



Household Financial Fragility, Debt and Income in a Dynamic Model

Giorgio Calcagnini^{1,2} · Federico Favaretto^{1,2}  · Germana Giombini² · Fabio Tramontana²

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Abstract

We develop a novel dynamic model for household debt and household income change studying the interaction between financial fragility and financial literacy. We compare the results to the U.S. data under several parameterizations. Households react pro-cyclically to income shocks and are better able to represent aggregate data when financial literacy is low.

Keywords Debt · Income · Piecewise defined maps · Financial fragility · Financial literacy.

JEL classification C61 · C62 · C63; E21 · G51 · G53

1 Introduction

According to the IMF's Global Debt Database (IMF, 2022), overall global debt has reached extremely high levels in recent years, standing at 247% of GDP in 2021, with private household and corporate debt accounting for three-quarters of the total amount (Liu et al., 2020). Indeed, in 2020 the world experienced the largest one-year

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✉ Federico Favaretto
federico.favaretto@uniurb.it

¹ Department of Economics, Society and Politics, University of Urbino, Via Saffi 42, Urbino, Italy

² Department of Economics, Society and Politics, University of Urbino, Urbino, Italy

increase in global debt, when it rose by 29 percentage points of GDP. Meanwhile, the world's non-financial private debt dropped from 160 percent of GDP in 2020 to 153 percent of GDP in 2021, driven by a larger drop in corporate debt than that of household debt. This paper develops a theoretical model that links financial literacy and financial fragility to the dynamics of household debt and income change. Analysing the dynamics of debt and over-indebtedness of households is not only a social issue, but also an economic stability issue for financial intermediaries and the financial system as a whole (D'Alessio & Iezzi, 2016). Indeed, some evidence shows that high private debt and over-indebtedness lead to deep recessions, unlike high public debt (Batini et al., 2018). Moreover, evidence suggests that the effects of COVID-19 on the economic well-being of households depend on their initial financial fragility, income, wealth, and financial literacy (Clark et al., 2021). For Europe, pre-crisis data showed that a substantial proportion of households would not have been able to cope with unexpected expenses. In some countries, many households had savings equivalent to a few weeks' basic consumption (Demertzis et al., 2020). In this scenario, the pandemic crisis has exacerbated household over-indebtedness (Brickell et al., 2020), which appears as a multifaceted concept affected by different types of households, each with a different profile and a different mix of potential risk factors (Ferreira et al., 2021).

Since the financial crisis of 2007–08, financial fragility appears to be one of the sources of financial instability. Indeed, from the Great Recession, it has become evident that households unable to cope with economic and financial shocks could also be defined as financially fragile because of the state of their household balance sheets, including debt, and subjective characteristics such as perceived ability to rely on family and friends, education and level of financial literacy (Demertzis et al., 2020).

In analyzing household finance, theoretical models usually refer to the life-cycle hypothesis, according to which agents optimize their consumption over time and incur debt on the basis of long-term fluctuations in their permanent income. However, evidence suggests that the ability to increase debt-financed consumption in line with rising asset prices makes saving less urgent. Thus, while current models predict a positive correlation between household debt and the level of education and an inverted U-shaped relationship with age, we also believe that the interaction between consumer characteristics, economic dynamics, and market imperfections is worth studying.

An analysis of household debt that focuses on its interaction with income shocks and its dependence on financial fragility is of interest for many reasons. Firstly, from the consumer's viewpoint, the way in which various risk factors combine and interact to produce concrete situations of over-indebtedness is a crucial social issue since it involves people living in a state of economic distress from which they are unable to escape. Moreover, over-indebtedness is crucial for the stability of financial intermediaries and of the financial system as a whole. Secondly, the recent economic literature attributes a growing and distinct importance of the private sector debt in relation to income. Even more specifically, household debt to income ratio and Debt Service Ratio (DSR) emerge among the best predictors of financial crises and important indicators of the business cycle (Mian et al., 2017; Drehmann & Juselius,

2014; Schularick & Taylor, 2012). Broadly speaking, the dynamic relationship between household income and debt seems to be at the center of a renewed interest. Mian et al. (2017) even stress the importance of a “debt-driven consumption channel” for the dynamics of the business cycle.

This paper aims to study the relationship between household financial fragility, financial literacy, income and debt in a dynamic model. We start by analyzing some stylized facts for the U.S. economy, and focus on the evolution of household debt and real GDP, and their variation from 1956 to 2021 (the most recent data available). We discuss issues of stationarity for the household income change and household debt to be able to compare the theoretical moments with the empirical ones. Then, we propose a parsimonious model that entails a system of two difference equations and study its equilibria under different parametrizations, both in deterministic and stochastic environments.

The first dynamic equation relates the change in income to the previous-period debt payment and a cubic government component. The latter component depends on past changes in income, and is countercyclical with a lag, meaning that it is positive if the yesterday-change in income was negative, and *vice versa*. That is, the component models a subsidy (tax) to households that experienced a drop (an increase) in income in the previous period. The second dynamic equation relates linearly household debt level to past levels of debt, past change in income, and non-linearly to a component that arises when households are “financially fragile”. Being financially fragile means that the past ratio between debt service and income is higher than a specific threshold, which we label as a Financial Fragility Condition (FFC). If this is the case, households are vulnerable to adverse shocks and find difficulties in managing their debt.

We present two main scenarios. The empirical literature shows that, among others, financial literacy affects positively financial decision-making and investment outcomes, allowing for better savings and debt management (Lusardi & Mitchell, 2023). In our model, households increase debt when income increases (as data suggests) and they do so according to a degree of financial literacy (Guerini et al., 2024). Indeed less financially illiterate households increase debt less after a positive income shock, while more financially illiterate households increase debt more. In short, we model two scenarios with debt behavior procyclical to income shocks and with the degree of procyclicality proportional to the degree of financial illiteracy. On top of that, we model household debt to react to income shocks according to the financial fragility dimension. Indeed when households are financially fragile, households increase debt more for positive income shocks. Accordingly, we obtain a two-dimensional piecewise map (Day, 1982, 1983), Day and Shafer (1985), the properties of which are studied analytically and by means of numerical simulations.

Results from the simulation analyses allow us to find the best model specification and parameters, i.e. those for which the simulated means and the standard deviations of household income change and debt are close to the observed ones for the U.S. economy. Our model finds that a higher degree of financial illiteracy brings the model closer to the mean and volatility for household debt and the mean and volatility of income change.

We investigate the effect of other parameters that allow to capture the impact of fiscal and monetary policies on the stability of the system. The increase in government intervention in the first equation reduces the fluctuations of household income and debt, though without a decisive impact on stabilizing the system. By contrast, a tighter monetary policy, implemented as an increase in the interest rate of debt, has a destabilizing role. The system moves from one to many equilibria and to a larger support of them.

Finally, neither changes in the parameter that regulates the persistence of debt nor variations in the means of income changes and stock of debt affect the number of equilibria.

Our work is original for three main reasons. Firstly, it uses a simple, yet unused method to study the interaction between the two variables of interest, namely debt and income changes, while other approaches use OLG models or heterogenous agents DSGE models. Secondly, it embeds a nonlinear way to account for financially fragile households, in line with the idea of non-linear effects of surpassing a threshold (i.e. our FFC). Thirdly, the model encompasses both the financial literacy of households and their financial fragility i.e. both components matters for household debt decisions after income shocks.¹

The paper is organized as follows. Section 2 details the literature review, Sect. 3 the data for the U.S. economy. Section 4 presents the model, the stability analyses, and the results from the simulations. Section 5 concludes.

2 Literature Review

In this Section we present the empirical and theoretical literature on the relationship between debt and income dynamics.

As for the empirical findings, the build-up of household debt has an impact on the occurrence and intensity of recessions. Schularick and Taylor (2012) and Jorda et al. (2013, 2015, 2016) show that high household debt levels predict financial crises and the magnitude of subsequent recessions. In our paper, we model together both household income growth and debt levels and we originally relate them to financial literacy and financial fragility.

