $K_{1,i}^*$  increases while  $K_{2,i}^*$  decreases.

This proposition points out that developed economies are strongly affected by suppression policies because their capital levels are steadily declining. Since advanced economies have implemented strong mitigation measures, this results in partial and total closures of economic activities.<sup>10</sup>

The study of stability properties of the equilibria, as well as an analysis of the qualitative properties of the dynamic behavior around them, is necessary to understand the long-run behavior of the economic system. As we show below, the stability is determined by the parameters:  $\rho$  (suppression policies),  $\alpha$  (the infection rate), and  $\gamma$  (the recovery rate).

**Proposition 3 (Stability of Equilibria).** The low-level equilibria, or poverty traps (characterized by zero capital), are such that  $E_{0,1}$  is locally asymptotically stable while  $E_{0,2}$  is a saddle point with unstable manifold along the axis  $K = 0$ .

In other words, [Proposition 3](#) states that in times of pandemic, the poverty trap is a stable situation (remember that  $E_{0,1}$  is characterized by  $L_1^* < \bar{L}$  in which the level of the labor supply depends on the severity of the pandemics), from which it is difficult to get out. Indeed, in order to escape the basin of attraction of such poverty trap it is necessary to have initial capital  $K > K_c$ , but this may not be sufficient. In fact, a stable equilibrium characterized by positive capital should exist, with its own basin of attraction. As we shall argue, such locally stable equilibrium, if exists, is the stable node  $E_{2,1}$  whose basin of attraction is bounded by the stable set of the saddle  $E_{1,1}$ , as shown in the schematic picture in [Fig. 5](#), summarizing the results of the next Proposition concerning the stability of the equilibrium points characterized by a positive capital per capita (a proof is given in [Appendix](#)).

**Proposition 4 (Stability of Equilibria).** As the parameter  $\rho$  decreases through the threshold values  $\rho_2$  and  $\rho_1$ , defined in [Proposition 1](#), twofold (or saddle nodes) bifurcations occur at which, respectively,  $E_{1,2} = E_{2,2} = E_{M,2}$  for  $\rho = \rho_2$  and  $E_{1,1} = E_{2,1} = E_{M,1}$  for  $\rho = \rho_1 < \rho_2$ . Then

- $E_{2,1}$ , when it exists, is a locally stable node
- $E_{1,1}$  and  $E_{2,2}$ , when they exist, are saddle points
- $E_{1,2}$ , when it exists, is an unstable node
- $E_{M,1}$  and  $E_{M,2}$ , when they exist, are non hyperbolic (i.e. structurally unstable, or bifurcation cases)

Notice that, as the two bifurcation curves  $\rho = \rho_1$  and  $\rho = \rho_2$  are concerned, the more significant one is  $\rho = \rho_2$ : for  $z > \left(\frac{\gamma}{\alpha}\right)^{1-\beta}$ , even if  $\rho < \rho_2$ , the only attractive fixed point is the poverty trap while for  $z < \left(\frac{\gamma}{\alpha}\right)^{1-\beta}$  the economy might reach the attractive equilibrium characterized by positive capital per capita only if the policies to contain the virus spread are sufficiently low, i.e.  $\rho < \rho_2$ .

A summary of the qualitative results of this section are represented in [Fig. 5](#).

Let us now give an interpretation of the effects of the threshold value  $z$  on the existence of equilibria. Recalling that  $z$  increases with  $K_c$  it is easy to see that, no matter the applied policy, if  $K_c$  is sufficiently high, i.e. the threshold level to get positive output is high, then the economy converges to the poverty trap. In addition to the poverty trap, there could be an attractive fixed point if  $z < \left(\frac{\gamma}{\alpha}\right)^{1-\beta}$ , e.g.  $\gamma$  sufficiently high or  $\beta$  sufficiently low; conversely, in the event of extremely severe pandemics, the risk of falling into a poverty trap is specific to all economies.

<sup>10</sup> The economic consequences are amplified by various internal vulnerabilities, for example a large tourism and service sector, the proportion of jobs that cannot be performed remotely and in limited fiscal space to offset economic impacts. The Global Economic Outlook During the COVID-19 Pandemic: A Changed World, <https://www.worldbank.org/en/news/feature/2020/06/08/the-global-economic-outlook-during-the-covid-19-pandemic-a-changed-world>, <https://www.economicsobservatory.com/how-coronavirus-affecting-emerging-market-and-developing-economies>.

