Archive for History of Exact Sciences (2025) 79:4
https://doi.org/10.1007/s00407-024-00343-3

ORIGINAL PAPER

How to use Kepler's first and second laws in a geo-heliocentric system? Ask G.B. Riccioli

Flavia Marcacci[1] · Paolo Bussotti[2]

Received: 6 August 2024 / Accepted: 5 November 2024

Abstract
Kepler's laws provided sufficient geometry and kinematics to strengthen astronomers' preference for heliocentrism. While Kepler outlined some dynamic arguments, they were not rigorous enough to turn his laws into kinematic tools. As a result, some astronomers found ways to reconcile Kepler's findings with geo-heliocentrism. One of these was the Jesuit astronomer Giovanni Battista Riccioli, who proposed a method known as the "epic-epicycle" (Riccioli, *Almagestum novum*, 1651). This paper will explore how Riccioli received and interpreted Kepler's first and second laws within his own astronomical framework. This analysis will include a discussion of how Riccioli understood the concept of "physics" in his work, beginning with a study of the Sun's motion (Riccioli, *Astronomia reformata*, 1665).

1 Introduction[1]

The most quoted astronomer in the *Almagestum novum* (1651), the masterpiece of the Jesuit Giovanni Battista Riccioli, was without doubts Johannes Kepler. He quotes him profusely, reads his texts slavishly, understands him, and accepts his mathematical solutions. In the following work, *Astronomia reformata* (1665), it is not about

[1] This paper is the second part of a research project conducted collaboratively by Paolo Bussotti and Flavia Marcacci. Flavia Marcacci conducted the primary research on the sources and contents of the paper presented here. Paolo Bussotti directed the first part of the research and results are being finalized in the paper *Kepler's laws with and without dynamics*. Nevertheless, both parts were extensively and consistently reviewed by both authors, ensuring that this work reflects a genuinely collaborative effort.

Communicated by Jed Buchwald.

✉ Flavia Marcacci
flavia.marcacci@uniurb.it

Paolo Bussotti
paolo.bussotti@uniud.it

1 Department of Pure and Applied Sciences, University of Urbino Carlo Bo, Urbino, Italy

2 Department of Humanistic Studies and Cultural Heritage, University of Udine, Udine, Italy

Published online: 11 December 2024

astronomers but directly about astronomy: Kepler's presence is evident in the explication of the elliptical shape of planetary orbits. The fundamental Riccioli's source for Kepler is *Epitome astronomiae copernicanae* (Riccioli 1651, vol. I, pp. 526–535), but *Harmonice mundi* and *Astronomia nova* are mentioned, too.

Riccioli devised a kinematic and geo-heliocentric solution capable of integrating the innovations spotted by the telescope after 1609. The work carried out an exhaustive survey of the conceptual proposals and experimental and observational discoveries that occurred in the intervening decades. The overall framework remains that of classical astronomy, in the geometric-demonstrative framework and research themes. However, there is also ample room for evaluations of observational data and philosophical discussions on the nature of the heavens or the relationship between philosophy of nature and theology. The combination of all these elements raises the question of how and how much Kepler influenced Riccioli's solutions, which were so attentive to mathematical and empirical details while still unable to embrace a new vision of the cosmos. Riccioli was familiar with Kepler's planetary laws, which had been obtained in the context of heliocentric astronomy. How to reconcile this seemingly inconsistent, if not entirely contradictory situation?

Riccioli mainly considered kinematics aspects but not because he did not search for a dynamic. Riccioli largely inquired *why* planets move around the Earth, what causes their irregular circuits, and if a general celestial law explains all celestial bodies' motions (Riccioli 1651, vol. 2). He is known for his famous experiments at the Torre degli Asinelli in Bologna where he perfected the value of the acceleration of gravity. This result alone forced the abandonment of the Aristotelian law of motion. Until the end of his life, he carefully searched empirical proofs for Earth's rotation and revolution: observation and data were the necessary premises to any other consideration in his view. However, he found no evidence to support those motions and repeatedly acknowledged that without this proof, he could not seriously consider the arguments of the Copernicans. Therefore, he gave up the search for a causal explanation and cosmological structure and focused on placing greater emphasis on the role of mathematical demonstration and reduction to empirical data.

This paper aims to investigate how Riccioli interprets and makes his own Kepler's astronomy. The very concept of "physics" comes to be proposed as an alternative to Kepler's, to the extent that it becomes possible to adopt a geo-heliocentric view while still respecting the laws of ellipsis, areas, and periods. To emphasize the various ways in which he used the term "physical," we will try to distinguish four meanings (Bussotti, Marcacci, *in preparation*). Riccioli inherited and strongly manifested the empirical attitude of ancient astronomy ($physical_0$ approach): collecting data, making measurements, fixing observations were the maximally important operations that legitimized all other lucubration. These data provided the basis for producing a kinematic model ($physical_1$ approach) capable of describing the motion of the planets and predicting their positions. To this end, he developed an overall model (called *epic-epicycle method*) of the geo-heliocentric type, which was strictly consistent with the data he had ($physical_2$ approach). As a fine connoisseur of the philosophy of nature, Riccioli knew that this model had to be based on consistent assumptions about the nature of the heaven. Thus, he assumed that the law that explained all celestial motions must be unique, and he thought that this law imposed a spiral motion whose producing cause he could not decipher (*physical₃* approach).

In this context, the appropriation of planetary laws takes place in an entirely kinematic sense, particularly for the first two laws. Riccioli's strange case makes it particularly interesting to understand the complexity of the transition from geocentrism to heliocentrism, as well as that from positional astronomy to astronomy in the modern sense. In fact, the problem was not only accumulating data (empirical aspect) and then finding an algorithm that correctly predicted the positions of the planets (kinematic aspect). The further problem was finding explanations for the behavior of the planets (dynamical aspect) and integrating them within an overall view of the cosmos (cosmological aspect).

Section 2 of this paper will primarily focus on Riccioli's *physica repugnantia* and to the general use of the word "physica" in his work. Section 3 discusses the reception of the first and second Keplerian laws, after some premises about Riccioli's Sun theory. Conclusions will follow.

2 Do not *repugn* physics if you want to get the best world-system

The reason why Riccioli is sympathetic with Kepler lies in certain characteristics that made Kepler particularly appealing to Riccioli. First, though less relevant to our discussion, is their shared metaphysical sensibility. Despite different approaches, both maintained a connection with the philosophical context within which an astronomer interpreted celestial phenomena. A thorough reading of Riccioli's work reveals that his philosophical interests were influenced by his life circumstances, particularly his Jesuit training. As a member of an order with a strong emphasis on Aristotelian natural philosophy, Riccioli's views were shaped by these doctrines. Kepler, on the other hand, developed his philosophical ideas independently of theological and metaphysical obligations.

A second relevant aspect is Kepler's exceptional mathematical skill, derived from his experience as an astronomer "in the old sense," who focused intensively on explaining the kinematics of planetary motions. Kepler thoroughly understood the central problems professional astronomers were trying to solve and approached them using plane and spherical geometry. This approach resonated with Riccioli, who saw it as similar to his own methodology. Seeking to solve the first and second inequalities, Riccioli closely studied Kepler's proposals to gain insights into potential improvements or, at the very least, to compare them with his own solutions.

Nevertheless, Riccioli often denounced Kepler or others' demonstrations as inaccurate and inconsistent with any philosophy of nature (Riccioli 1651, vol. 2, 288) and criticized the necessity of adapting the principles of natural philosophy whenever a geometric model or explanation alters already established patterns. Riccioli describes his own system as Ptolemaic, or quasi-geocentric, or Tychonic.[2] Nevertheless, Tycho is often a target to attack. Notably, Tycho did not justify sufficiently why comets as

[2] Riccioli defines his system as reformed Ptolemaic, semi-geocentric or semi-Tychonic. Marcacci 2018, 90.

supralunar bodies—as he supposed the comets were—could have crossed the heavens: thank to Christoph Rothman he modified the heavens as concentric rigid shells into fluid matter (Granada 2002). If comets are subject to Earth physics, then Tycho should have come up with solutions not repugnant (*physica repungantia*) to experience and observations. Riccioli is inclined toward Galileo and Kepler, who denounced the limitations of his physics, respectively, in the *Assayer* and the commentary against Scipione Chiaramonti (Kepler 1625). Kepler complained that Tycho's physics was inadequate to support the planetary motions: beyond the goodness of the intuition that comets were supralunar bodies and were not made of ether, a geo-heliocentric model needed a more advanced physical system. In other words, any *physical*$_2$ consideration must be found on *physical*$_3$ justifications. Similar criticisms had arisen from many other astronomers who accused Tycho of making planetary motions more intricate than a spider's web (Schofield 1981, 222–230), having added too many circles, creating confusion, and making physical justification more difficult.

Riccioli turned out about how philosophical assumptions on nature must be changed if data and demonstrations do not support a model. Observations suggested Tycho that comets' trajectories were rectilinear, and they passed through superlunar heavens. These ideas clashed with the Aristotelian cosmos, and Brahe proposed a new conception of these celestial bodies and a new world model. However, this did not lead to a dynamic revision of his theory, but only to a new cinematic description and philosophical understanding about what heavens were made of. Consequently, he considered a new geo-heliocentric world-system. Analogously, Riccioli denounced *physica repugnantia* but it did not lead to a dynamic conception of astronomy (*physical*$_3$). Instead, it led to a new variance of Tychonian world-system (*physical*$_2$).