There is a strand of the literature documenting the relevance of accounting for the relative importance of household debt payment. Indeed, debt servicing costs have been considered crucial to predict bad macroeconomic outcomes (Drehmann & Juselius, 2014). In particular, the best early warning indicators of banking crises are the credit-to-GDP gap and the Debt Service Ratio (DSR). In addition, Johnson and Li (2010) find that high DSR and low liquid assets-to-income ratio make households

¹ In our model we consider debt as nominal debt, but de facto introducing an "inflation" element that corrects for this dimension every period. Indeed, the autocorrelation parameter on past debt implies a decreasing value of it over time, as nominal variables are valued less due to inflation. From a definition standpoint, standard measures of DSR typically consider disposable income, while in our FFC measure, we consider income per household and at the same time account for public intervention.

more likely to be turned down for credit (with a non-linear effect, such that having a high DSR implies a much higher probability of being turned down).

From a macroeconomic perspective, Rinaldi and Sanchis-Arellano (2006) find that a rise in debt-to-income ratios in advanced economies between 1989 and 2004 increased the probability of families being in arrears since there was not enough income growth during the period to offset the increase in debt. In addition, Iacoviello (2008) finds that U.S. debt as a percentage of disposable income has been rising steadily between 1970 and 2007, while income growth and debt growth have been fluctuating around a positive mean, with a slightly bigger variance for debt growth. We employ a different methodology than Iacoviello, but we follow the approach of comparing our theoretical results with empirical data for the U.S.. The same pattern of increasing household debt is found in Mason and Jayadev (2014) and in our paper.

On the relation between debt levels and income growth, Mian et al. (2017) show that for 30 countries (1960–2012) shocks by household debt to GDP ratios have a non-linear and switching effect on GDP growth. First, the positive shock increases household debt, consumption and imports, and eventually GDP; but then GDP reverts and declines after three or four years. The same patterns do not hold for corporate debt. These findings are confirmed by Lombardi (2021), using data for 54 economies between 1990 and 2016.

There is evidence that the distribution of household assets and debts is decisive for household economic decisions, in particular consumption decisions. Indeed during the Global Financial Crisis, consumption was cut because highly indebted households were the ones with a higher propensity to consume, thus with higher difficulty in saving (Mian et al., 2013). Similarly, Baker (2018) finds that the elasticity of consumption is higher at higher debt levels for U.S. data. Moreover, Cloyne and Surico (2014) confirm that households with higher mortgage debt increase consumption more after positive income shocks due to tax changes. In our model, these three contributions ground our model such that increasing debt when a positive income shock occurs (i.e. assuming the increase in consumption implies a diminished capacity to repay debt).

Zabai (2017) and Zabai (2020) document that countries fall into four groups depending on household debt and its variation, i.e.: high and rising debt, high and flat/falling debt, low and rising debt, and low and flat/falling debt. Regarding the within-country debt distribution in the Euro area, the debt-to-asset ratio and mortgage-debt-service-to-income ratios are especially high for the lowest income quintiles, while the net-liquid-assets-to-income ratios are low (André (2016)).

Some papers focus on the relationship between households' indebtedness and financial fragility. The pioneering work of Lusardi and Mitchell (2011) defines financial fragility by examining households' ability to access emergency funds from any source. The authors find widespread financial weakness in the U.S., since one-quarter of U.S. households surveyed reported that they were certain they could not come up with \$ 2,000 within 30 days. Moreover, Jappelli et al. (2013) find that insolvencies tend to be associated with greater households' indebtedness. Indeed their panel data analysis of insolvencies shows that European countries that experienced relatively fast debt growth also featured larger increases in insolvency rates.

On the theoretical side, the literature is scant. Iacoviello (2008) builds a macro dynamic general equilibrium model with heterogeneous agents where short-run fluctuations in household debt are explained by business cycle movements, while the trend increase in debt is explained by rising income inequality. Similarly, Miranda-Pinto et al. (2023) use a heterogeneous agents Bewley model with capital and stochastic time-varying consumption thresholds that, if violated, yield substantial utility costs and represent adverse shocks. They model consumers differently than our model, since when an adverse shock hits, they accumulate debt. Other approaches entail the use of OLG models (OverLapping Generations). Barnes and Young (2003) use an OLG model to explain the increase in household debt to income in the U.S.. As OLG models usually dictate, shocks to real interest rates, borrowing constraints, and demographic factors are the main drivers of the results. Tudela and Young (2005) employ a similar OLG for the U.K.. Finally, Azariadis and Lambertini (2003) use a deterministic three-period OLG model to study household consumption rationing through debt constraints. With respect to these theoretical contributions to the literature, our approach is original since, firstly, it models an income-debt relation based on financial literacy and, secondly, it adds a financial fragility mechanism that modifies it.

3 Stylized Facts for the U.S. Economy

In this Section we present some stylized facts for the U.S. economy and we discuss comparability to our model. We focus on some economic variables of interest, that is household debt and income, as we will simulate our theoretical model to understand under which conditions (if any) the model is able to explain these data dynamics. The data are our own calculation starting from FRED datasets (FRED datasets n.d., USA)² in thousands of dollars.

Figure 1 shows the dynamics of household debt (HH Debt) and of the change of Debt per HH in the U.S. from 1956 to 2021. The figure shows an exponentially increasing debt per household until the Great Financial Crisis in 2008, while the change in debt per household peaks in 2007 at around a ten thousand dollar increase in one year.

Similarly, Fig. 2 represents the time series of Real GDP per household and of the Change in Real GDP per household. The figure shows a strong increase in real GDP per household throughout the considered period, in a similar fashion to the Debt per Household. The change in real GDP per household shows many more periods of negative values than the change in debt per household, with a highly negative peak during 2020.

Figure 3 represents the debt and the income change plane with the addition of the time dimension, starting with value 1 for the first annual observation of 1956. The figure shows that the U.S. economy has experienced some years of negative

² In particular using datasets number A939RX0Q048SBEA, CMDEBT, GDPC1, TTLHH, TTLH-HM156N, W398RC1A027NBEA.

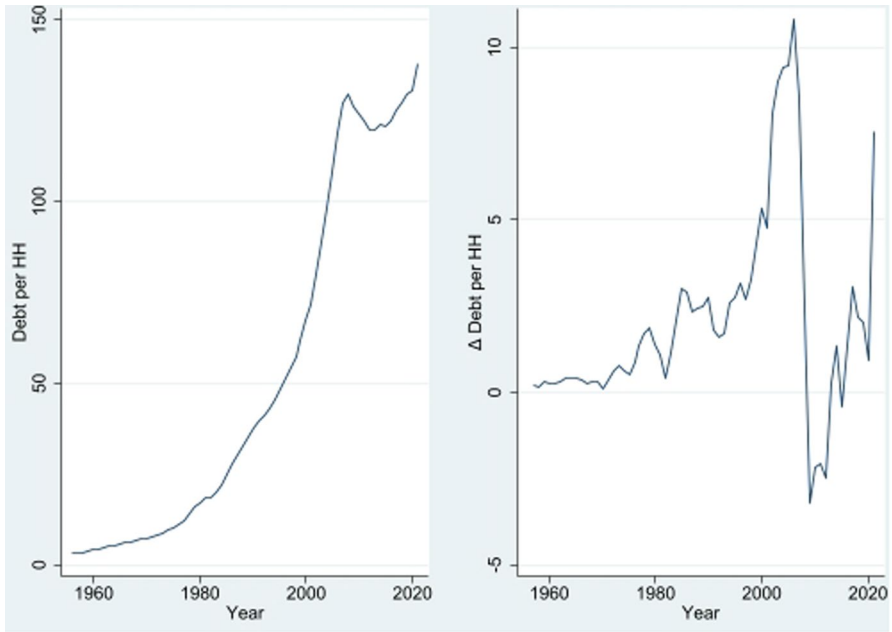


Fig. 1 Household Debt dynamics. Data from the United States of America. Thousands of dollars