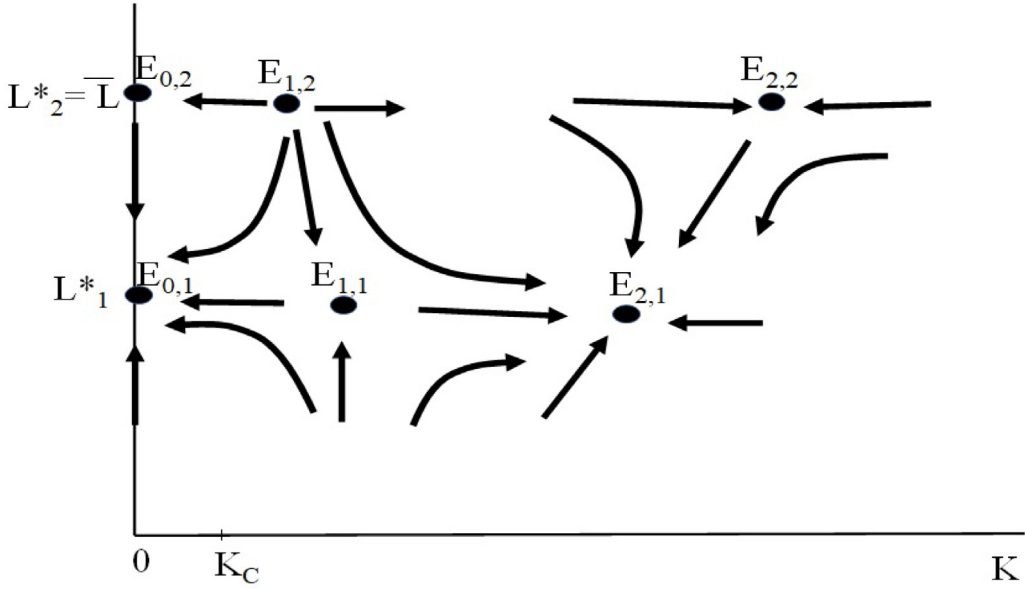


Fig. 5. Qualitative sketch of the phase diagram in the case of six equilibrium points.

**Corollary 3.** For avoiding to enter into a poverty trap it is necessary to have an attracting equilibrium characterized by positive capital. In the model proposed such equilibrium is identified with  $E_{2,1}$ , it exists when  $z < \left(\frac{\gamma}{\alpha}\right)^{1-\beta}$  and  $\rho < \rho_1 = 1 - z^{\frac{1}{\beta}} \left(\frac{\alpha}{\gamma}\right)^{\frac{1-\beta}{\beta}}$  where  $z = \left[\frac{K_c}{L(1-\beta)}\right]^{1-\beta} \frac{\delta}{s\beta\beta}$ . In other words, for sufficiently low values of  $K_c$  and sufficiently low values of  $\rho$ . Moreover, whenever the equilibrium  $E_{2,1}$  exists it is a stable node, and its basin of attraction enlarges as its distance  $K_2^* - K_1^*$  from the twin-saddle  $E_{1,1}$  increases. This happens when

$$G(K_M, L_1^*) = \left(\frac{1-\beta}{\beta}\right) \left[\frac{\beta s(1-\rho)}{\delta\beta}\right]^{\frac{1}{1-\beta}} \frac{\gamma\bar{L}}{\alpha(1-\rho)} - \delta K_c$$

increases (see the Proof of Proposition 1 in Appendix), i.e. for increasing values of the saving rate,  $s$  and the recovery rate,  $\gamma$ .

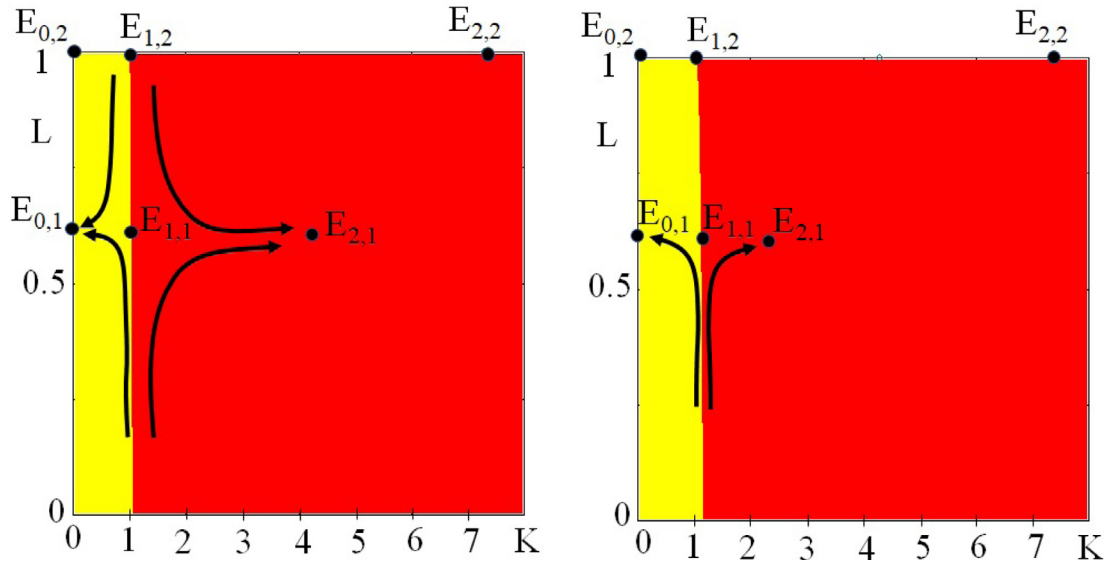
What all the above tells us is that economies must be careful with policies to suppress leisure economic activities, because if those policies are too rigorous and for increasing values of the infection rate  $\alpha$ , such economies can fall into the poverty trap. However, if the suppression policies are effective in such a way that the disease recovery rate is high and the infection rate is low, then the economies can converge towards the path of economic growth with positive capital, which is also stable if, together with the effectiveness of the policies to contain the pandemic, there are also high levels of savings rates. We present below some numerical simulations to corroborate the results given above.