However, Riccioli's criticism against Tycho goes beyond the circumscribed topic of comets. To understand Riccioli's geo-heliocentrism, we must first consider the significance of astronomy during a time of profound change. As well known, the teaching of philosophy of nature in Jesuit schools underwent significant changes (Giard 1995), just as mathematics teaching reform was being discussed (Romano 1999). Aristotle's *Physics* still provided the building elements for the philosophy of nature, but it was precisely the meaning of "physics" that was compromised. A recurring problem, both for philosophy of nature and for mathematics, was determining whether the mathematical and applied disciplines (*mathesis mixta*) pointed to natural objects.[3] The philosophy of nature provided the principles of reality. However, they could clash

[3] During the latter half of the sixteenth century, the teachings followed by the Order were primarily based on the works of Aristotle and Thomas Aquinas. The *Constitutiones* of 1558, *Ratio studiorum* of 1599, and *Ordinationes* of 1651 detail the education and training process, a process which evolved over a century. The criteria for education were periodically reviewed and confirmed, considering the guidelines of the Holy Office and the latest scientific discoveries. These two needs were polarizing. Thus, so that they did not become paralyzing, the possibility of transforming uses and meanings of mathematical reasoning was explored. Indeed, physics and mathematics continued to cover their role. Physics had natural beings as objects with quantifiable and connecting properties and it had to show causal reasons about natural phenomena, with respect to a more general metaphysical discourse (*explicatio per causas*). In dependence of such physical argumentations, mathematics wore the form of typically medieval practice *mathesis mixta* and furnished the demonstrative schemes to natural facts, so that its methods and applications were always heteronymic and addressed to solving problems of a non-mathematical nature. In this context, one problematic area was the study of statics (or *centrobaryca*), which was internal to *mathesis mixta* but directly related to experimental practice. A second and fundamental problem for now, stemmed from the predictive

with geometric descriptions and the data of experience: many telescopic observations precisely overturned those principles in the intention of scientists like Galileo. Similarly, more geometric descriptions could agree with the data and not allow discernment as to the most appropriate reality: the case of the phases of Venus is exemplary, for while it allowed Ptolemy's system to be ruled out, it did not allow Tycho's system to be abolished.

Riccioli frequently invokes the concept of *physica repugnantia*, as well as denounces the risk of contradiction between geometrical descriptions and physical experiences. When mathematical demonstrations fail to align with observational data, the argument is contradictory (*argumentum à repugnantia*; Riccioli 1651, vol. 2, 323). Since a purely *physical*$_0$ explanation requires that observations and theoretical models converge (Riccioli 1651, vol. 2, 259–260), unresolved contradictions necessitate a shift in debate towards the principles of natural philosophy. Every instance of *physica repugnantia* is directed against the deepest assumptions of a system, involving the principles by which that system interprets nature.

Nevertheless, while *physica repugnantia* prompted Riccioli to move beyond traditional philosophical explanations, he did not embrace any concept of force to explain the celestial motion. He adopts a geo-heliocentric model, with Earth in the center, Moon, Sun, Jupiter and Saturn around it, and Mercury, Venus and Mars around the Sun. It is evident as Riccioli's system is intermediate between Ptolemy and Tycho's ones, similar to the so-called Egyptian system but adding Mars around the Sun. Mars stands out due to its proximity to both the Sun and Earth. Even in Ptolemy's system, Mars is shown to be much closer to Earth and the Sun compared to the other outer planets. Anyway, Riccioli's choice is now the consequence of observations, comparisons with other systems, and application of his method of *epic-epicycle* (Marcacci 2023), explained later in this paper with reference to the first Keplerian law. This method plans to use epicycles and combine its diameter with the Earth-planet variable distances, in correspondence to the variable eccentricity of the orbit. The epicycle's radius length according to Ptolemy (65,833 parts of planet's orbit radius) was around the average value (66,057 parts of planet's orbit radius) attributed to it by Riccioli. Indeed, epicycle's semidiameter is variable in this last, and Riccioli always provides maximum and minimum values (Riccioli 1651, vol. 1, 542). This is the cause of planetary retrogradations. The general comparison is more complex, because one must also consider the changing positions of the eccentric's apogee and perigee, which causes variations in planetary speeds. Despite these challenges, Mars consistently appears around the

Footnote 3 continued
ability of algorithms, particularly in astronomy. The new telescopic discoveries had made the latter problem more bitter. In such case, did demonstrative nexuses have any connection with causal nexuses? There was no unanimous opinion in the Society of Jesus. Some denied mathematics as a science deeply connected to something real (B. Pereira), some wanted to keep it closely connected with physical explanations instead (G. Biancani). In these attempts, each brother explored autonomously how and how much to keep together of the old and new methods and approaches. No unified perspective on the nature of mathematical knowledge emerged. Because of the maintenance of the traditional disciplinary system, Jesuits remained in great difficulty in passing from the *Mathesis mixta* to that sort of *Physico-Mathesis* prompted by the new Galilean approach (Baldini1992, 36-56; Feingold 2003; Dinis 2017, 237–260). Instead, they were clear in claiming the interlace of mathematics with some philosophical principles, so that the two were at least weakly compatible. On the other hand, this was possible because mathematical forms were the result of a process of abstraction that started from physical forms. The speculative process, until now perfectly Aristotelian-Thomist, allowed to trace also the inverse path from the experience to its mathematization.

Sun in all comparisons.[4] Because of this, Riccioli classifies Mars as a satellite of the Sun. For similar reasons, Jupiter and Saturn revolve around the Earth.

His model did not contrast any $physical_0$ and $physical_1$ explanation and, simultaneously, admitted a geometric demonstration: consequently, an astronomer could use epicycles as tools for computation, but only the result was real, that is the spiral orbit of planets, as we shall see in a while. This form of mutual adaptation between *mathematica* and *physica* was typical of the Jesuit context. Riccioli interacts with this path of maturation (Marcacci 2019a): in his opinion, astronomy is intermediate between physics and mathematics, and it requires both. It investigates celestial objects by their properties. Some of those properties are reduced to quantities and examined to predict and calculate their paths: astronomy is first interested in kinematics. $Physical_1$ results are susceptible to refinement or even error but are the very purpose of astronomy. Any other corporeal properties of celestial bodies—e.g., nature of light, heat, or matter—-must adhere or at least concord with such $physical_1$ properties. However, Riccioli is aware that the old approach to astronomy, as knowledge for saving phenomena, is now inadequate. Thus, astronomy as only $physical_1$ science can no longer satisfy. As a matter of fact, although subordinate to mathematics and physics, astronomy challenges both: it poses new problems and requires new explanations. In order to explain celestial observations, it is necessary to confirm the principles of some philosophy of nature by *physico-mathesis*, with its deductive and geometrical justifications. The epithet *physico-mathesis* is common in Riccioli's major works and entitles two little but dense treatises (Riccioli 1668, 1669).

In the wake of this idea, twofold point most closely approximates and separates Riccioli from Kepler. First, the latter learns, at least in principle, the lesson of looking for a cause in natural phenomena that is translatable into a mathematical description and capable of supporting kinematics: $physical_1$ analysis must stand alongside $physical_3$ analysis in order to choose one world-system. Kepler choose the Sun as the cause of each planetary motion, and from that he searched for something as a "force" translatable in mathematics form. Riccioli will evaluate either the cause of planetary motions in the Sun or that of general celestial motions in angelic intelligences, but he will not condescend to either hypothesis. Therefore, eventually he decided not to deal with the $physical_3$ aspect, because the available data were not sufficient to reach

[4] In the table *Synopsis mensuram et equipollentiae hypotesium. In partibus, qualium Radius Orbis Planetae est 100000* (Riccioli 1651, I, 542) Riccioli offers the mean values of the geometrical tools for Mars, Jupiter, and Saturn motions in models proposed by Ptolemy, Copernicus, Lansbergus, Longomontanus, Kepler, Bullialdus, Riccioli. In such a table Riccioli compares values as the length of epicycle's radius, anomalies of eccentricity, and so on. At the end he calculates the absolute mean and integrates recent corrections. In this way he balances the various geometrical solutions. "Quoniam autem poset aliquis velle ex mensuris omnibus illam eligere, quæ media esset inter extremas, & hanc in quavis forma hypothesium adhibere, táquam securior erroris minoris; adiecimus duas ultimas columnas, in quarum penultima sunt mensuræ mediæ absolutè inter antiquas ac recentes; & in ultima sunt mediae inter recentissimas à Tychone inclusive. Quae quidem hypothesis huncupari potest Electiva mediocritatis"; "However, since someone might wish to choose from all the measurements the one which lies in the middle between the extremes and apply it to any form of hypotheses as the one most secure from error, we have added two final columns. In the next-to-last column are the measurements that are absolutely in the middle between the ancient and recent ones, and in the last column are the ones that lie in the middle between the most recent ones, starting with Tycho. This hypothesis can indeed be called the 'Elective Mean'." (Riccioli 1651, I, 541).

a satisfying dynamical theory. For this reason, he will give up founding a dynamic and limit himself to physico-mathesis in its *physical*$_1$ aims.