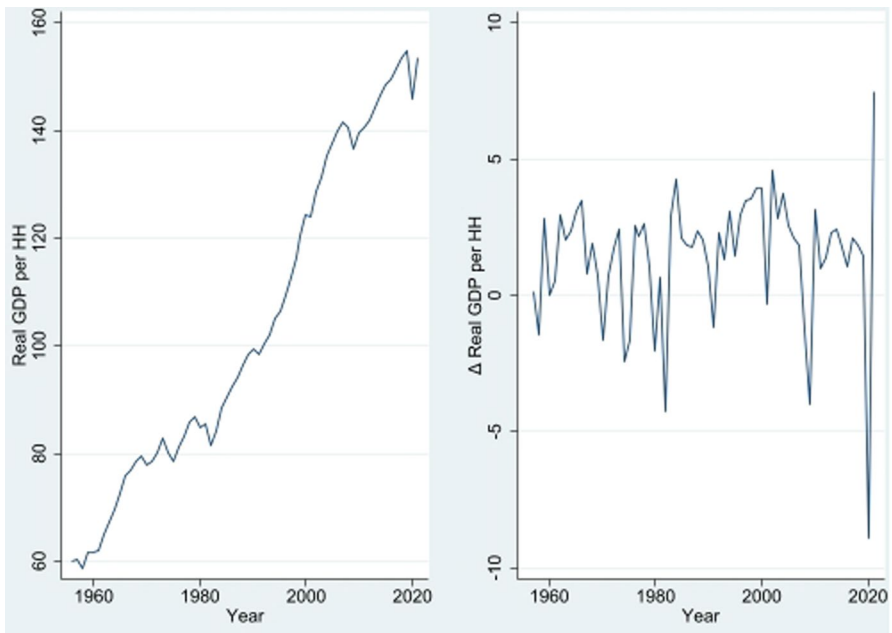


Fig. 2 GDP per Household dynamics. Data from the United States of America. Thousands of dollars

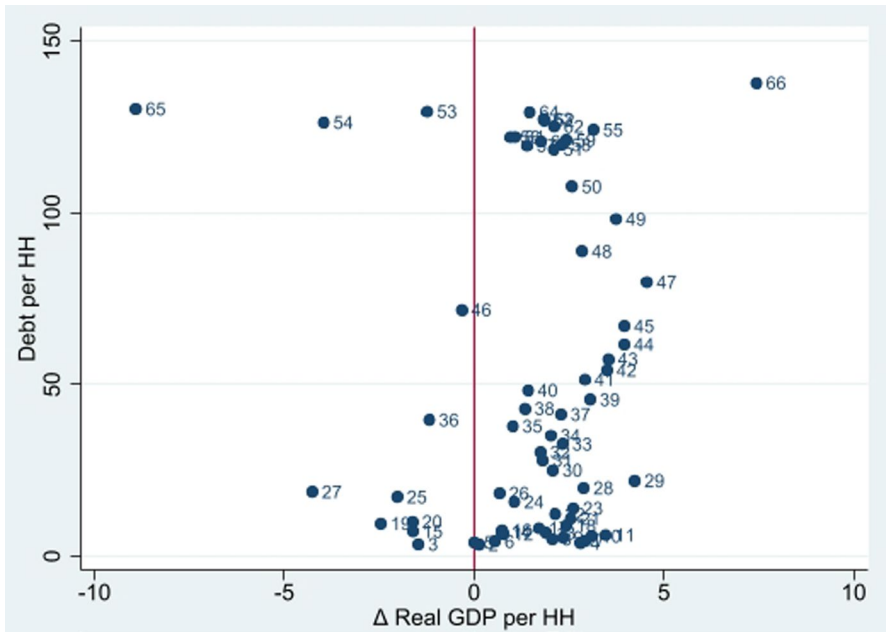


Fig. 3 (Debt-income change) plane with labels depicting the time dimension, with value 1 for starting year 1956. Data from the United States of America. Thousands of dollars

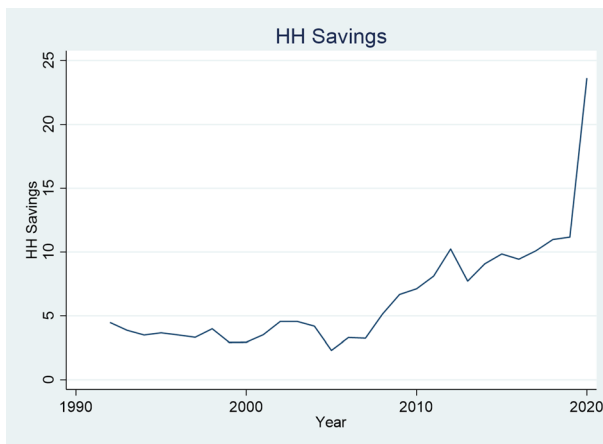


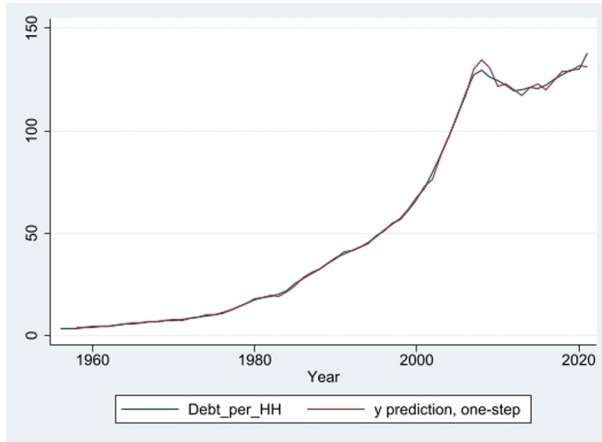
Fig. 4 Household Savings dynamics. Data from the United States of America. Thousands of dollars

income variations and that the largest income variations both positive and negative, are associated with very high debt levels.

Figure 4 shows the evolution of household savings (HH Savings). Due to data limitations, the time span goes from 1992 to 2020. The figure suggests that

Table 1 U.S. Economy.
Summary statistics in thousands
of dollars

	Mean	Sd	Min	Max	Count
Debt per HH	51.738	48.159	3.092	137.855	66
Real GDP per HH	103.644	30.269	58.725	154.909	66
Δ Debt per HH	2.073	2.877	-3.203	10.796	65
Δ Real GDP per HH	1.436	2.381	-8.906	7.438	65
HH Savings	6.451	4.365	2.305	23.618	29

**Fig. 5** Debt per HH and its stationary counterpart**Table 2** Summary statistics for
Debt per HH and its stationary
counterpart

	Count	Mean	Sd	Min	Max
Debt per HH	66	51.74	48.16	3.09	137.86
Stationary Debt per HH	64	53.25	48.11	3.67	134.68

Summary statistics for the Debt per HH and the one step ahead forecast of the empirical model ARIMA (0,2,1)

savings per household have been increasing since around 2007 and increased significantly in 2020.

We summarize these statistics of the U.S. economy in Table 1.

In comparing these data to our model, we point out that the time series of Debt per HH is not stationary, while the Change in Real GDP per HH is stationary. To overcome this issue, we selected and then estimated an ARIMA model (0,2,1) for Debt per HH, i.e. twice differentiated until the empirical model is stationary. We present the one-step ahead forecast for the stationary empirical model in comparison to our data to show the closeness of the two (Fig. 5).

We report the statistics of the stationary empirical model compared to the original data (Table 2). This evidence indicates that the original data may be represented

by a stationary empirical model and that the mean and standard deviation comparison to the theoretical model is feasible.³

4 The Model

We present a system of two difference equations to study the relationship between household financial fragility, income, and debt in a dynamic setting. To this aim, we model the dynamics of the change in income per household, and the evolution of the level of debt per household, respectively.