#### 4. Numerical simulations

In this section we carry out some numerical simulations of the dynamical system (7) in order to confirm the analytical results of the previous section concerning existence and local stability of the equilibrium points and to extend this analysis to include some global properties of the dynamical system, such as the structure and extent of the basins of attraction of the coexisting stable equilibria.

Indeed, as we have argued in the previous section, for proper parameters' constellations, two locally stable equilibrium points coexist:  $E_{0,1} = (0, L_1^*)$ , denoted as a pandemic poverty trap (poverty trap because it is characterized by the disappearance of capital, pandemic because  $L_1^* < \bar{L}$ ) and  $E_{2,1} = (K_{2,1}^*, L_1^*)$ , a stable equilibrium characterized by a higher level of capital  $K_{2,1}^* > K_{M,1}$  even with the same labor force reduced due to the pandemic situation, each with its own basin of attraction. Of course, to understand which of these two attractors will prevail in the long run, for a given initial condition  $(K_0, L_0)$  of the economy, a global delimitation of the basins is necessary. This information cannot be obtained by means of a local analysis of the equilibria based on a linear approximation (that is, the Jacobian matrix) but rather it requires a global vision of the dynamic scenario of the non-linear system, a task that can only be approached by a dialogue between numerical and analytical methods.

As stated by Corollary 3 in the previous section, information on the extension of the basins and, consequently, on the robustness of the stability of the equilibrium  $E_{2,1}$  with respect of an exogenous displacement of the state of the system (i.e. of the initial condition of the economy) can be obtained by considering the difference  $K_{2,1}^* - K_{1,1}^*$ .



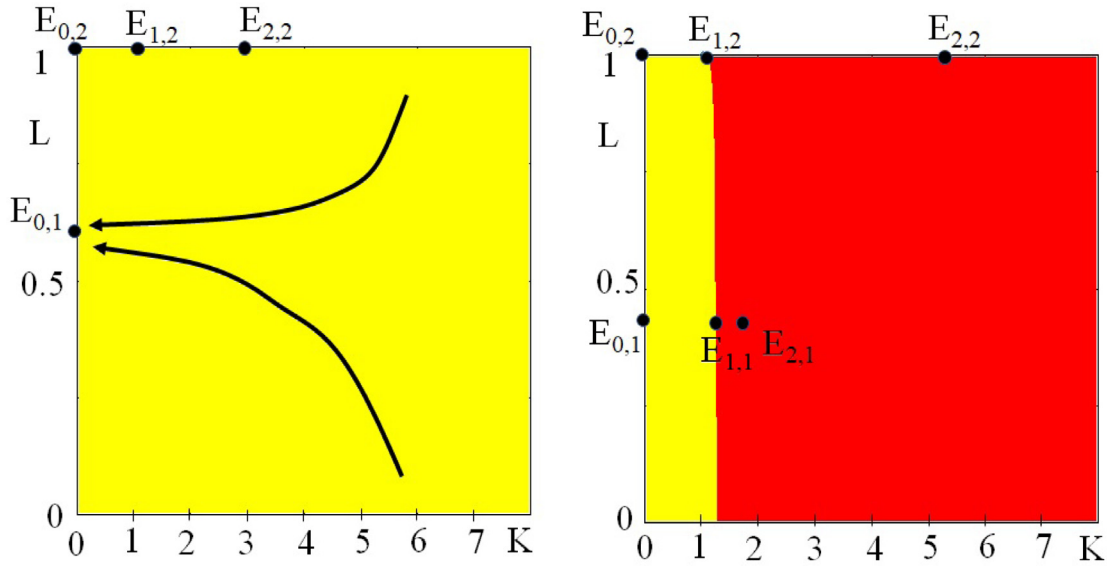
**Fig. 6.** Phase diagram of the dynamical system (7) with the set of parameters described in the text and saving rate  $s = 0.07$  in the left panel,  $s = 0.05$  in the right panel. Basins of attraction are represented with different colors.

Indeed, as explained in the previous section, the couple of equilibrium points  $E_{1,1} = (K_{1,1}^*, L_1^*)$  and  $E_{2,1} = (K_{2,1}^*, L_1^*)$ , with  $K_c < K_{1,1}^* \leq K_{M,1} \leq K_{2,1}^*$ , are created together through saddle-node bifurcation, hence the stable set of the saddle  $E_{1,1}$  constitutes the boundary that separates the basin of attraction of the corresponding stable node  $E_{2,1}$  and the stable poverty trap  $E_{0,1}$  (see also Fig. 5). The global shape of this stable set can only be determined numerically, because its analytic expression cannot be determined, in general, for a nonlinear dynamical system.