Second, observation and data—that is, *physical*$_0$ issue—fitted multiple interpretations. Even if every astronomical argument needs its mathematical counterpart, the latter is only necessary, not sufficient, to ensure the goodness of such argument. In addition, it was problematic for Riccioli. He distinguishes between *ratio* a posteriori and *ratio* a priori (Marcacci 2016): the *ratio* a posteriori is the result of data, measurements, numerical reprocessing, and all concerning the practical experience of observation (*physical*$_0$ and *physical*$_1$); the *ratio* a priori indicates the principles on which a model can be based and their consequences (*physical*$_3$). This approach is everywhere in *Almagestum novum*, even to introduce Kepler. For instance, the practice of drawing the line of apsides passing through the Sun and the planet has two kinds of justification: first, the observations are better explained that in other models; second, the Sun is the center because it spreads heat and light, then Kepler's reasons are more solid than those of Copernicus (Riccioli 1651, vol. 1, VII, 528). Therefore, in principle, astronomy is the science that provides a *physical*$_3$ explanation to a *physical*$_2$ model based on *physical*$_0$ and *physical*$_1$ arguments. For its best designation one must call it *physico-astronomia* (Riccioli 1651, vol. 1, 204, 752). However, if it is not possible to edify a satisfying *physical*$_3$ theory, this does not mean that the other aspects of astronomy have also to be dismissed.

Consequently, Riccioli's crucial problem is choosing a model without discriminating data. The chronology of his works is valuable for understanding his speculative path. After Riccioli retired from his position as a professor of theology and an official within the Order, he dedicated himself entirely to astronomy. In a short period, he produced a remarkable work that earned him recognition and fame: the *Almagestum novum* of 1651. This monumental work implants a timely, verbose, and dense review of the leading astronomical topics of the time, comparing alternative solutions. Flipping through the pages, one also observes at a glance this comparative work (Marcacci 2021b) that irrigates all of Riccioli's research. The work, planned in three volumes, was not completed and only the first volume in two tomes obtained publication. In 1665, however, a second work, *Astronomia reformata*, resumed and partly completed the project. The pages of this work are parched with words and full of tables helpful to astronomers. Here is one crucial novelty: the terms "ellipse", "semi-major axis", and "semi-minor axis" appear regularly. We will shortly understand how Riccioli fits the ellipse into a geo-heliocentric system. Though he felt politically and theologically compelled to support the idea that the Earth was the center of the universe, he never considered his own system as the ultimate truth. Nevertheless, his system is only the best possible system (Marcacci 2021a). But why the best and not the ultimate? Because the most important thing was missing evidence about the Earth's motion. Therefore, the final decision regarding the Earth's movement could not be made with the available data. They made such a movement implausible, but not impossible.

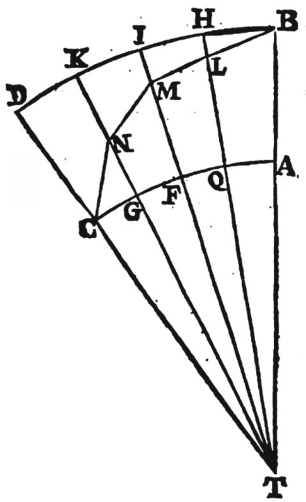

Fig. 1 Trajectory of falling body (Riccioli 1668 39)

Riccioli's geo-heliocentric system had only a contender[5]: the heliocentric system with elliptical orbits: *alias*, Kepler's system. In his final years, he engaged in a prolonged and heated debate with Giovanni Alfonso Borelli, a pupil of Benedetto Castelli, and Stefano degli Angeli, a Jesuat, regarding the rotation of the Earth (Riccioli 1668, 1669; Degli Angeli 1667, 1668a, 1668b, 1669). Riccioli has gained fame for conducting experiments on the free-falling motion of bodies at the Asinelli Tower in Bologna. Through these experiments, he had improved Galileo's value and determined that a body, under uniform acceleration (*uniformiter difformiter*), travels 4.62 m in the first second of free fall.

A broken line is assumed by Riccioli as the trajectory of a falling body in case the Earth moved: in such a case, the falling body felt down with a real increment of the distance from the perpendicular to the ground (*reale incremento*; Riccioli, AFM, 38). With reference to Fig. 1, be Earth's center in T. AB is the Torre degli Asinelli in Bologna. DB is a small arch, valuable 1'. Let us trace the segment DT and divide in four equal parts the curve DB: BH, HI, IK, KD is each of 15''. By drawing the corresponding lines and arc CA, determine the points G, F, and Q. If Earth moves, the point A moves, too. First, A passes in Q and the falling body from B in L. The terrestrial motion continues in F and G and the body occupies the following positions M and N and finally both are in C. Having divided the spaces equally, Riccioli remembers that the fall times along the broken line will be different because of the accelerated motion. Besides, the intervals are very small, then straight and not curvilinear intervals can be assumed. The conclusion is that if the Earth is rotating on itself, then the distance BL + LM + MN + NC should be measured directly and the increase in speed along those stretches. However, the latter is not appreciably different from that obtained by assuming a fall along the vertical trajectory. Therefore, the Earth does not move (Riccioli, AFM, 39–43).

[5] Riccioli's proofs against the Copernicanism were considered not trival also by G. Leibniz (Bertoloni Meli 1988).

Borelli (1667) reacted by stressing that the transversal motion has no role in the body's falling: consequently, the falling body cannot trace a spiral and, in the case of its speed had decreased on the descent, then the curve would still have been irregular. He was attacking Riccioli with a correct opinion, but he also admitted the wrong idea that experimental proof would never be possible. Degli Angeli (1667, 1668a, 1668b, 1669) attacked the proof, too: Riccioli neither considered nor understood the principle of composition of motion. He denied that an object moves in accordance of a double motion, accelerating towards the ground and moving straight and uniformly towards the east. Degli Angeli argued that supporters of the heliocentric model had to provide a convincing justification of the principle of composition of motions. While the composition of motions is widely accepted in celestial mechanics, such as in epicycle–eccentric models, it is not as readily applied to terrestrial phenomena. For terrestrial applications, physical evidence is required to support its use, particularly measurements of the displacement of falling objects on the ground due to Earth's rotation. If they fail to do so, Riccioli's objections to heliocentrism stand, suggesting that the Copernican model (heliocentrism) would be defeated or disproved. Without strong physical evidence or logical reasoning to support how motions combine under the heliocentric model, doubts about its validity remain (Lerner 1980; Bertoloni Meli 1988). Riccioli (1668) insisted on his findings for a long time. Still in 1669, he summarized old results and produced new examples, also employing inclined planes (Riccioli 1669, 35), but without changing his ideas.

Up to here, we considered arguments about terrestrial rotation. As to terrestrial revolution, arguments are controversial and complex at the same time. Stellar parallaxes should have been the desired *physical*$_0$ proof. Unfortunately, it will arrive only later. This is what Riccioli named "Copernicans' *Achille*" (Riccioli 1651, vol. 2, 321–322; Lerner 1988). On the other way round, assuming an Earth-centered world, and establishing the fixed stars–Earth distance far less than Copernicus thought, then the calculation of such velocity became not so high. Nevertheless, the crucial point remains the observation of the stellar parallax (Riccioli 1651, vol. 2, 613–648).

The Tychonic method is analyzed and refined, with a thorough examination of refractive effects. Beyond Saturn, extending toward the fixed stars, these refractive effects significantly limit the validity of any assumptions (Riccioli 1651, vol. 2, 641). The universe's dimension is a cautious extension of the largest planetary distance (Riccioli 1651, vol. 1, 419; Marcacci 2018, 94–108). It is only through the detection of a significant parallax value for a fixed star that we may conclude that observing such a vast distance is only possible due to Earth's revolution. But such a value was not yet detected.

Thus, Riccioli was a strong advocate of the idea of terrestrial immobility. However, his arguments were challenged by Jesuits such as Fabrì or Tacquet (Gambaro 2021), as well as mathematicians and astronomers such as John Wallis and Adrian Azout, who criticized his interpretation while acknowledging its mathematical rigor (Dinis 2017, 256). These criticisms were not limited to Italy but circulated outside. But Riccioli repeated and insisted: the Earth does not move, and the opposite thesis cannot be assumed a priori and without *physical*$_0$ evidence. He was obsessively experimental in his thinking, which seems to border on paranoia at times. However, from the point of view of an astronomer so hell-bent on relying on data, such a methodological principle was indispensable. Data had to bend the whole speculative system, both ontology and epistemology.

First, ontology: Riccioli discussed how the Sun, Moon, and planets were conceived in their qualities and substance over time; new observations or discoveries are mentioned; finally, a personal interpretation is attempted. Most of the qualities of terrestrial matter is extended to celestial bodies: light, color, density, dimensions, all these properties deal to admit their nature as those of terrestrial elements, although somewhere he remains ambiguous (Dinis 2017, 159–166). Anyway, his distance from Tycho appears when he discusses about the origin of celestial motions. Catholic astronomers and confreres had overlooked the inconsistency of Tycho physics in order to hold on to a system more reconcilable with Scripture. The cracks left open of unresolved doubts were useful in linking the idea of the world with the celestial powers: for if Tycho's kinematics were more advanced than contemporary ones, one could gloss over the problem of the substance of the heavens in order to move on to that of the origin of their motion. Thus, the celestial intelligences became the holders of motion, as argued, for example, by Jesuit Christopher Borri (Schofield 1981, 226–230; Carolino 2023, 97–104). The other way round, Riccioli raised many perplexities to the point of demolishing the government of celestial motion to the angelic intelligences (Dinis 2017, 164–166; Marcacci 2018, 185–203; Marcacci 2024).

Second, the obsessive search for evidence bent epistemology: if Riccioli exaggeratedly has been labeled as incapable of taking his own position (Ashworth 1986), he chooses every time. For each observational problem and each solving method, he judges whether it is good or not, whether it contains errors, and whether it is taken or abandoned. He provides his method of calculation, cosmological view, and philosophy of nature. These choices, however, are never judged as absolute: they remain the best available solution, taking into account the equally fundamental tool of experience and mathematics. For these reasons, his world-system is "only" simply the best.