Equation (1) is a modified flow equation for the household income. We define the income change as $H_t = I_t - I_{t-1}$ and model it as follows:

$$H_t = -r_d D_{t-1} - \alpha_2 H_{t-1}^3 + \epsilon \quad (1)$$

According to Eq. (1), H_t depends on the debt repayment from previous period $-r_d D_{t-1}$; a fiscal policy component $\alpha_2 H_{t-1}^3$; and a ϵ parameter that captures exogenous income variations that depend on the structural characteristics of the economic environment, such as efficiency and quality of markets and institutions. Larger ϵ values identify countries equipped with better institutions than countries characterized by lower ϵ values (Calcagnini et al., 2021). The fiscal policy is modeled as a government intervention that is countercyclical and affects the income change in a non-linear manner: it is positive (i.e. a subsidy) when the household experienced a reduction of income in the previous period; it is negative (i.e. a tax) when the income change of the previous period is positive. The strength of the channel depends on the parameter $\alpha_2 > 0$, which regulates the magnitude of this government intervention. Debt repayment from the previous period ($r_d D_{t-1}$) consists of interest payments and principal repayments so that r_d is actually the sum of two components $r_d = \beta + r$, i.e. the share of debt repaid at the end of each period and the interest rate component. The interest rate on debt $r_d > 0$ is, therefore, an average interest rate, that we assume constant and exogenous. Thus, r_d captures the monetary policy, i.e. the larger r_d , the stricter the monetary policy, for any given β .

The main parts we want to account for in our modeling of the stock of debt consist of inflation (in an indirect way), a repayment component that depends on past income changes, and a component that captures financially fragile households. Thus, Eq. 2 models the evolution of the stock of the household debt:

$$D_t = (1 - \beta)D_{t-1} + \alpha_1 H_{t-1} + \mathbb{1}\left(\frac{r_d D_{t-1}}{I_{t-1}} \geq FFC^*\right) p H_{t-1} + \mu \quad (2)$$

The stock of debt D_t is equal to the portion $(1 - \beta)$ of debt of the previous period that did not expire, plus two components that represent debt repayment ($+\alpha_1 H_{t-1} + \mathbb{1}\left(\frac{r_d D_{t-1}}{I_{t-1}} \geq FFC^*\right) p H_{t-1}$) and a parameter μ . We interpret μ as an inverse

³ Differently, our theoretical model cannot match the full bivariate distribution of the two variables .

risk aversion parameter: the larger μ , the lower the household risk aversion, the larger D_t , all else equal. Accordingly, most of the theoretical and empirical analyses on how households manage short-term risks and their exposure to shocks focus on precautionary savings and asset levels. According to the theory, risk-averse households tend to accumulate wealth (reduce debt) to shield themselves against shocks (Deaton, 1992; Carroll 1997). However, many empirical studies (among others, Lusardi and Mitchell (2011) find that, in fact, many households hold few or no assets and no emergency funds and are very vulnerable to shocks (Caner & Wolff, 2004). To take into account these alternative household behaviors, the first debt repayment component, $+\alpha_1 H_{t-1}$, allows for the dynamics of the stock of debt to depend on the dynamics of the previous period income variation. How debt repayment reacts to income variation depends on the magnitude of the crucial parameter $\alpha_1 > 0$ which measures the degree of financial illiteracy. Recent evidence (Clark et al., 2021) suggests that households who were more financially literate were better protected against negative shocks. This is probably because the more financially literate made better saving and spending decisions in the past, so they could more easily withstand economic shocks and make better decisions in times of crisis. Moreover, financial literacy shapes the liability side of the balance sheet: in the United States, debt has risen across generations, while more financially literate households are also better able to manage this debt. The underlying mechanism is based on the assumption that becoming financially knowledgeable permits to reap the benefits of having access to a better saving technology (see Lusardi and Mitchell (2023), for a review of the main theoretical and empirical findings). To account for these findings, therefore, we model $\alpha_1 > 0$ as a financially illiterate parameter such that low levels of it imply a smaller increase in debt given an increase in income, while high levels imply larger increase in debt. In short, the debt dynamics is procyclical to income dynamics, so we assume that households increase their stock of debt when the income increases.

The second debt repayment component, $\mathbb{1}(\frac{r_d D_{t-1}}{I_{t-1}} \geq FFC^*) p H_{t-1}$, defines the condition according to which households are financially fragile (hereinafter FFH). To this aim, we focus on changes that affect households in relation to debt and income and use a financial fragility condition (FFC) that considers the household debt service ratio. According to this specification, households are FFH if they are above the threshold.⁴ Finally, we assume $p > 0$ that is if the FFC holds, then fragility renders the debt response to income variation more extreme. Indeed, in line with the evidence in the United States, in which 30 or 40 percent of wealth inequality near retirement can be accounted for by financial literacy (Lusardi et al., 2017), we are assuming that FFH implies a greater heterogeneity of income and debt dynamics in the presence of low financial literacy.

Our model may be summarized by the two-dimensional piecewise map (Avrutin et al., 2019) $(H_t, D_t) = P(H_{t-1}, D_{t-1})$:

⁴ Our FFC follows standard definitions of the Debt Service Ratio (DSR, such as Attinà et al. (2019)).

$$P : \begin{cases} H_t = -r_d D_{t-1} - \alpha_2 H_{t-1}^3 + \epsilon \\ D_t = \begin{cases} g_H : (1 - \beta)D_{t-1} + \alpha_1(H_{t-1}) + p(H_{t-1}) + \mu & \text{if } \mathbb{1}\left(\frac{r_d D_{t-1}}{I_{t-1}} \geq FFC^*\right) \\ g_L : (1 - \beta)D_{t-1} + \alpha_1(H_{t-1}) + \mu & \text{otherwise} \end{cases} \end{cases} \quad (3)$$

The system (3) depends on the main exogenous variables and parameters of the model, namely: the interest rate r_d , the magnitude of the public intervention α_2 , the assumption about household financial literacy α_1 and the magnitude of the p parameter, as well as the portion, $1 - \beta$, of previous-period debt. Other keys to the analysis are the income and debt parameters, given by ϵ and μ , respectively.

4.1 Number of Equilibria

Map (3) is piecewise defined, because of the two possible evolutions of the debt. Actually there is a third dynamic variable, the income I , but it influences H and the debt D only via the condition that determine if there is financial fragility and this must be taken into consideration in the analysis. So we can separate the study of the dynamics of D and H from the study of what happens to I , and this is what we do in what follows.

Concerning the number of equilibria we can state the following Proposition:

Proposition 1 *For positive values of α_1 map (3) has only one equilibrium.*

Proof By applying the equilibrium condition $H_t = H_{t-1} = \bar{H}$ and $D_t = D_{t-1} = \bar{D}$ to the dynamical system (3) we get the following system of equations when the financial fragility condition does not hold:

$$\begin{cases} \bar{H} = -r_d \bar{D} - \alpha_2 \bar{H}^3 + \epsilon \\ \bar{D} = (1 - \beta)\bar{D} + \alpha_1 \bar{H} + \mu \end{cases}$$

and the following system of equations when the financial fragility condition holds:

$$\begin{cases} \bar{H} = -r_d \bar{D} - \alpha_2 \bar{H}^3 + \epsilon \\ \bar{D} = (1 - \beta)\bar{D} + (\alpha_1 + p)\bar{H} + \mu \end{cases}$$

Let us consider the first case. From the second equation, we obtain:

$$\bar{D} = \frac{\mu + \alpha_1 \bar{H}}{\beta}$$

that we can use in the first equation to get:

$$\alpha_2 \beta \bar{H}^3 + (\beta + r_d \alpha_1) \bar{H} + r_d \mu - \epsilon \beta = 0$$

which is a cubic equation. A necessary condition to have three real solutions requires the presence of a local minimum and a local maximum, that is a first derivative with two real solutions. By calculating and putting equal to zero the first derivative we get:

$$3\alpha_2\beta\bar{H}^2 + (\beta + r_d\alpha_1) = 0$$

whose roots are:

$$\bar{H} = \pm \sqrt{\frac{-r_d\alpha_1 - \beta}{3\alpha_2\beta}}, \text{ which, given the restrictions on the parameters' values, are}$$

real only if $\alpha_1 \leq -\frac{\beta}{r_d} < 0$ and it cannot occur given that we only consider positive values of α_1 . So the map always admits only one real equilibrium.