To fix some suitable parameter sets, we consider Rawson et al. [41], who applied a Susceptible–Exposure–Infection–Recovery (SEIR) model to study the efficacy of lockdown measures. Based on their evidence to Covid-19 daily data, we take  $\gamma = 0.22$ ,  $\alpha = 0.6$  as baseline values. We also choose a suppression policy value of  $\rho = 0.4$  [see 42]. As economic parameters are concerned, and considering daily rates, we assume as baseline values  $s = 0.07$  i.e. 7% as an approximate average saving rate for OECD countries,<sup>11</sup>  $\beta = 0.3$ ,  $\delta = 0.01$  [see 44]. We normalize  $\bar{L} = 1$  and a threshold level of capital  $K_c = 1$ . Note that the assumption  $K_c = 1$  is arbitrary and, as discussed in Section 2, a more appropriate alternative would be to consider an S-shaped production function that avoids the presence of a kink. For this set of parameters we get that  $z = 0.26$ , hence we are in the situation  $z < \frac{\gamma}{\alpha} = 0.37$ . This implies, according to Proposition 1, that in addition to the two equilibrium points with zero capital  $E_{0,1} = (0, 0.61)$  and  $E_{0,2} = (0, 1)$ , there are four equilibrium points with positive capital, given by  $E_{1,1} = (1.03, 0.61)$ ,  $E_{2,1} = (4.22, 0.61)$ ,  $E_{1,2} = (1, 1)$ ,  $E_{2,2} = (7.29, 1)$ . The computation of the eigenvalues of the Jacobian matrix at the equilibrium points confirms that  $E_{0,1}$  and  $E_{2,1}$  are stable nodes, as the two eigenvalues of  $E_{0,1}$  are  $\lambda_1 = 0.86$  and  $\lambda_2 = 0.99$  and analogously for  $E_{2,1}$  we have  $\lambda_1 = 0.86$  and  $\lambda_2 = 0.99$ . Instead  $E_{1,1}$  is a saddle, being  $\lambda_1 = 0.86$  and  $\lambda_2 = 1.09$ . The corresponding structure of the basins of attraction is shown in the left panel of Fig. 6, where the red region represents the basin of  $E_{2,1}$  and the yellow region is the basin of the poverty trap  $E_{0,1}$ . Notice that besides the saddle  $E_{1,1}$  also the unstable node  $E_{1,2}$  (eigenvalues  $\lambda_1 = 1.14$  and  $\lambda_2 = 1.34$ ) is located along the boundary that separates the two basins. Moreover, it is evident as expected that the horizontal distance between the pair of equilibria  $E_{1,1}$  and  $E_{2,1}$  gives an approximation of the robustness of the equilibrium with positive capital, that is, the maximum displacement (exogenous reduction of the capital level) that the system can recover spontaneously without falling into the poverty trap. Therefore, any variation in parameters that reduces this distance will lead to a situation of greater vulnerability.

For example, if *ceteris paribus* we consider a lower propensity to save, e.g.  $s = 0.05$  (i.e. 5% instead of 7%) with all other parameters set to the same values, the dynamic situation obtained is shown in the right panel of Fig. 6, where it is evident that even if the stable equilibrium  $E_{2,1} = (2.30, 0.61)$  has the same eigenvalues  $\lambda_1 = 0.86$  and  $\lambda_2 = 0.99$  and a large basin, nevertheless it is more vulnerable being much closer to the boundary of its basin, so an exogenous perturbation causing a small decrease of the capital may lead to cross the basin boundary and consequently enter the poverty trap where irreversible transition to the equilibrium  $E_{0,1}$  occurs.

<sup>11</sup> See OECD [43] <https://data.oecd.org/natincome/saving-rate.htm>. Saving is equal to the difference between disposable income (including an adjustment for the change in employment-related pension entitlements) and final consumption expenditure. It reflects the part of disposable income that, together with the incurrence of liabilities, is available to acquire financial and non-financial assets. The saving rate presented here corresponds to net saving, which is saving net of depreciation, as percentage of gross domestic product (GDP).



**Fig. 7.** Phase diagram of the dynamical system (7) with the set of parameters described in the text and reduced savings (4%). In the left panel the suppression policy  $\rho = 0.4$  like in Fig. 8, and in the right panel  $\rho = 0.15$ .

If a slightly lower propensity to save is considered, such as  $s = 0.04$  (i.e. 4% instead of 5%) with the same values of all the other parameters, we get  $z = 0.46$ , hence  $z > \frac{\gamma}{\alpha} = 0.37$  and  $z < (\frac{\gamma}{\alpha})^{1-\beta} = 0.49$ . So, in this situation the value of the parameter  $\rho$  is crucial for deciding the number of positive equilibrium points, according to Proposition 1. As  $\rho_1 = 0.22$  and  $\rho_2 = 0.54$ , a suppression policy  $\rho = 0.4$  corresponds to the case  $\rho_1 < \rho < \rho_2$ , and this implies that the couple of equilibria  $E_{1,1}$  and  $E_{2,1}$  no longer exist, and any initial condition (except the ones starting from the unstable equilibrium points) generates a trajectory converging to the globally stable poverty trap  $E_{0,1}$ , as shown in the left panel of Fig. 7.