3 Riccioli's premises to Kepler: a theory for the Sun and other assumptions

Putting the Sun at the center of the world might have been an attractive idea. However, Riccioli would need official permission from the Catholic Church and physical evidence of the Earth's motion of rotation and revolution. Unfortunately, he never received the permission he needed, and his attempts to find evidence of the Earth's motion were unsuccessful. In the perspective of his *physico-astronomia*, he decided to focus on mathematics and ensure that whatever the center of the world might be, it would have to have a rigorous mathematical explanation. Ultimately, Riccioli placed the Earth at the center of the world even if he reviewed carefully the principal world-systems (Riccioli 1651, vol. 1, 101–105; Riccioli 1651, 536). However, the charisma of astronomer forced Riccioli through the *physical*$_3$ considerations. He wanted to establish the best technique derived from a system and inherent any single astronomical issue.

The ellipse was first introduced arguing about the solar anomalies (Riccioli 1651, vol. 1, 149) and planetary motion (Riccioli 1651, vol. 1, 534 ff.) and quoting Kepler alongside Ismael Boulliau (1605–1694). About the way Riccioli comments on Kepler, we will come back to it later. Reference to ellipse becomes completely explicit in AR. Ellipses are mentioned in tables, with their major and minor semiaxis, and used in the demonstrations. According to Riccioli, ellipses are useful tools for calculating the positions of planets. However, the crucial problem remains the immobility of the Earth. As said, he and his collaborators demonstrated the lack of physical evidence for the Earth's rotation and revolution. The gravitational acceleration, he argued, disproves Copernicus' system. Assuming the Earth's immobility, it is impossible to follow Kepler's hypothesis that the planets' motions are truly eastward, even when they appear to move westward. While ellipses are effective for astronomical calculations, they cannot be considered the "real trajectory" of the planets.[6]

Riccioli pretends a single law for the celestial bodies that can represent their actual motion, but such a law depicts celestial motions as widening and narrowing spirals.[7] It is necessary to keep in mind that the term "spiral"[8] was used widely and variously, but it has nothing to do with the meaning that today we ascribe to this word. Stars and planets move along spiral paths ("per unicam lineam spiralem", Riccioli 1668, vol. 2, 339, 340; see also 253, 324, 326). These spirals are what observers actually see in the sky, ensuring simplicity and harmony in their movements. In order to describe these motions mathematically, he resorts to ellipses. In AN, as we will see, successive ellipses result from the displacements of the planets obtained by the method for epic-epicycles. In such a method, planets' motion is computed in consideration to the distances with the Earth and the Sun.

[6] Riccioli 1665, VI (italic ours): "Atque ita in hoc dissentimus ab Astronomis Recentioribus, *quòd Ellipticam viam Planetae, sub mera Hypothesi usurpamus* ad calculum ineundum; illi autem putant esse realem viam Planetæ, per quam moveatur continuè Orientem versùs, cum revera moveatur Occidentem, versùs"; "In this way, we differ from recent astronomers. We *usurp the elliptical path of the planet merely as a hypothesis* for our calculations, whereas they believe the planet truly follows an elliptical path, moving continuously eastward, when it actually moves westward".

[7] Riccioli 1665, VIII (italic ours): "Circa Terram autem Sphæramque elementarem moventur tam Sol, quàm Luna duobus quidem, quoad Longitudinem, motibus apparenter contrarijs, uno versùs Occidentem, altero versùs Orientem obliquè, *sed realiter, unico motu* semper versùs Occidentem per Spiras in Aethere fluido descriptas ea lege, ut fundamentum praebeant imaginationi humanae çoncipiendi eorum motum, tanquam factum in unica peripheria quasi circulari, seu Elliptica circa Conum scalenum designabili"; "Regarding the Earth and the Elemental Sphere, both the Sun and the Moon appear to move in two opposite directions concerning longitude: one westward and the other obliquely eastward. However, *in reality*, they move westward *in a single motion* through spirals described in the fluid ether. This allows the human imagination to conceive their motion as occurring in a single, almost circular or elliptical path around the scalene cone".

[8] Spiral-like orbits were hypnotized since Xenophanes and Plato (Heat 1913, 109). The German historian Ernst Hugo Berger in its *Geschichte der wissenschaftlichen Erdkunde der Griechen* (quoted by Heath 913, 109) believed that Xenophanes argued against the idea of the "sun's 'going-forward ad infinitum as contrasted with circular motion; as, on his theory, it cannot be *in a straight line* without limit": the motion is in a *spiral* which the Sun exhibits owing to the combination of its two motions, that of daily rotation, and its yearly motion in the ecliptic (…). This motion in a spiral is not motion forward ad infinitum for the spiral returns on itself in a year just a simple circular motion would in 24 h". Beyond the correctness of Berger's interpretation, the idea is similar to the one Riccioli had for the motion of the Sun in AR. Other ancient scholars about spiral motions were Theon of Alexandria, Averroes, Alpetragius, Johannes de Sacrobosco (Thordnike 1949), Roger Bacon, Albertus Magnus, Nicole Oresme (Grant 1971; Thijssen, Luthy 2007, 179–180).

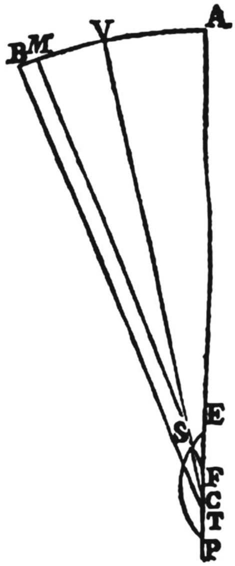

Fig. 2 Mean anomaly and elliptical planetary path (Riccioli 1665, 32)

3.1 Riccioli's loop theory for the Sun

As in the ancient astronomy, any explanations needed the reference to the Sun because of the second irregularity, which depends on it. For a good theory about position and motion of the Sun was necessary. As seen, Kepler had replaced the mean Sun with the true Sun. Riccioli was fascinated by this technique, although he had to modify it to transition to a geo-heliocentric system. In this way, the main reference remained the true Sun, although the mean Sun is integrated as a valid tool. More than in AN, the theory of solar motion is fully expressed in AR. To understand the original way Riccioli reconfigures Kepler's laws, we will give here a summary of the explanation of solar motion. It, in fact, consists of quasi-elliptical loops along a truncated cone, obtained by partial elliptical paths. In other words, Riccioli wants to express the solar motion in relation to three points: 1. as if its elliptical orbit around the Earth were the true path, given that the Earth is supposed to be at rest; 2. as if the immobile Earth occupies one of the two foci; and 3. the mean, true and eccentric anomalies are calculated in reference with the ecliptic and the apogee and perigee. About this third point, values for the anomalies are given considering not the center of the ellipse, but one focus in Earth and one "blind" focus as the center of mean motion (Fig. 2).[9] Riccioli has no

[9] About the calculus of the mean anomaly including elliptical path, see Riccioli 1665, 32. With reference to Fig. 2 below, and considering F the blind focus, C the ellipse's centre: "Sit enim in sequenti figura ex Terrae centro T, descripta infinito propemodum intervallo Eclipticae portio AB, sitque Solis Apogeum A, sit deinde semi Ellipsis ESP, in qua sit Sol S, et per eum ducantur recte FM, pro loco medio, et TV, pro loco vero agaturque; TB, parallela ipsi FM, nam propter ingentem distantiam Eclipticae arcus BM, insensibilis erit, et BT, licet Mathematicè distincta sit ab FM, Physicè tamen coincidet cum FM, et arcus AB, insensibiliter maior erit arcu AM, quare sicut AB, est mensura anguli ATB, ut pote descriptus ex centro T, ita arcus AM, (hoc est physicè AB), mensura erit *Anguli AFM, hoc est Anomaliae mediae*"; "For in the following figure, from the centre T of the Earth, be described the portion of the ecliptic AB with an almost infinite radius, and

problem admitting "nonphysical" focus as a purely computational tool. Finally, those anomalies give parameters[10] to calculate the right eccentricity.

In fact, collected numerous observations, particularly Cassini's ones,[11] Riccioli focused at length on calculating the small eccentricity of the Sun's orbit. A first piece of data to be acquired relates to the need to bisect the eccentricity of the Sun's orbit. Whereas in AN Riccioli did not bisect the eccentricity values of the orbits of celestial bodies, including the Sun, in AR he does it. The rule of dividing eccentricity in two, which enabled Kepler to visualize the ellipse, is assumed here in order to calculate Sun's motion.[12] However, the first focus is Earth and the second focus is "blind". It is the Sun to revolve around the Earth and the astronomer must determine its true location by its equation of motion and collecting pieces of information as the distance of the Sun from the two foci of the ellipse or its speed.