We can easily extend this result for the second case by replacing α_1 with $(\alpha_1 + p)$. \square

Something more can be said in the particular case when $\epsilon = \mu = 0$ and this is what we do in the next subsection.

4.2 Equilibrium and Its Local Stability Properties (case $\epsilon = \mu = 0$)

By assuming $\epsilon = \mu = 0$ it is possible to compute the equilibrium of the system. We have the following result:

Proposition 2 *Map (3), limited to D and H , has one equilibrium $E_0(0, 0)$ always existing.*

Proof By applying the equilibrium condition $H_t = H_{t-1} = \bar{H}$ and $D_t = D_{t-1} = \bar{D}$ to the dynamical system (3) we get the following system of equations:

$$\begin{cases} -r_d\bar{D} - \alpha_2\bar{H}^3 - \bar{H} = 0 \\ -\beta\bar{D} + \alpha_1\bar{H} = 0 \end{cases}$$

From the second equation, we obtain:

$$\bar{D} = \frac{\alpha_1}{\beta}\bar{H}$$

then we can use in the first equation to get:

$$-\alpha_2\bar{H}^3 - \left(1 - \frac{r_d\alpha_1}{-\beta}\right)\bar{H} = \bar{H}\left[-\alpha_2\bar{H}^2 - \left(1 + \frac{r_d\alpha_1}{\beta}\right)\right] = 0$$

Table 3 Parameters values

r_d	α_2	β	p	FFC^*
0.06	0.04	0.005	0.05	0.3

which is solved for $\bar{H} = 0$. The term into square brackets provides two further solutions, namely $\bar{H} = \pm \sqrt{-\frac{r_d \alpha_1 - \beta}{\alpha_2 \beta}}$, which, given the restrictions on the parameters' values, are real only if $\alpha_1 \leq -\frac{\beta}{r_d}$, and it never happens if α_1 is positive (as proved in Proposition 1). \square

The role of α_1 is also crucial for the stability of the equilibrium, as emphasized in the next Proposition:

Proposition 3 *The equilibrium E_0 is locally stable provided that:*

$$\alpha_1 < \frac{1}{r_d} \quad (4)$$

In particular, at $\alpha_1 = \frac{1}{r_d}$ the equilibrium undergoes a Neimark-Sacker bifurcation.

Proof To study the stability of the equilibrium we must compute the Jacobian matrix of the map (3) which is the following:

$$J : \begin{bmatrix} -3\alpha_2 H^2 & -r_d \\ \alpha_1 & 1 - \beta \end{bmatrix}$$

The Jacobian matrix calculated at the equilibrium E_0 becomes:

$$J_0 : \begin{bmatrix} 0 & -r_d \\ \alpha_1 & 1 - \beta \end{bmatrix}$$

with $Trace = 1 - \beta$ and $Det = \alpha_1 r_d$. By using the three stability (or Jury) conditions:

- (i) $1 + Tr + Det > 0 \implies 2 - \beta + \alpha_1 r_d > 0$;
- (ii) $1 - Tr + Det > 0 \implies \beta + \alpha_1 r_d > 0$;
- (iii) $Det < 1 \implies \alpha_1 < \frac{1}{r_d}$.

Conditions (i) and (ii) always hold for feasible values of the parameters, while by increasing the value of α_1 condition (iii) can be violated. When it happens two complex and conjugated eigenvalues cross the unit circle and a Neimark-Sacker bifurcation occurs. If the bifurcation is supercritical, after the bifurcation, the trajectories converge to a closed invariant curve around the equilibrium, causing permanent fluctuations of the two dynamic variables. Note that this bifurcation value (i.e. $\alpha_1 = \frac{1}{r_d}$) is always positive. \square

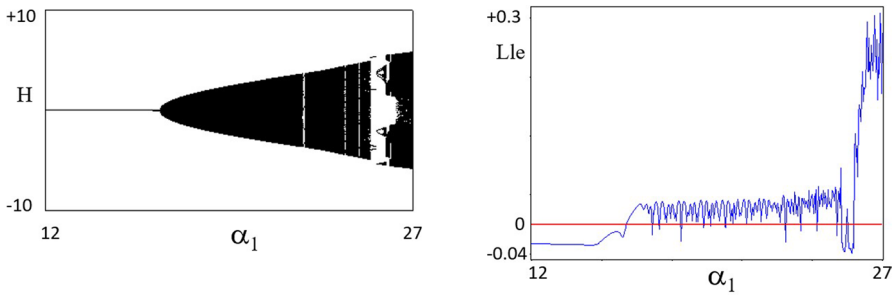


Fig. 6 Bifurcation diagram, and maximum Lyapunov exponent, for H in the range between 12 and 27 of α_1 values. $\epsilon = 0$ and $\mu = 0$

Stability condition (4) clearly shows the destabilizing role of the interest rate on debt r_d . This important result for monetary policy will be investigated in detail in 4.4.

4.3 Simulation Analyses of the Deterministic Model

4.3.1 The Role of Financial Illiteracy

In the next paragraphs, we will separately consider the role of the most important parameters, starting from α_1 . As stated above, α_1 is one of the crucial parameters of the model and it measures the degree of financial illiteracy of households. For all the simulations we fixed some parameters' values as indicated in Table 3.

The parameter values have been fixed according to actual values and opting for a conservative approach, as follows. r_d represents the average interest rate paid by households on debt contracts (mortgages, consumer credit, and credit cards), divided into a principal component β and an interest rate component r . The value we chose is supposed to be a weighted average of an important amount of mortgage contracts with lower interest rates and lower amounts of consumer credit and credit cards with higher interest rates. α_2 is a fiscal policy coefficient that we set to 0.04. In the case of having H equal to its mean 2.36 in both time t and $t-1$, the total fiscal policy component amounts to 0.53, 22.46 % of H . Of course is possible to set higher α_2 for higher government intervention, but we decided to adopt a conservative parameterization.

$1 - \beta$ is the portion of debt that did not expire. We set it very close to one, capturing the effect of inflation on nominal debt for modeling purposes. p is the additional component present only for financially fragile households and we set it low with respect to the size of α_1 especially for being conservative on the relative size of the non-linear effect, while it is possible that the penalty for being financially fragile might be much higher, maybe close to a single digit percentage. FFC^* is set to 30 % to follow the literature on Debt Service Ratios.

Figure 6 shows the impact of α_1 on income dynamics.