In order to have a situation similar to the ones shown in Fig. 6 with the lower propensity to save  $s = 0.04$  it is necessary to reduce the degree of suppression policy. For example, the dynamic situation shown in the right panel of Fig. 7, similar to the one of the right panel of Fig. 6, is obtained with  $s = 0.04$  and  $\rho = 0.15$  instead of  $\rho = 0.4$ .

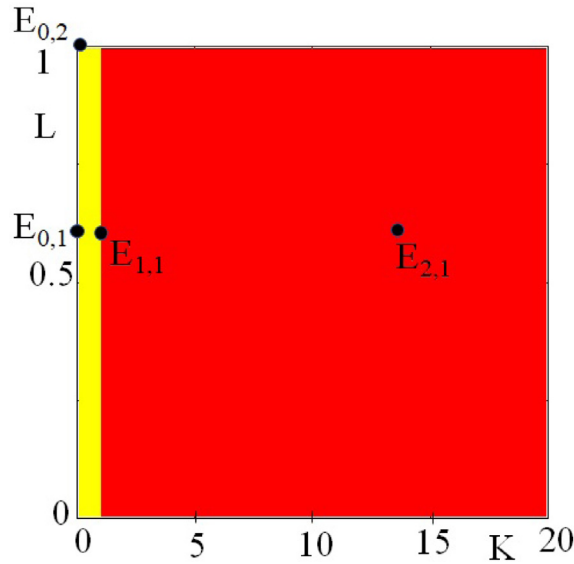
Finally, we consider a much higher propensity to save (Denmark and South Korea are advanced countries with high savings rates of around 15% of GDP), that is  $s = 0.15$  and a repression policy level of  $\rho = 0.4$ , and all other parameters remain the same values. The result, reported in Fig. 8, exhibits a stable equilibrium with a much higher capital value,  $K_{2,1}^* = 13.65$  and a quite enhanced robustness with respect to exogenous perturbations of the capital level, as its distance from the basin boundary has increased considerably. With this set of parameters we have  $z = 0.12 < \frac{\gamma}{\alpha} = 0.36$ , hence  $\rho_1 = 0.99 > \rho_2 = 0.87$ .

This last numerical exercise clearly shows us that in times of pandemic countries with higher savings rates have had faster economic growth than those with lower savings rates, even if they have applied strict policies to contain the pandemic. It is well known that higher rates of saving imply greater accumulation of capital creating greater opportunities for production and productivity by providing an additional income stream for such saving economies. While saving is known to be important, when an economy is going through tough times, one is surprised to discover how much a high savings rate can accelerate the economic recovery (see, for instance, [45,46]). And this is what in the end we show here, considering that you have a greater capacity to cope with socioeconomic difficulties which ultimately means that the economy recovers much faster.

## 5. Concluding remarks

This paper develops a model incorporating an epidemic SIS population into a Solow growth model. The model exhibits multiple steady state equilibria. When the propensity to save is low and the suppression policy does not work to combat the spread of pandemic, the economy may fall into a poverty trap characterized by low and locally stable steady state level of capital accumulation. In the opposite case, the economy may converge to a steady state characterized by high capital accumulation. The suppression policy affects capital accumulation through its direct impact on labor supply in production. We have identified analytically some segregation thresholds of capital accumulation and of degree of suppression policy at which a bifurcation occurs. These analytical results were illustrated and supported by some numerical exercises.

We also show that the propensity to save in an economy, that is its levels of productive investment, influences the existence of the positive equilibrium. In case of a low saving rate, it is very likely that such an economy, in times of a pandemic, will converge towards a poverty trap; conversely, if the propensity to save is high and the initial capital level



**Fig. 8.** Phase diagram of the dynamical system (7) with the set of parameters described in the text with suppression policy  $\rho = 0.4$  and increased saving rate (15%).

condition is high and the suppression policy works in such a way that there are low infection levels and high recovery levels from disease high, then the economy converges towards a positive capital level.

A higher-than-normal savings rate also imply that there will be a demand gap that must be covered by some other component of GDP, such as higher public spending, if the economy is to operate near its full employment production level. In this context, the public deficit can be essential to sustain demand. But this is something that we do not address in this paper but that undoubtedly emerges as economic policy. Another important aspect of the latest consumer data is that it can give us more insight into the post-pandemic economy. We know that many more people worked from home during the pandemic than before. In many cases, firms are now implementing their plans to operate in a post-pandemic world. This is sure to include more opportunities for people to work from home, although obviously less than at the peak of the pandemic. In other words, the labor supply will not stop and even increase, such that the economies that have managed to combat the pandemic efficiently will have positive economic growth, characterized by demand (supported by savings) and levels of labor supply.