Perhaps out of practical conviction, or perhaps to appease "old school" astronomers, Riccioli intends to establish a comparison between the representation of the Sun's motions by means of an ellipse for the true motion, and by means of a circle for the mean motion. If one opts to use the geometry of the circle, one needs to divide the eccentricity, which is the distance between the Earth as a center of the world and the eccentric point, into two. Thus, the Earth as a center and the eccentric point deviate from the center of the circle by exactly the same distance.[13] Figure 3 does not reproduce the correct proportions but emphasizes the roles of circle and ellipse. Let us consider a great distance from the Ecliptic and the paths A6 and P14. Be T the Earth (one focus) and F (second focus). AP is the apsides' line, A is at the apogee (point E on the ellipse EBDC), P at the perigee (point D on the ellipse EBDC). Sun describes its mean motion around F, so that we can consider the circle HIKL as the equant circle. Such circle is divided in 12 equal arches (individuated by the points H, M, N, I, O, Q, K, R, S, L, V, X). Lines FM2, FN4. FI6, and so on are drawn to the ecliptic. This division allows the sun's path to be ideally followed along identical travel times along portions of the ecliptic. Thereafter, those lines intercept on the ellipse EBDC a series of points (α, β, γ, δ, etc.), which must be connected with T and extend (by lines Tα1, Tβ3, Tγ4, Tδ5, etc.) to the Ecliptic (Zodiac) at the points 1, 2, 3, 4, 5, etc. In this way, it is evident that true motion is slower around apogee, faster around perigee. For accuracy, true motion continuously accelerates in the background of the arc AP, decelerates in

Footnote 9 continued
let A be the Apogee of the Sun, let then be the half-ellipse ESP, in which the Sun S stands, and through it be conducted for the middle locus the segment FM, and TV for the true locus, and let this be done; TB, parallel to the FM segment itself, for because of the great distance of the ecliptic, the arc BM will be insensible, and BT, though mathematically distinct from FM, nevertheless coincides physically with FM, and the arc AB will be insensibly greater than the arc AM, so as AB, is the measure of an angle ATB, as can be described from the centre T, so the arc AM, (this is physically AB), will be the measure of the Angle AFM, which is the Mean Anomaly."

[10] See Riccioli 1665, 33.

[11] The variability in the Sun's speed along its trajectory is real, not apparent: indeed, Riccioli was familiar with the results of Giandomenico Cassini with the sundial of San Petronio in Bologna, fundamental to understand the motion of the Sun (Riccioli 1665, 6–7; Di Teodoro et al. 2010; Bònoli 2001).

[12] Riccioli 1665, 33: "…hinc pariter constat necessitas bissecandi eccentricitatem illam, quae necessaria est ad Aequationes Motus Solaris"; "so it is equally obvious the need to bisect that eccentricity, which is necessary for the Equations of Solar Motion".

[13] See Riccioli 1665, 35 (Problema IX).

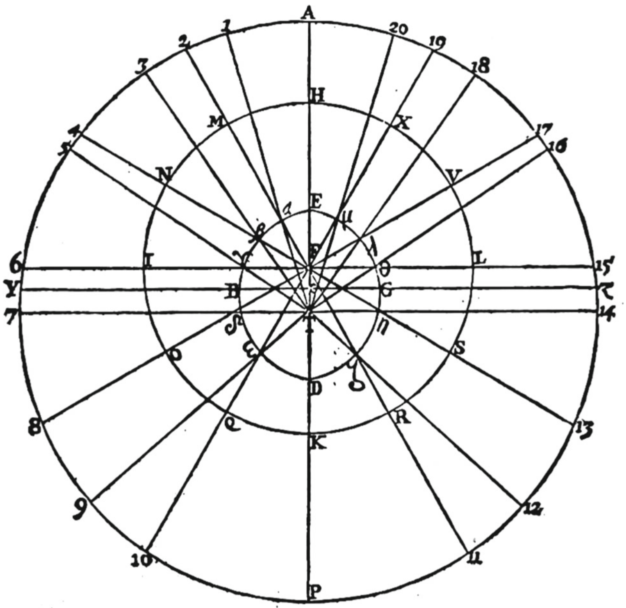

Fig. 3 The elliptical path of the Sun EBDG in the background of the ecliptic AYPZ (Riccioli 1665, 36)

Table 1 Difference between arches in elliptical and circular methods (Riccioli 1665, 37)

	Maxima Aequatio		
	G	I	II
In Ellipsi	I	59	38
In Circulo non bissecta Eccentricitate	I	59	40
In Circulo bissecta Eccentricitate	I	59	40 1/3

the background of the arc PA. By comparing the arcs, it can be seen that the average motion may be greater than the actual motion (arc A10 is longer than arc A9) or less (arc AP11 is less than AP12). Mean motion and true motion coincide for the points corresponding to B and C, minor axes of the ellipse. Riccioli quantifies the maximum value of the sun's motion in the model based on the circle and that based on the ellipse (see Table 1).[14]

The eccentricity of solar motion is very small. Therefore, the ellipse is very close to the circle. However, some inaccuracies lead Riccioli not to follow Kepler beyond adopting the law of the ellipse. The most serious is that neither annual motion on the ellipse, nor a motion on an eccentric circle, justifies the diurnal motion of the Sun. This is not a problem for Copernicans, because they attribute it to the motion of the

[14] Riccioli finds the difference of 2,3" for the arch 23 when calculated by the ellipse and by the circle, considering a circle with both entire and halved eccentricity. See Table 1 below (Riccioli 1665, 37).

Earth. But having found no evidence for such a motion, Riccioli seeks a reconciliation: annual motion and diurnal motion must have a single representation. This is achieved with a spiral motion around a conic trunk, and spirals result from the combination of annual and diurnal solar motions.[15] As early as AN (I, 3, 149 ff.), Riccioli adopted the basic conic model from Boulliau, who refined Kepler's methods and heliocentric system by combining two circles and supposing the planets rotating around a conic section (Boulliau 1657).[16] This approach was based on observations and geometrical calculations. As we will see shortly, the same reference to a conic section appeared in planetary theory.

Riccioli pretends that such a loop theory, which includes the elliptical figure, accounts for the variability of the Sun's diurnal motion, the variation of declination along the ecliptic, the different duration of daylight hours due to the season of the year, the large divergence between true and mean motion at the apogee and perigee. The data collected to construct the geometrical description are various: the classic argument of the apparent diameter of the sun, the calculation of the maximum shadows of the Earth and Moon observed during eclipses, and the determination of the eccentricity of the orbit (Riccioli 1665, 62–65). The basic diagram (Fig. 4a) refers to the year 1157, putting the Sun in the Cancer. The sun travels around the surface of two truncated cones, having in common the base (RQ in transversal section, Fig. 4a), from G to I, I to L, L to M, and so on. With reference to Fig. 4a, AB is the Equatorial plane. C is the Earth, DE the celestial horizon. QRIG and QRSh are two trunks of a cone with the common basis QR. FAG is a portion of the solar meridian, equivalent to the Tropics' distance and divided in two parts by A. Such a point signs the beginning the Aries. Symmetrically, HBI is the nocturnal arch with B in the middle, which signs the beginning of the Libra. Sun moves though Aries's constellation in A, Capricorn in F and H, Cancer in G and I, and Libra in B. AQ and BR are the halves of the solar eccentricity referring to the mean motion on the circle (see Fig. 3). Fh on the diameter FI, and HS, on the diameter HG, be like equal to the entire solar eccentricity referring to the mean motion. Lines GQ e Qh represent the sides of two truncated Cones, around which the Sun, having its Apogee at the beginning of Cancer, describes its diurnal spires. The length of these loops changes with the seasons. Both cones have a common basis in QR on the Equatorial plane. The longest section of the Cone will be QRIG, and the shortest section QRHF.[17] The Sun moves from its apogee westward by traversing ellipses on the cone's surface. Riccioli considers the case when the Sun's apogee is in the constellation Cancer. Then Sun goes in Libra, and finally in Capricorn. At the beginning, the Sun's diurnal spires stand on the truncated major cone, at first

[15] Riccioli 1665, 63: "…quod per Ellipsim immò nec per simplicem excentricum orbem non conciliatur motus diurnus apparens in Sole ab Oriente in Occasum, cum altero proprio ab Occasu in Orientem. Etsi enim Copernicanis cura hæc non incumbit, quippe qui motum Primi Mobilis Coelestibus corporibus pernegant, et uni Terrae adscribunt. (…) non videtur melius conciliari posse duos illos motus continuè contrarios quoad apparentiam, nisi per motum spiralem, omnibus Planetis communem"; "… the diurnal motion of the Sun from East to West is not explained by the Ellipse or the simple eccentric globe, nor is it consistent with the proper motion from West to East. The Copernicans disregard this issue since they attribute the motion of the celestial bodies not to themselves but to the Earth. (…) It seems that these two seemingly opposed motions can best be reconciled through a spiral motion common to all the planets".

[16] See Riccioli 1651, I, 149; 534 ff.; 541.

[17] See Riccioli 1665, 66.