The left panel of Fig. 6 shows that while α_1 becomes more positive (i.e. the higher is the degree of financial illiteracy) a supercritical Neimark-Sacker bifurcation

Fig. 7 Phase space for H and D. $\alpha_1 = 17.76$. The black points are obtained with $\epsilon = 0$ and $\mu = 0$, while the grey points with $\epsilon = 6$ and $\mu = -7$

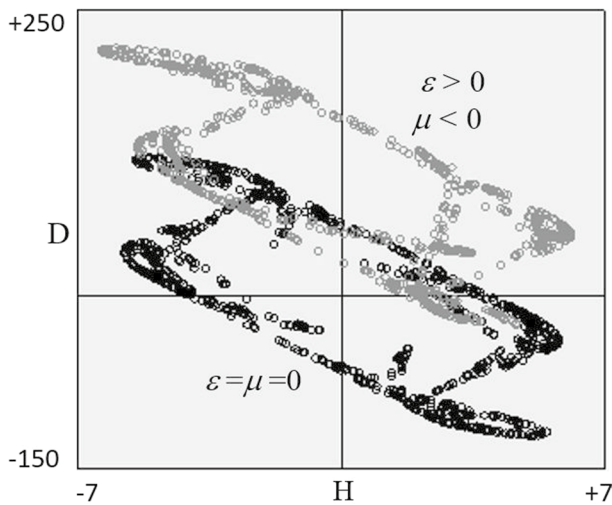
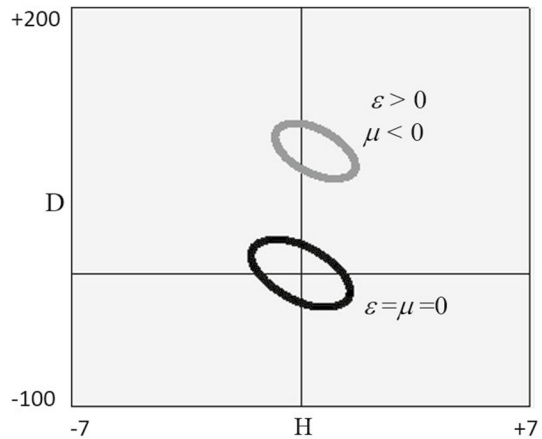


Fig. 8 Phase space for H and D. $\alpha_1 = 27.64$. The black points are obtained with $\epsilon = 0$ and $\mu = 0$, while the grey points with $\epsilon = 6$ and $\mu = -4$

occurs. This means that even if the equilibrium is now unstable, the dynamic variables oscillate around it, forming in the state space a closed invariant curve encircling the equilibrium. The higher is α_1 the larger is the curve, so fluctuations have a larger amplitude. Moreover, for high values of α_1 dynamics can be chaotic, as proved by the plot of the Largest Lyapunov exponent in the image on the right side of the panel.

The bifurcation diagrams confirm what has been proven in the context of Propositions 1 and 3.

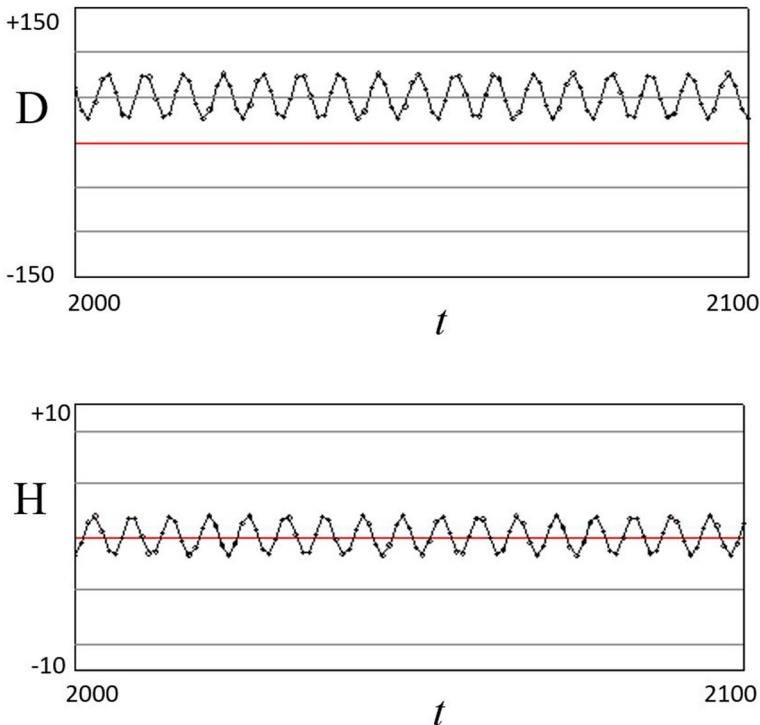


Fig. 9 Simulations for “Financially illiterate” HH. $\alpha_1 = 17.76$, $\epsilon = 3.2$ and $\mu = -1.74$. Time series of H and D. $\bar{x}_H = 2.36$, $\sigma_H = 2.48$. $\bar{x}_D = 51.2$, $\sigma_D = 17.98$

4.3.2 Financially Illiterate Households

Figure 7 is obtained by using a value of α_1 higher than the Neimark-Sacker bifurcation value (i.e. a violation of the left inequality of the stability condition (4)). The dynamics in the (H, D) plane follows a closed invariant curve, shifting between debt and saving on one side and increase and decrease of income on the other. Being a financially fragile household renders the curve flat suggesting that two opposite situations happen more frequently, i.e.: positive debt and income change, negative debt and income change. By assuming that the parameters ϵ and μ are equal to zero and that the household is financially fragile we get the closed invariant curve centered at the origin of the axes. If we assume a positive value of ϵ and a negative one of μ we obtain a similar closed invariant curve but characterizing a positive mean for income change (that is better institutions than in the previous case) and for the debt.

Continuing to increase the value of α_1 new simulations show that the closed curve becomes larger and less regular, eventually becoming a chaotic attractor, as it is possible to see in Fig. 8. Also in this case a negative mean for debt (i.e. $\mu < 0$), that is, according to our interpretation of the μ parameter, by assuming a

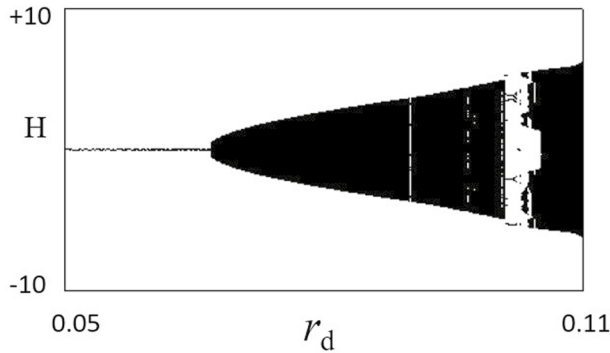


Fig. 10 Bifurcation diagrams for H and parameter r_d . $\epsilon = 0$, $\mu = 0$ and $\alpha_1 = 15$

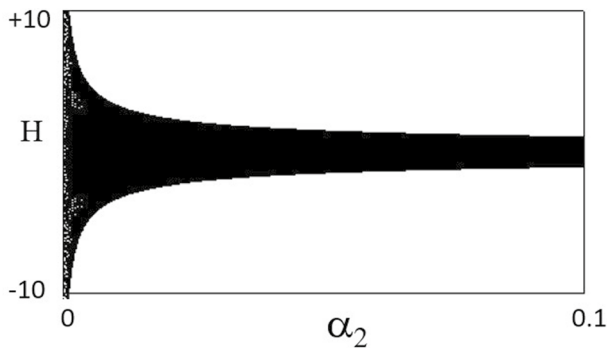


Fig. 11 Bifurcation diagrams for H and parameter α_2 . $\epsilon = 0$, $\mu = 0$ and $\alpha_1 = 18$

risk-averse household, the dynamics (periodic in this case) occur for positive debt levels, regardless of the initial savings.

Now that we know the effects of parameters ϵ and μ we can calibrate them with a simple trial and error method some features of real data. Figure 9 shows time series simulations corresponding to the case with the closed invariant curve (i.e. $\alpha_1 = 17.76$). Results (in thousands of dollars) show values similar to those reported in Table 1 for the mean of debt (model 51.2, data 51.74) and standard deviation of income change (model 1.91, data 2.48). The mean of income change is 2.36, compared to data 2.07, while the standard deviation of debt is 17.98 compared to data 48.16. There is two/three-period persistence in the time series.