We conclude that future research of the above model should go in a direction such that it is possible to analyze the dynamics of suppression policies considered as a time-dependent variable. Important may be also to analyze that during the COVID-19 pandemic, there are indeed many forms of remote working (home offices, online meetings, etc.) which contribute to determine labor supply under the suppression policy. We therefore claim that it is important to consider these facts in the above model, and so implicitly or explicitly account these facts in determining labor supply. Hence we should be able to analyze how the results of the model change if these factors are introduced into the model. For instance, the suppression policy, which leads to remote working and learning, may generate mental health diseases for current labor force generation and/or negatively affect the educational achievements for the young generation. These undesirable effects may reduce labor productivity in both short and long run. Hence, it seems very important to take into account these channels. Thus we should be able to study how suppression policies dynamically affect economic growth over time. It would also be important to analyze the structures of the labor supply by age groups. That is, labor supplies of the elderly and of young people may be affected by the pandemic through different levels of infection and recovery rates.

#### CRediT authorship contribution statement

**Gian Italo Bischi:** Methodology, Software, Formal analysis, Writing, Supervision. **Francesca Grassetti:** Methodology, Software, Formal analysis, Writing. **Edgar J. Sanchez Carrera:** Conceptualization, Formal analysis, Writing.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.



## Appendix A. Proof of Proposition 1

The equilibrium points of the two dimensional map (7) are given by couples  $(K, L)$  such that

$$\begin{cases} \mathcal{K}(K, L) = K \\ \mathcal{L}(L) = L \end{cases} \quad (9)$$

The second degree algebraic equation  $\mathcal{L}(L) - L = 0$  has two solutions,  $L_1^* = \frac{\gamma \bar{L}}{\alpha(1-\rho)}$  and  $L_2^* = \bar{L}$ ,  $L_1^* < L_2^*$ , as  $\gamma < \alpha(1-\rho)$  follows from the condition  $\rho < 1 - \frac{\gamma}{\alpha}$ .

For  $K \leq K_c$ , equation  $\mathcal{K}(K) - K = 0$  becomes  $-\delta K = 0$ , hence the unique solution  $K_0^* = 0$  exists.

For  $K > K_c$  let

$$G(K, L^*) := \mathcal{K}(K, L^*) - K = s(1-\rho)L^{*1-\beta}(K - K_c)^\beta - \delta K \quad (10)$$

where  $L^* \in \{L_1^*, L_2^*\}$ . The function  $G(K, L^*)$  is such that  $\lim_{K \rightarrow K_c} G(K, L^*) = -\delta K_c < 0$  and  $\lim_{K \rightarrow +\infty} G(K, L^*) = -\infty$ .

Moreover  $G'(K, L^*) = \frac{\beta s(1-\rho)L^{*1-\beta}}{(K-K_c)^{1-\beta}} - \delta$ , hence  $G'(K, L^*) \geq 0$  iff  $K \leq K_c + \left(\frac{\beta s(1-\rho)}{\delta}\right)^{\frac{1}{1-\beta}} L^*$  and  $G'(K, L^*) < 0$  otherwise.

Therefore  $G(K, L^*)$  has a maximum in  $K_M = K_c + \left(\frac{\beta s(1-\rho)}{\delta}\right)^{\frac{1}{1-\beta}} L^*$ , with  $G(K_M, L^*) = \left(\frac{1-\beta}{\beta}\right) \left[\frac{\beta s(1-\rho)}{\delta}\right]^{\frac{1}{1-\beta}} L^* - \delta K_c$ . So, if  $G(K_M, L^*) > 0$  the equation  $G(K) = 0$  has two solutions, say  $K_1^* < K_M < K_2^*$ .

Notice that  $G(K, L_1^*) < G(K, L_2^*)$ ,  $\forall K$  and assuming  $z = \left[\frac{K_c}{L(1-\beta)}\right]^{1-\beta} \frac{\delta}{s\beta\beta}$  it has  $G(K_{M,1}, L_1^*) > 0$  if  $\rho < \rho_1 = 1 - z^{\frac{1}{\beta}} \left(\frac{\alpha}{\gamma}\right)^{\frac{1-\beta}{\beta}}$  and  $G(K_{M,2}, L_2^*) > 0$  if  $\rho < \rho_2 = 1 - z$ , where  $K_{M,i} = K_c + \left(\frac{\beta s(1-\rho)}{\delta}\right)^{\frac{1}{1-\beta}} L_i^*$ . Moreover

- for  $z \geq 1$  it has  $\rho_1 < \rho_2 \leq 0 < \rho$  and  $G(K_{M,i}, L_i^*) < 0 \forall i \in \{1, 2\}$ , therefore no positive equilibria exist;
- for  $z \in \left[\frac{\gamma^{1-\beta}}{\alpha^{1-\beta}}, 1\right)$  it has  $\rho_1 \leq 0$ ,  $0 < \rho_2 < 1 - \frac{\gamma}{\alpha}$  therefore  $G(K_{M,1}, L_1^*) < 0$  while  $G(K_{M,2}, L_2^*) > 0$  iff  $\rho < \rho_2$ ;
- for  $z \in \left(\frac{\gamma}{\alpha}, \frac{\gamma^{1-\beta}}{\alpha^{1-\beta}}\right)$  it has  $0 < \rho_1 < \rho_2 < 1 - \frac{\gamma}{\alpha}$  therefore  $G(K_{M,1}, L_1^*) > 0$  iff  $\rho < \rho_1$  and  $G(K_{M,2}, L_2^*) > 0$  iff  $\rho < \rho_2$ ;
- for  $z \leq \frac{\gamma}{\alpha}$  it has  $\rho < 1 - \frac{\gamma}{\alpha} \leq \rho_2 \leq \rho_1$  therefore  $G(K_{M,i}, L_i^*) > 0 \forall i \in \{1, 2\}$ .