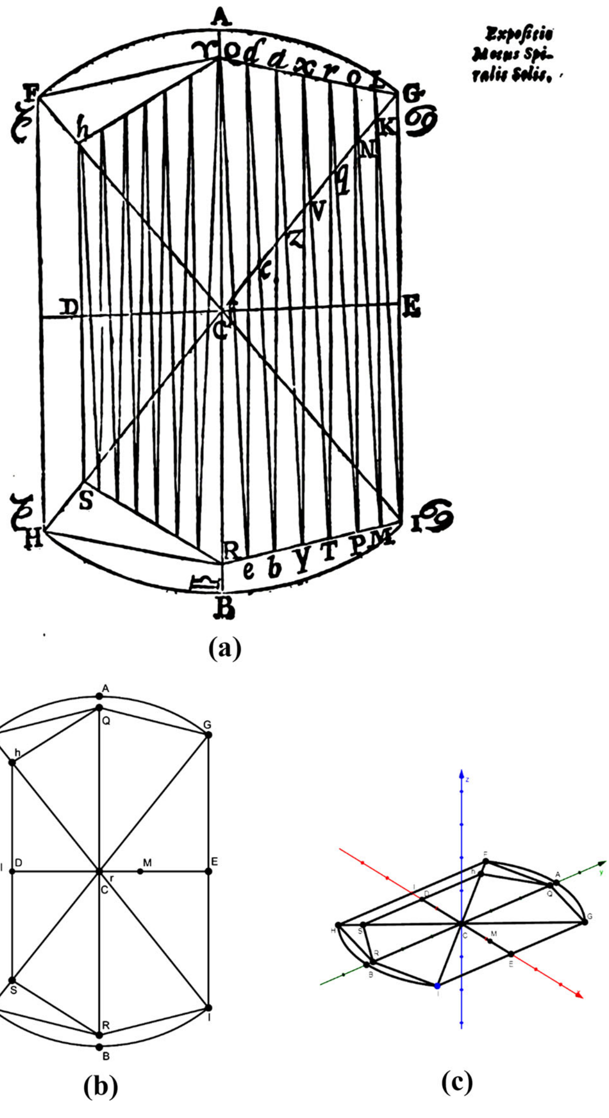

Fig. 4 **a** The Sun motion starting from the apogee in the Cancer (Riccioli 1651, vol. I, p. 66). **b** (in 2D) and **c** (in 3D). Variation in solar declination. In this picture, the transversal section of the major truncated cones' bases are reproduced (QhSR and QRIG). The points correspond at those in **a**. The auxiliar point M has been introduced to illustrate as it moves daily along the celestial horizon DE and determines the variation in solar declination. Animated 3D gif at URL: https://www.geogebra.org/m/vtgsn37t

narrower, then wider and wider, until they guzzle at the equator (QR). At that time the sun stands at the average distance from the center of its motion, Earth C (on the opposite, when it is in G it stands at the maximum distance). Therefore, the Sun will continue its course toward the base of the narrower cone to Sh, when its distance from the Earth is Ch and the distance between the true motion and mean motion is highest.

Figures 4b, c and 5 are given to better understand the spiral movement of the Sun, resulting from its daily and annual motions. Summarizing, every day the Sun rotates around the Earth. However, its orbit remains open at the end of 24-h period. The Sun cuts the orbit of the previous day in a point. This happens for everyday, and therefore, we obtain a set of points. All these points shape the ecliptic.

A last issue remains about the Sun and $physical_3$ implications. Riccioli conducted a lengthy investigation on the Sun's substance and confirmed through telescopic observations that it is made of fire. However, the key issue is the origin of motion. The motion of celestial objects is not necessarily an indication of their own internal animation. Riccioli believed that the driving force that unites the Sun with the other planets is external. Although the ultimate cause of planetary motion is unclear, one hypothesis is that it arises from some angelic intelligence (Marcacci 2018, 159–170). The last point is a critical against Kepler, who granted the possibility of angels as celestial movers (Dinis 2017, 172–176). However, Riccioli seems attribute a kind of efficient causality to the Sun in the transport of planets rotating around it. Yet he does not recognize Kepler's quasi-magnetic force as the cause of planetary motion as supposed by Kepler (Dinis 2017, 188–189).

3.2 All about Kepler in four propositions

Riccioli reads Kepler's astronomical work in order to solve the two most important inequalities that had been attempted for centuries: the first and the second inequalities. The genius of Kepler is in the mathematical methods, which are able to explain planetary kinematics very well. The arguments of both are useful in the hypothesis with a moving Earth as well as with a stationary Earth.[18]

In order to understand the way in which Riccioli was influenced by Kepler and to understand how the former interpreted the latter, it is necessary to recall the following four propositions for introducing Keplerian astronomy (Riccioli 1651, vol. 1, 527–531).

Proposition 1 *The line of the apsides of the five minor planets transits the center of the body of the planet and the sun.*

Riccioli recalled that the ancients made the line of the planets pass either through the center of the Earth or the center of the Sun's annual orbit. Instead, Kepler overlapped the planetary system's center with the world's center: he started from observations and attested that a planet transited with minimal velocity at aphelion. Since the velocity value corresponds to that observed only when the line of apsides transits the Sun's

[18] Riccioli 1651, vol. 1, VII, 536: "…quorum argumenta militant tam in stantis, quam in circumventis Terrae hypothesi"; "the arguments of which militate both in favor of an immobile Earth and the hypothesis of an Earth in circular motion."

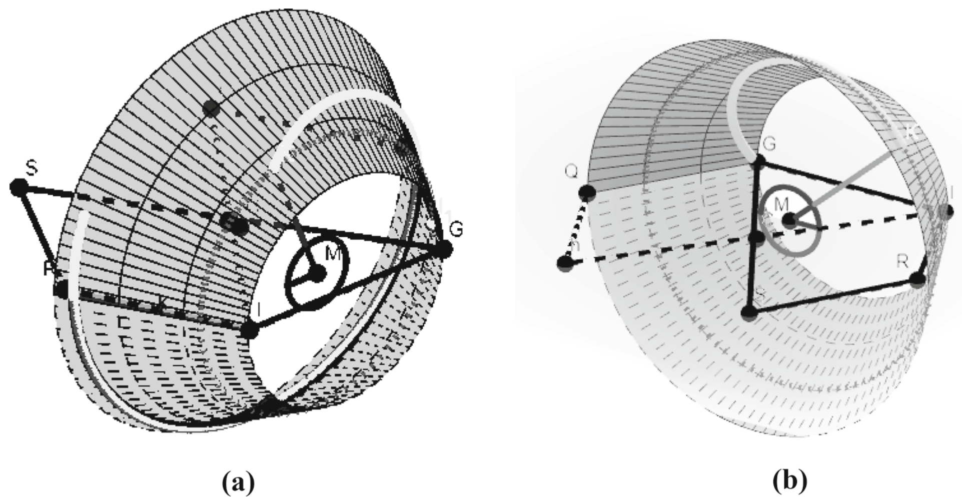

Fig. 5 a, **b** The spiral-like motion of the Sun according to Riccioli. With reference to Fig. 4a, the Sun goes on spire after spire through the following points L, M, N, and O starting from Cancer. The sun apogee is in G and Sun moves westwards along spirals: each spire is ellipse-like. Riccioli measures the solar distance to the cone's axis DE (coincident with the horizon). The trunk QRIG contains the solar spiral during the boreal motion, the trunk QRSh those of the austral one. For the eastwards, motion is apparent, the only true motion is westwards. Besides, the spiral-like motion has the kind of motion of all the planets, too. In **a** and **b**, the point M is a point on the solar plane and indicates the variation of its declination. The point K traces Sun's spirals on the truncated cone's surface. Distances are enlarged here to be the image clear. Vistas are from outside (**a**) to inside (**b**) the major section of the truncated cone. For the animated gif, URL: https://www.geogebra.org/m/rtfnggtn center, Kepler made it the planetary and world's center, as we have seen in the second section.

Proposition 2 *In the second inequality, the true opposition or conjunction of the planets with the Sun and the true or apparent motion of the Sun must be shown.*

The distinction between true and mean motions explains this assumption. As seen, Kepler chooses the true motion of the Earth to construct his model. Riccioli followed the same path and opted for the true motion rather than the mean one when he described and solved the second inequality.

Proposition 3 *The planet's path is along an ovoid or elliptical line; the deferent of that orbit is not a perfect circle, but an ellipse.*

Riccioli collects the best and most recent observations investigating the planet's eccentric location and distance from the Sun: the best results are obtained using the ellipse, not the circle as a mathematical device. Mercury's path was already associated with an oval from Tycho to Gassendi, up to Boulliau and Kepler. In fact, Ptolemy justified some valuable elongations of Mercury with the addition of a small circular mechanism which made the orbit oval. Peurbach and Regiomontanus worked hardly on this problem (Malpangotto 2013), all dependent on the impossibility of improving observations (Gingerich 1993; Poulle, Gingerich 1967). Riccioli pointed out how in some cases (12 December 1630, 1 December 1635), the error between the observation and the predicted position appeared significantly lower by assuming the ellipse as a model (5 s and 23 s, respectively) (Riccioli 1651, 529).

Proposition 4 *Eccentricity, upon which the first inequality of planetary motion is founded, is to be divided in two; that is, half of that inequality is to be attributed to the eccentricity of the deferent, the other half of the cause to something else that physically and actually retards or accelerates the motion of the Planet; and for this the equant is introduced.*

Hipparcus let the Sun move on a circle with the center no longer coincident with the Earth but slightly displayed from that. The eccentric point makes the planetary trajectory equivalent to that detected by a model with a concentric deferent with an epicycle. Ptolemy added the equant to the eccentric. Generally, the eccentric and the equant points were spaced from the center, depending on several parameters based on observations. Copernicus replaced the equant by adding a little epicycle whose radius was half the eccentricity. For some values, Ptolemy's and Copernicus' models were equivalent. Moreover, both adopted a bisection of eccentricity in some circumstances, for instance, for Mars. Before Tycho and Kepler, it was pretty impossible to determine the slight difference by observations. However, the bisected eccentricity had no physical basis or theoretical justification, and it was used only for calculation. Riccioli regarded the Keplerian method as a kind of continuity with the past—about the eccentricity of the deferent, that is, the component due to the mean motion—and a new component—about the role of the second eccentric point—that is, the equant.