4.4 The Role of Fiscal and Monetary Policies

Income dynamics are directly affected by both monetary and fiscal policies that enter into Eq. (1) through the debt repayment from previous period $-r_d D_{t-1}$, and the fiscal policy $\alpha_2 H_{t-1}^3$ components, respectively. Figure 10 shows how income dynamics

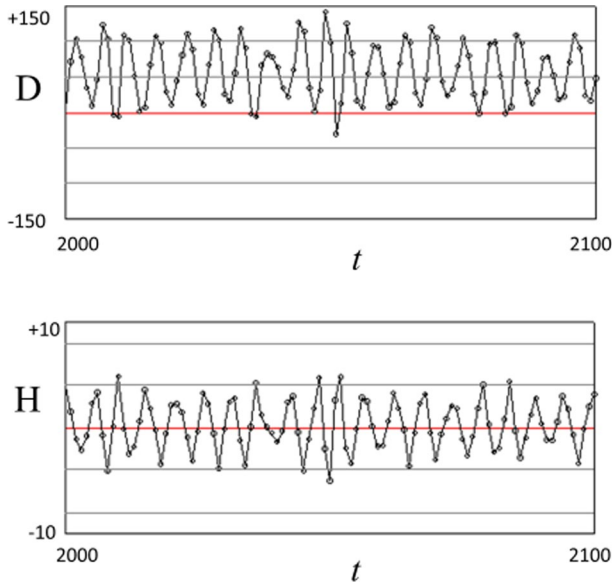


Fig. 12 Simulations with additive noise. Time series of H and D. $\sigma_{\xi}^H = 0.1$, $\sigma_{\xi}^D = 1$, $\epsilon = 0$, $\mu = 0$, $\alpha_1 = 24$

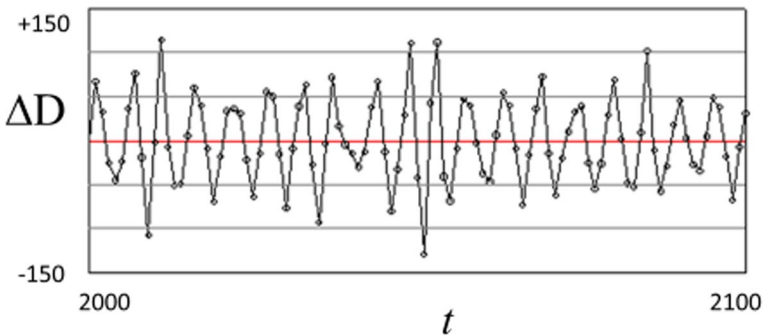


Fig. 13 Simulations with additive noise. Time series of ΔD

depends on r_d . We obtain that a tighter monetary policy, summarized by a larger r_d , has a destabilizing role on income, as it increases the cost of debt. These numerical results confirm what is already implicit in the stability condition (4). On the other hand, the deeper the fiscal policy (the larger α_2) the lower the income oscillations. This can be seen in Fig. 11. In other words, both fiscal and monetary expansive policies have a stabilizing role, dampening income variations (Bacchiocchi & Giombini, 2021; Bischi et al., 2022).

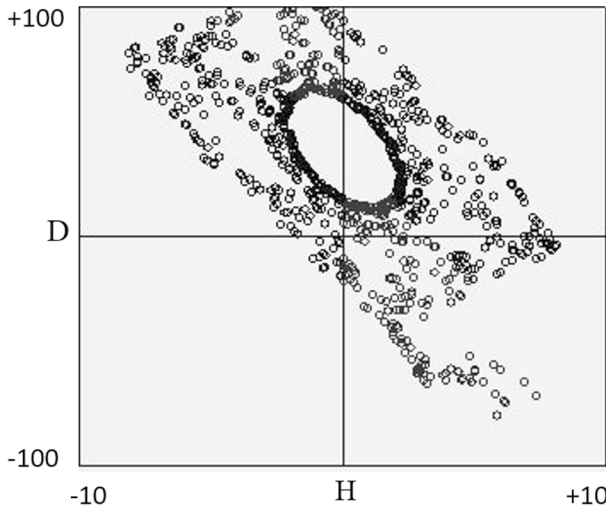


Fig. 14 Phase plane with α_1 negative and stochastic, such that $\xi_{\sigma}^{\alpha_1} = 1.5$. There are lower (17) and upper (27) limits to fluctuations to avoid divergence. $\epsilon = 2.3, \mu = 0$

4.5 Addition of Stochastic Elements

The deterministic model we have studied so far allows to replicate fluctuations in debt and income and to analyze the role of monetary and fiscal policy. In order to better fit the data shown in Table 1 we build some versions of our model by augmenting the deterministic skeleton given in (3) by stochastic elements.

4.5.1 Noise in the Case of the Financially Illiterate Household

We have seen that if the equilibrium is unstable, persistent fluctuations of debt and income take place (periodic, quasi-periodic, or chaotic). We have also studied the map P_2 (5) with additive noise terms to both the dynamic variables:

$$P_2 : \begin{cases} H_t = -r_d D_{t-1} - \alpha_2 H_{t-1}^3 + \epsilon + \xi_t^H \\ D_t = \begin{cases} g_H : (1 - \beta)D_{t-1} + \alpha_1(H_{t-1}) + p(H_{t-1}) + \mu + \xi_t^D & \text{if } \mathbb{1}\left(\frac{r_d D_{t-1}}{I_{t-1}} \geq FFC^*\right) \\ g_L : (1 - \beta)D_{t-1} + \alpha_1(H_{t-1}) + \mu + \xi_t^D & \text{otherwise} \end{cases} \end{cases} \quad (5)$$

where $\xi_t^H \sim N(0, \sigma_{\xi}^H)$ and $\xi_t^D \sim N(0, \sigma_{\xi}^D)$.

Figures 12 and 13 show time series simulations in this case. Results (in thousands of dollars) are close to the mean of debt (model 53.64, data 51.74) and the standard deviation of income change (model 2.34, data 1.44). The mean of income change is 0.2, compared to data 2.07, while the standard deviation of debt go closer to 41.55 compared to data 48.16. There is two/three-period persistence in the time series.

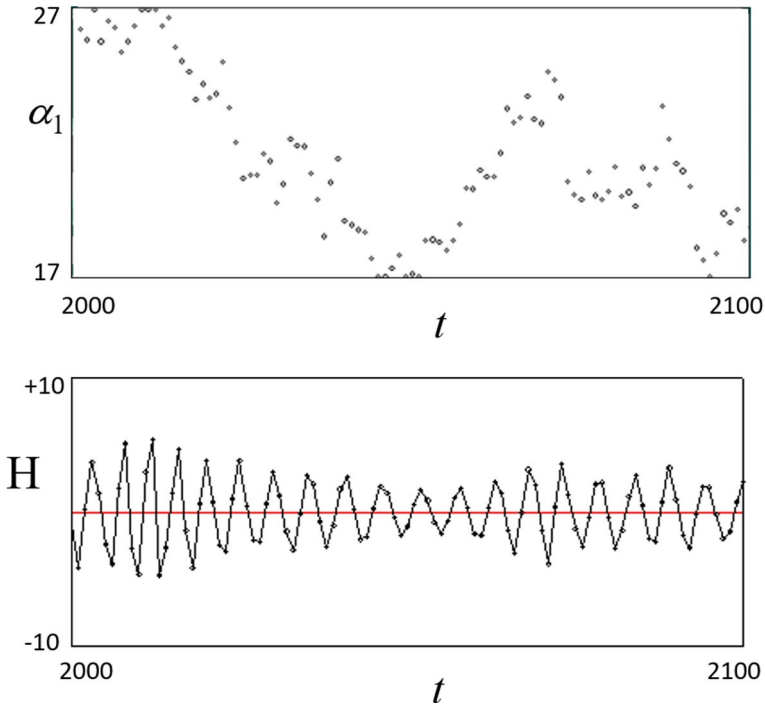


Fig. 15 Times series of H in relation to stochastic values of α_1 (upper panel). $\sigma_{\xi}^{\alpha_1} = 1.5$. There are lower (17) and upper (27) limits to fluctuations to avoid divergence. $\epsilon = 2.3, \mu = 0$

4.5.2 α_1 Stochastic Dynamic Variable

Finally, we also want to take into consideration the role of α_1 in regulating the amplitude of fluctuations when the equilibrium is unstable. To this aim, we simulate a version of our model where α_1 is not a parameter anymore but a dynamic variable, endowed with a Brownian motion. We expect that when α_1 is quite high, fluctuations have higher amplitude than when it is not so high (to remain in feasible cases and avoid divergence we fix a range of variation for α_1 only made up of positive values).