It is possible to conclude that map (7):

- always has two fixed point characterized by zero capital per capita:  $E_{0,1} = (0, L_1^*)$  and  $E_{0,2} = (0, L_2^*)$ ;
- in case  $z \geq 1$  no additional equilibria exist;
- in case  $z \in \left[\frac{\gamma}{\alpha}, 1\right)$ 
  - for  $\rho > \rho_2$  no additional equilibria exist;
  - for  $\rho = \rho_2$  one positive fixed points exist given by  $E_{M,2} = (K_{M,2}, L_2^*)$ ;
  - for  $\rho < \rho_2$  two positives fixed points exist given by  $E_{1,2} = (K_{1,2}^*, L_2^*)$  and  $E_{2,2} = (K_{2,2}^*, L_2^*)$ , with  $K_{1,2}^* < K_{M,2} < K_{2,2}^*$ ;
- in case  $z \in \left(\frac{\gamma}{\alpha}, \frac{\gamma^{1-\beta}}{\alpha^{1-\beta}}\right)$ 
  - for  $\rho > \rho_2$  no additional equilibria exist;
  - for  $\rho = \rho_2$  one positive fixed point exists given by  $E_{M,2}$ ;
  - for  $\rho_1 < \rho < \rho_2$  two positives fixed points exist given by  $E_{1,2}$  and  $E_{2,2}$ ;
  - for  $\rho = \rho_1$  three positive fixed point exist given by  $E_{M,1} = (K_{M,1}, L_1^*)$ ,  $E_{1,2}$  and  $E_{2,2}$ , with  $K_{1,2}^* < K_{M,1} < K_{2,2}^*$ ;
  - for  $\rho < \rho_1$  four positive fixed point exist given by  $E_{1,1} = (K_{1,1}^*, L_1^*)$ ,  $E_{2,1} = (K_{2,1}^*, L_1^*)$ ,  $E_{1,2}$  and  $E_{2,2}$ , with  $K_{1,2}^* < K_{1,1}^* < K_{2,1}^* < K_{2,2}^*$ ;
- in case  $z \leq \frac{\gamma}{\alpha}$  four positive fixed point exist given by  $E_{1,1}$ ,  $E_{2,1}$ ,  $E_{1,2}$  and  $E_{2,2}$ .

The bifurcation conditions are given by  $G(K_{M,i}, L_i^*) = 0$ ,  $i \in \{1, 2\}$  equivalent to  $\rho = 1 - z$  and  $\rho = 1 - z^{\frac{1}{\beta}} \left(\frac{\alpha}{\gamma}\right)^{\frac{1-\beta}{\beta}}$ , so all the cases given in Proposition 1 are obtained.

## Appendix B. Proof of Proposition 2

$K_{1,i}^*$  and  $K_{2,i}^*$  are solutions of  $G(K, L^*) = 0$ , where  $G(K, L^*)$  is given by (10) and  $i \in \{1, 2\}$ . Function  $G$  is unimodal with  $\frac{\partial G}{\partial \rho} < 0$ , consequently - as long as  $\rho < \rho_i$  - increasing  $\rho$ ,  $K_{1,i}^*$  increases while  $K_{2,i}^*$  decreases.

### Appendix C. Proof of Proposition 3

The Jacobian matrix evaluated at the fixed points  $E_{0,1}$  and  $E_{0,2}$  becomes

$$\mathbf{J}_T(E_{0,1}) = \begin{bmatrix} 1-\delta & 0 \\ 0 & 1+I^* \end{bmatrix}, \quad \mathbf{J}_T(E_{0,2}) = \begin{bmatrix} 1-\delta & 0 \\ 0 & 1-I^* \end{bmatrix},$$

with

$$I^* = \gamma - \alpha(1 - \rho) < 0. \quad (11)$$

Since the matrices are both diagonal, the eigenvalues are given by the diagonal entries: the first eigenvalue is equal for both the equilibria  $\lambda_1 = 1 - \delta$  with associated eigenvector  $\mathbf{v}_1 = (1, 0)$  along the horizontal axis, while for the second eigenvalue it has  $\lambda_{2,1} = 1 + I^*$  for  $E_{0,1}$  and  $\lambda_{2,2} = 1 - I^*$  for  $E_{0,2}$ , both with associated eigenvector  $\mathbf{v}_2 = (0, 1)$  along the invariant vertical axis  $K = 0$ . So, the equilibria are always attracting along the horizontal direction, whereas in the vertical direction they are locally asymptotically stable if  $-1 < \lambda_{2,i} < 1$ ,  $i \in \{1, 2\}$ . As  $I^* < 0$ ,  $0 < \lambda_{2,1} < 1$ ,  $\lambda_{2,2} > 1$  for all parameter values therefore  $E_{0,1}$  is a stable node while  $E_{0,2}$  is a saddle, with unstable manifold along the vertical direction.