Riccioli dwells on the reasoning that led Kepler to support proposition 4. Kepler's arguments were based on aspects of planetary motion, either specific to individual planets or common to some or all of them. Riccioli's disagreement arises from his inability to verify the key data underlying this conclusion, specifically the one-minute difference in the Sun's apparent diameter at apogee and perigee. He also finds insufficient evidence in lunar eclipses to support Kepler's theory. While Kepler and others like Bullialdus demonstrated the halving of eccentricity for Mars and Saturn through geometric calculations, but Riccioli finds their solution unconvincing. After all, he notes that Copernicus did not bisect Mars' eccentricity directly and "attributed the reason for not bisecting it not to the errors of Ptolemy—for in Ptolemy's time he thought it should be bisected—but to the varied and diminished eccentricity of the Earth". Riccioli concludes: "Therefore, up to this day, the bisection remains controversial."[19]

The arguments supporting the proposition that the Sun's eccentricity should be bisected are partly specific to individual planets and partly apply to several. Riccioli critiques Kepler's claim, based on a 1-min difference in the Sun's apparent diameter at aphelion and perihelion, as it contradicts his own observations and others. He also finds insufficient evidence in lunar eclipses to support Kepler's theory. While Kepler and others like Bullialdus demonstrated the halving of eccentricity for Mars and Saturn through geometric calculations, Riccioli notes that Copernicus did not bisect Mars' eccentricity but attributed differences to Earth's changing eccentricity. Tycho's observations also led to a different measurement of Mars' eccentricity.

3.3 Law of ellipses

The third proposition above already contained the law of ellipses. Riccioli cited widely it in the *Almagestum novum*. Obviously, in the sections dedicated to Kepler in *Almagestum novum*, it is dealt with specifically. Also elsewhere, however, such as in the third book entitled *De Sole*, he mainly takes up the studies of Boulliau, by integrating Keplerian discoveries with Boulliau' method (AN, I, 541). The law of the ellipse becomes increasingly essential and assumes absolute centrality in *Astronomia reformata*.

In the ancient approach, the equation of the center was the difference between the true and the mean anomaly measured from the apogee. Kepler demonstrated that an ellipse allows for more accurate predictions by considering observations from the Sun rather than the Earth. Thus, his laws provided a more precise and comprehensive way to describe the path of a planet. Up to here, Riccioli expressed his favor toward Kepler. Nevertheless, Riccioli did not accept the explanation of the prograde motion (towards eastward) as the only true motion for planets. He recognized that Keplerian orbits produced more exact astronomical tables and previsions. But the method was not rigorous (*Imperfectione Methodi Kepleriana*, Riccioli 1651, vol. 1, 532), insofar an elliptical line (*circumferentia elliptica*) cannot be divided in equal parts, unlike a circumference. But that division is fundamental for any calculation: in agreement with this, in the demonstration of solar motion Riccioli gives the twelfths of the circumference on the ellipse. Analogous doubts by Boulliau and Magini comforted him, as well as the examen of works of Fabricius, Cavalieri e De Rheyta. Thus, Riccioli did not adopt directly the ellipse's law in AN, albeit he used ellipses backing to the calculation based on the eccentric circle plus an epicycle.

[19] See Riccioli 1651, vol. I, 531.

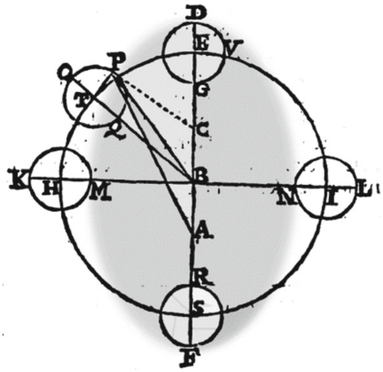

Fig. 6 Approximation between epicycles and ellipses: the planet goes through the points D, P, M, F, and N which trace an oval trajectory (see Riccioli 1651, 533)

The planet's mean motion is distributed not from one focus to the other focus of the ellipse (as one would expect) but from the ellipse's center (Fig. 6).

Riccioli gives relevance to the ellipse's center. Solar theory considered in *Astronomia reformata*, which recalled above, was formulated later than the planetary theory exposed in *Almagestum novum*. While the two foci were fundamental in solar theory in *Astronomia reformata*, both the center and the two foci are crucial points in planetary theory in *Almagestum novum*. From such center and not from the two foci, the distribution of the mean motion takes place. This is a crucial point in understanding the development of his own *epic-epicycles method*. This method combines two variable elements: the eccentric point's variable position and the epicycle's variable radius. Thus, the planetary motion results from two motions: 1. the planet's motion along the epicycle, whose center rotates along the deferent and whose radius's length continuously changes; 2. the motion of the eccentric point oscillates between a maximum and minimum distance from Earth, that is, respectively, B (apogee) and Z (perigee) (see Fig. 7). The principle of motion composition is largely used here.

The way to combine those motions depends strictly on the quadrant that the planet is traveling. Indeed, the link between the variation of the epicycle's radius and the eccentric is based on both observations and the rule of the supplementary of mean anomaly's angle and true anomaly's one of the eccentrics: the overlap between observational elements related to experience and mathematical justification is perfect within the spirit of the *physico-mathesis*. The result is determining the distance planet-Earth and, consequently, the planetary position and the shape of the planetary orbit. This latter is a spiral motion on spires of variable size. The first inequality descends from the variability of the epicycle's radius, and the second inequality is from the variation of the eccentric point: obviously, each component directly influences the other (Marcacci 2023).

So far, this is the method invented by Riccioli. Let us understand shortly how Keplerian astronomy influenced Riccioli in the elaboration of such method. A summarized comparison is provided in Table 2.

1. First, on the one hand, Kepler adopts elliptical orbits, on the other hand, Riccioli spirals. Riccioli appeals to ellipse as a tool for spirals. Ellipses are not mentioned in the description of Riccioli's method in *Almagestum novum*. Eccentric oscillates

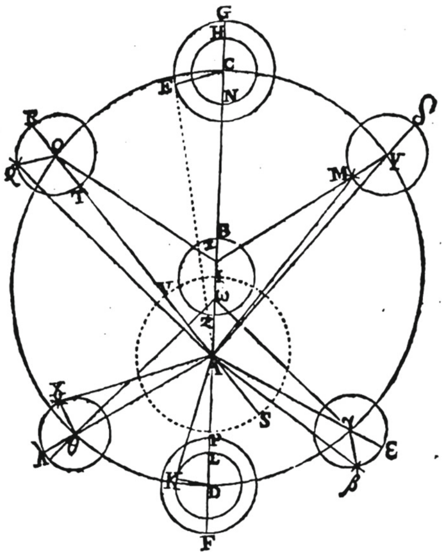

Fig. 7 The epic-epicycle method by Riccioli (1651, vol. 1, 538)

Table 2 A comparison between tools asked for explaining the shape of planetary orbits in Kepler and in Riccioli

Keplerian elements for describing planetary orbits	Original in Kepler	Modified in Riccioli
Shape of annual planetary orbit	Ellipse	Spiral
Empty focus	For the determination of the mean motion	Occupied by variable eccentric; suitable for determining the mean motion
"Full" focus	Sun	Earth
Distance Sun–planet	Variation of the vector Sun–planet	Sun–Earth distance (VS in Fig. 7) ± planet–Earth distance, dependent on eccentricity and epicycle's radius
Distance Earth–planet	–	Mean anomaly (segment AP, Fig. 6)
First anomaly (variation in planetary velocity)	Second law	Variable epicycle's diameter
Second anomaly (stations and retrogradiations)	First law	Variable eccentricity

up and down along BZ, the diameter of the circillus which contains the anomaly of the eccentric.[20] Four fundamental positions for the eccentric are considered, associated with mean anomaly (AI), as the eccentric transitions from the major (AB) to the minor anomaly (AZ). During the transit, the epicycle variation is contained within the maximum elongation of its center in *true conjunction* with the Sun, and the minimum one in *true opposition* with the Sun (Riccioli 1651, vol. 1, 538).

Riccioli uses the word "ellipse" as little as possible in *Almagestum Novum*, unlike in *Astronomia Reformata*. Despite this, there are ellipses everywhere. Take, for example, the case of a superior planet. Consider the eccentric's anomaly CxO (Fig. 8). The angle AxO is its supplement. Depending on whether it is in a range below or above 180, or in the intermediate ranges (90 and 270 degrees, quadratures), then the rule for calculating the eccentricity of the orbit changes, in dependence of the quadrant (Riccioli 1651, vol. 1, 538; Marcacci 2023). Of utmost importance: this is equivalent to saying that the deferent changes, *a deferent which has, approximately, always the shape of an ellipse.* To be more precise, the deferent is instantaneously an ellipse. Thus, if the eccentricity varies continuously, then an (infinite) series of ellipses is determined. The elliptical deferent (Fig. 6) is obtained (see Fig. 7) in correspondence to the variation of the anomaly of eccentric and to the variation of the epicycle's radius (see Tables 2 and 3). In reference to Figs. 7 and 8, the points signed by a cross or a star are crucial planetary positions during the revolution of the planet. The resulting annual trajectory of the planets has the shape of a curve, which is a part of an ellipse up to the point that part after part it becomes a spiral. Nevertheless, we must remember that the center of the orbit is the Earth (A). This depends on the fact that, if we would express the apparent motion of the Sun holding the Earth steady, then the solar orbit appears as approximately elliptical. The same in the case of a planet, with details which are not relevant now. In addition, Riccioli opts for annual spiral orbits, as we will comment below at point 5.

Let us go more quickly to other remarks

2. Second, Kepler distributed the longitudinal planetary motion in two components:

   - The first component concerned the planetary mean motion;
   - The second component regarded the planetary approaching and receding from the Sun, and it was established in respect to the focus where the sun was located.