So the dynamical system becomes:

$$P_3 : \begin{cases} \alpha_{1,t} = \alpha_{1,t-1} + \xi_t^{\alpha_1} \\ H_t = -r_d D_{t-1} - \alpha_2 H_{t-1}^3 + \epsilon \\ D_t = \begin{cases} g_H : (1 - \beta)D_{t-1} + \alpha_1(H_{t-1}) + p(H_{t-1}) + \mu & \text{if } \mathbb{1}\left(\frac{r_d D_{t-1}}{I_{t-1}} \geq FFC^*\right) \\ g_L : (1 - \beta)D_{t-1} + \alpha_1(H_{t-1}) + \mu & \text{otherwise} \end{cases} \end{cases} \quad (6)$$

where $\xi_t^{\alpha_1} \sim N(0, \sigma_{\xi}^{\alpha_1})$.

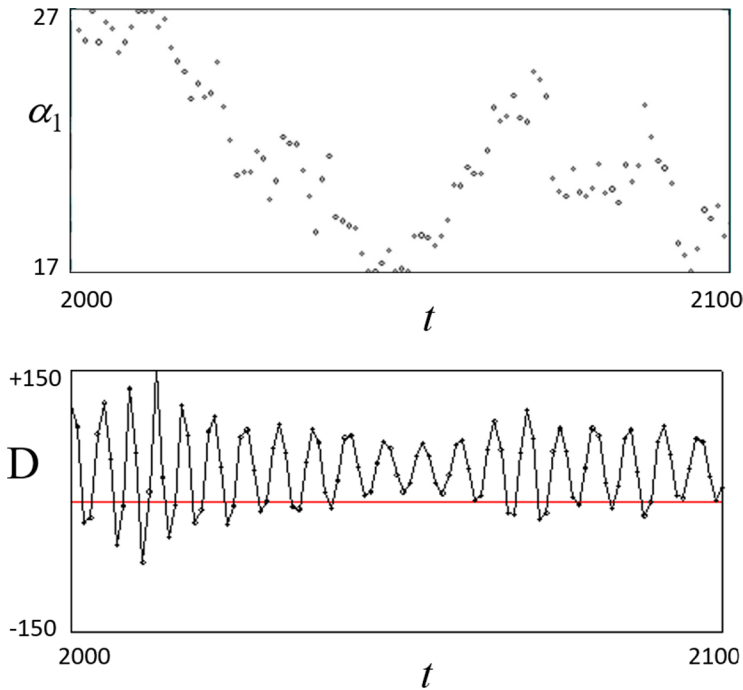


Fig. 16 Times series of D in relation to stochastic values of α_1 (upper panel). $\sigma_{\xi}^{\alpha_1} = 1.5$. There are lower (17) and upper (27) limits to fluctuations to avoid divergence. $\epsilon = 2.3$, $\mu = 0$

Assuming a stochastic α_1 varying in the realm of the unstable equilibrium, implies that dynamics may occur either around a chaotic or a quasi-periodic attractor, producing a sequence of values of D and H as shown in Fig. 14.

Moreover, Figs. 15, 16 and 17 show the time series of income and debt variation in relation to the stochastic values of α_1 . The latter needs to fluctuate within boundaries to avoid income and debt divergence.

The simulated time series confirms that when α_1 is close to its lower limit, fluctuations are characterized by a lower amplitude than in the case where the value of the parameter approaches its upper limit. Our results show that a model encompassing two noise terms fits U.S. data better than model 6 in which α_1 becomes a dynamic stochastic variable, with the added Brownian motion component. Indeed, the latter case performs worse in fitting $\sigma_D = 35.45$ (previously 41.55, data 48.16), $\bar{x}_H = 0.02$ (previously 0.2, data 2.07), $\bar{x}_D = 37.92$ (previously 53.64, data 51.74), while being substantially identical in fitting $\sigma_H = 2.2$ (previously 2.34, data 1.44).

5 Conclusion

This paper studies a dynamic model of household income change and household debt. The model incorporates a standard way to account for financially fragile households, while originally modeling the interplay between financial fragility and financial literacy.

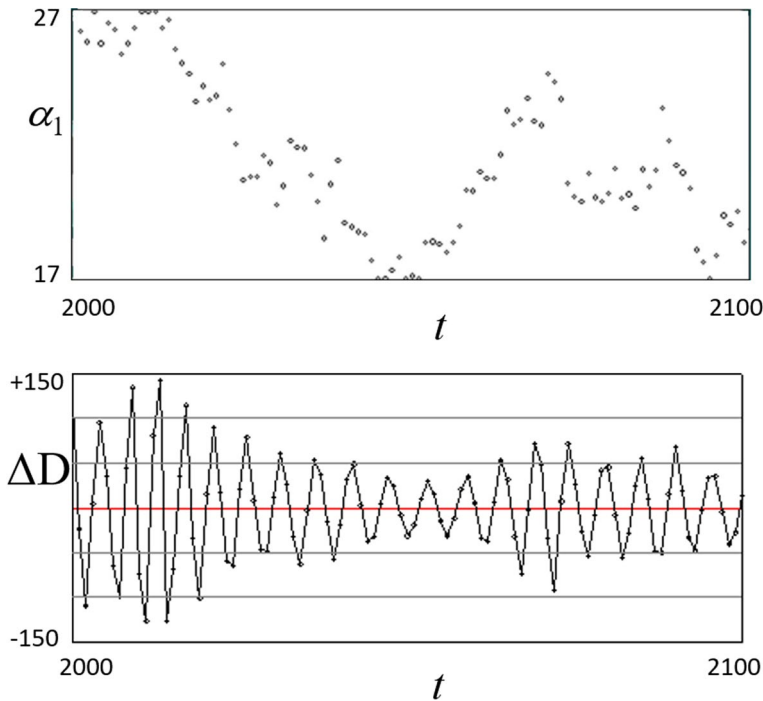


Fig. 17 Times series of ΔD in relation to stochastic values of α_1 (upper panel). $\sigma_{\xi}^{\alpha_1} = 1.5$. There are lower (17) and upper (27) limits to fluctuations to avoid divergence. $\epsilon = 2.3$, $\mu = 0$

Household debt reacts procyclically to income shocks depending on the degrees of financial illiteracy: the greater the illiteracy, the greater the procyclicality. On top of that, financially fragile households are the ones that experience a debt service ratio above a critical threshold FFC^* , i.e. those households with a low repayment capacity. Being financially fragile further exacerbates the procyclical response to income shocks.

This model partially reproduces the mean and volatility of debt and income change for U.S. data. Changes in parameters that track the persistence of debt (β) and the behavioral difference of financially fragile households (p) do not alter dramatically the dynamics of the system. Macroeconomic expansionary fiscal and monetary policies are effective in limiting income fluctuations by reducing the household's probability of being financially fragile.

With respect to the theoretical literature, we introduce a novel way of modeling the relation between household income change and household debt through a system of difference equations. Moreover, our parameterizations rest on the interplay between financial fragility and financial literacy i.e. assuming an influence on household debt decisions after income shocks. This way of modeling allows us to render the theoretical analysis closer to the empirical dynamics and to disentangle potential non-linearities and interactions of income and debt with household financial fragility and heterogeneity.

This approach brings both strengths and limits. The strengths rely on the originality of modeling and the ability to partially reproduce some U.S. statistics. The limits are the

inability to match the full bivariate distribution of household income change and debt and the complexity of the model.

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Declarations

Conflict of interest This work has been developed in the framework of the research project on “Consumers and firms: the effects of the pandemic on financial fragility and over-indebtedness”, financed by DESP-University of Urbino, Italy. The authors have no relevant financial or non-financial interests to disclose. The authors have no Conflict of interest to declare that are relevant to the content of this article. Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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