### Appendix D. Proof of Proposition 4

For  $K > K_c$  the Jacobian matrix of the map (7) is

$$\mathbf{J}_T(K, L) = \begin{bmatrix} 1 - \delta + \beta s(1 - \rho) \left( \frac{L}{K - K_c} \right)^{1-\beta} & s(1 - \beta)(1 - \rho) \left( \frac{K - K_c}{L} \right)^\beta \\ 0 & 2 \frac{\alpha(1 - \rho)}{L} L + 1 - \gamma - \alpha(1 - \rho) \end{bmatrix}.$$

Notice that the Jacobian matrix is triangular, hence its eigenvalues are always real, given by the diagonal entries. When it is computed at the equilibria characterized by  $L_1^*$ , i.e.  $E_{i,1}^* = (K_i^*, L_1^*)$ , with  $K_i^* > K_c$  according to Proposition 1, the two eigenvalues are given by

$$\lambda_{1,1} = 1 - \delta + \beta s(1 - \rho)^\beta \left[ \frac{\gamma \bar{L}}{\alpha(K_i^* - K_c)} \right]^{1-\beta}$$

with corresponding eigenvector parallel to the  $K$  axis, and

$$\lambda_{2,1} = 1 + I^*$$

where  $I^*$  is given by (11). So, the stability condition of  $E_{i,1}^*$  along the  $L$  direction is the same as the one discussed in the Proof C for the equilibrium  $E_{0,1}^*$ . Indeed, it has  $0 < \lambda_{2,1} < 1$  and the eigenvalue does not depend on  $K_i^*$ , so it is the same at any equilibrium point.

Concerning the stability of  $E_{i,1}^*$  along the  $K$  direction, being  $0 \leq \delta \leq 1$ , we have  $\lambda_{1,1} > -1$ , hence the condition of local asymptotic stability along the  $K$  direction is given by  $\lambda_1 < 1$ . This condition is equivalent to

$$K_i^* > K_c + \left[ \frac{\beta s(1 - \rho)^\beta}{\delta} \right]^{\frac{1}{1-\beta}} \frac{\gamma \bar{L}}{\alpha}. \quad (12)$$

For  $\rho = 1 - z^{\frac{1}{\beta}} \left( \frac{\alpha}{\gamma} \right)^{\frac{1-\beta}{\beta}} = \rho_1$  we have  $K_{1,1}^* = K_{2,1}^* = K_{M,1}$  and  $\lambda_1 = 1$ . So, this condition states that a unique equilibrium point with positive capital exists and is a typical fold (or saddle-node) bifurcation at which the two fixed points  $E_{i,1}^*$ ,  $i = 1, 2$ , are created, with  $K_{1,1}^* < K_M < K_{2,1}^*$ , hence  $E_{1,1}^*$  unstable and  $E_{2,1}$  stable. As usual in this kind of bifurcation, the stable set of the saddle point  $E_{1,1}^*$  constitutes the boundary of the basin of attraction of the stable node  $E_{2,1}^*$ .

Then we can conclude that, considering fixed points in which  $L^* = L_1^*$ , for  $\rho > \rho_1$  no fixed points with positive capital exist; for  $\rho = \rho_1$  a non-hyperbolic (hence structurally unstable) fixed point  $E_{M,1}^* = (K_{M,1}, L_1^*)$  is created through a fold bifurcation; for  $\rho < \rho_1$  two fixed points  $E_{i,1}^*$ ,  $i = 1, 2$ , emerge:  $E_{1,1}^*$  is a saddle point while  $E_{2,1}^*$  is a stable node.

As the equilibria characterized by  $L_2^*$  are concerned, note that the same considerations hold, despite that, in this case the eigenvalues are given by

$$\lambda_{1,2} = 1 - \delta + \beta s(1 - \rho) \left[ \frac{\bar{L}}{K_i^* - K_c} \right]^{1-\beta}$$

and

$$\lambda_{2,2} = 1 - I^*$$

therefore  $\lambda_{2,2} > 1$  for all parameter values while concerning the stability of the points  $E_{i,2}^*$  along the  $K$  direction the bifurcation (saddle-node) exist for  $\rho = \rho_2 = 1 - z$  and we can conclude that, considering fixed points in which  $L^* = L_2^*$ , for  $\rho > \rho_2$  no fixed point with positive capital exist; for  $\rho = \rho_2$  a non-hyperbolic (structurally unstable) fixed point  $E_{M,2}^* = (K_{M,2}, L_2^*)$  is created through a fold bifurcation; for  $\rho < \rho_2$  two fixed points  $E_{i,2}^*$ ,  $i = 1, 2$ , emerge:  $E_{1,2}^*$  is an unstable node while  $E_{2,1}^*$  is a saddle point.

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