   The epic-epicycle method is displayed along the following elements (Figs. 6 and 7):

   - Planetary path's deferent is eccentric with Earth at the center. There is a fundamental deferent centered on I, that on the eccentric's mean anomaly AI, but the deferent is actually a variable ellipse, as an infinite series of ellipses;

[20] Riccioli 1651, vol. 1, VII, 537: "Anomalia Eccentricitatis, in cuius diametro BZ, libretur sursum ac deorsum centrum Eccentrici…"; "Eccentricity anomalies, in whose diameter BZ, the centre of the Eccentric is balanced up and down…".

- The planetary mean motion is computed on the "empty" focus, that is the variable point x, which oscillates along BZ and around the center of the principal deferent, plus the variation of epicycle's radius.

3. Third, the variable epicycle's diameter explains the variation in planetary velocity (first inequality), as the Keplerian second law accurately describes how the variation occurs. However, epicycles help to give more detailed orbits. In fact, the rule that reduces the technique with a minor epicycle to that of equant[21] cannot be used here because it would not explain retrogressions and apparent stations. Such a rule is admissible only if the annual orbit of the Earth around the Sun is substituted in place of epicycles, as in the heliocentric hypothesis (Riccioli 1651, vol. 1, 540).
4. Fourth, Riccioli employs true opposition and conjunction with the Sun to establish eccentricity at the quadratures, then to explain the second irregularity, strictly following Kepler's instructions (see Proposition 2 above). He also used the mean Sun (Riccioli 1651, vol. 1, 140, 152–157): comparison between true and mean Sun is important to specify variations along the epicycle and prevent the actual value from deviating too much from the predicted value. The variable eccentricity explained the planets' variable trajectory (second inequality). However, the two variations (epicycle's radius and eccentric' variations) continuously interact with each other. In the second inequality, the true opposition or conjunction of the planets with the Sun and the true or apparent motion of the Sun must be shown.
5. Fifth, Kepler intends to describe the annual revolution by the ellipse law. As shown, Riccioli uses ellipses to describe the annual motion too, but no single ellipse closes. Because of the variation of the epicycle's radius, the planet actually traces successive portions of ellipses. In fact, Riccioli does not refer to an annual ellipse as Kepler, but daily portions of ellipses. Above all, Riccioli has got a "three-dimensional" view of the motion of the planets: he wants to describe their motion in the shape of a spiral. For this purpose, he needed a valid model to describe the motion in latitude of the planets (Riccioli 1651 VII, 624 ff.) and, therefore, receptive to data on the inclination of the orbital planes. Although still respectful of Kepler's work, he raises doubts for instance about calculating the latitudes of Venus' boreal and meridional epicycles, for he prefers to use his epic-epicycle method (Riccioli 1651, 627). Riccioli exalts the idea that the ellipticity of the orbits really persists only by assuming that the planet carries out its motion on a scalene cone (Riccioli 1651, 149).
6. Sixth, the relative distance planet–Sun is computable in Kepler system by the vector between the center of the planet and of the Sun. Riccioli must compute both the distance planet-Sun and the distance Earth-Sun. For the first, he compiled many tables comparing different proposals by astronomers. For the second, he must give the geometric way (Riccioli 1651, vol. I, 685). The rules are different

[21] The reference is to the Copernicus' technique of a minor epicycle equivalent to Ptolemaic equant. See Evans 1998, 430–434. Besides, Copernicus should have been influenced by Arabian tradition and particularly from the Maragha school. Indeed: "The method of the Maragha planetary models was to break up the equant motion in Ptolemy's models into two or more components of uniform circular motion, physically the uniform rotation of spheres, that together control the direction and distance of the centre of the epicycle, so that it comes to lie in nearly the same position it would have in Ptolemy's model, and always moves uniformly with respect to the equant" (Swederlow, Neuegebauer 1984, 46; see also 41–48 and 566).

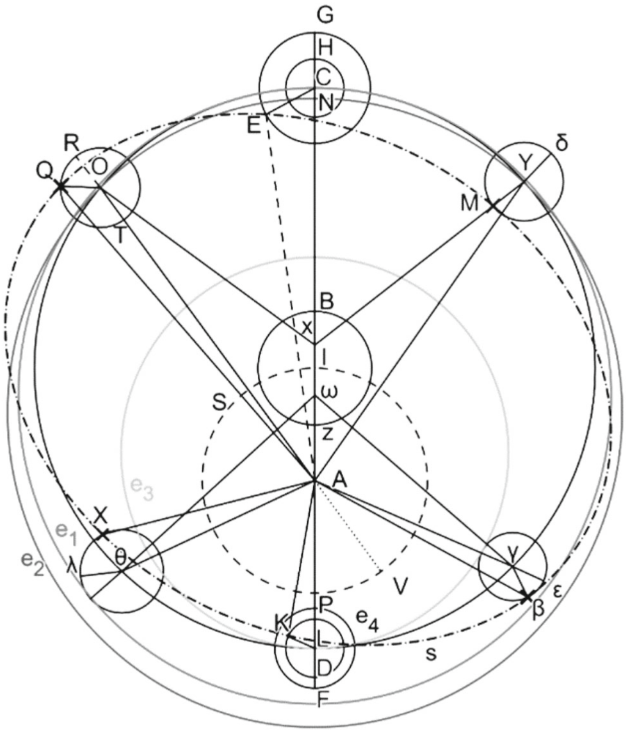

Fig. 8 From ellipses to spiral in the planetary motion: interpretation of Riccioli's method in comparison with Fig. 6. DAC apsides' line, A Earth's center, I center of the deferent and of the ZB, BZ Circellus which contains the anomaly of eccentricity, *x* eccentric and oscillating point, AZ minimum value of the apogee. AB maximum value of perigee, VS annual solar orbit, The conic S results by joining the "starred" points (Q, X, β, M) obtained by the supplement of the mean anomaly and concerning the Sun's motion in S, all computed with respect to the eccentric oscillating along the BZ diameter. The other points (E, K) are obtained by finding apogee (in B) and perigee (in A) of the eccentric, corresponding to the center of the epicycle (in C and D, respectively). Such a conic is an ellipse. The planet moves on successive portion of ellipses (as e_1, e_2, and e_3, see Table 3). The final spiral-like planetary motion is obtained spire by spire

for superior and inferior planets, but all of them are a combination of distances considering the variation of the epicycle's radius and the eccentric's position.

Some points are everywhere stressed: (1) the stillness of the Earth; (2) the fluidity of the heavens; (3) the uniqueness of the shape of planetary orbits. For that, and nevertheless Riccioli's dependence and admiration to Kepler, his words are[22]:

> And so in this we differ from the most recent astronomers, in that we use the elliptical path of the planet as a mere hypothesis so that we can begin the calculation; those in fact think that it is the true path of the planet, moving continuously eastward, when in fact it is moving westward. Because of the difficulty of understanding and explaining this spiral motion, we should neither disapprove of it

22 See latin test in the footnote 41. See also Riccioli, AR, 3.

Table 3 Crucial positions for determining annual planetary orbits

Center of the epicycle	Center of the eccentric	anomaly	Elliptical deferent	With respect to the Sun
C	B (apogee)	Maximum (AB)	e_1	True conjunction
–	I	Mean (AI)	e_2	
D	Z (perigee)	Minimum (AZ)	e_3	True opposition

> nor fear the censure of Kepler, who tried to bury it with all his contrivances, undermining the basis of his discoveries.

Riccioli makes his interest in Kepler's hypothesis of elliptical motion, but being the Earth immobile, it is not possible to believe that Kepler is right. Thus, he usurps the method of calculation for ellipses (*sub mera Hypothesi usurpamus ad calculum ineundum*. Riccioli 1665, 6). These ellipses are approximately a circle for the Sun, Venus, Jupiter, and Saturn, less similar to a circle for Mercury and Mars. Riccioli knows that this may not please the followers of Kepler and hopes, for this, not to be censored (Riccioli 1665, 6).

For all these reasons, the term ellipse is widespread in *Almagestum novum*. It is evident in the tables summarizing the motion of the planets in which the measures of the major and minor semiaxes and the eccentricity of the ellipses appears (ex. Riccioli 1651, vol. V, 301; VI, 305; VII, 328; VIII, 341; IX, 352). As seen above, Riccioli understood how to resolve inequalities by resorting to the distinction between optical and real causes.

3.4 Law of areas

The determination of the correct position and orbit of the Sun is crucial: the variation of the planetary eccentricity depends on it, because one planets' focus is the Earth while the other one is the Sun, which moves[23]:

> [Kepler] identified an imperfection of the planetary path belonging to the [second] class, namely a deviation from the perfect circle. He broke away from the traditional concept of perfect circles and the complex system of eccentric and epicycles, instead proposing the simple periphery of an ellipse. Kepler placed the Sun at one of the two foci of each planet's elliptical orbit.

[23] Riccioli is refereeing to various groups of mathematical imperfections in the dealing with planetary orbits. Among those imperfections there is the difference from a perfect circular path and an elliptical path: Riccioli, AN, vol. 1, 526: "[A secunda] verò classe imperfectionem viae Planetariae, seu deviationem à perfecto circulo [Keplerus] hausit, aususque circulos perfectos, ac totam Eccentricorum Epicyclorumque struem infringere, per simplicem peripheriam Ellipsis unius, singulos Planetas circumtulit, in cuius Ellipsis altero focorum seu umbilicorum sit Sol".

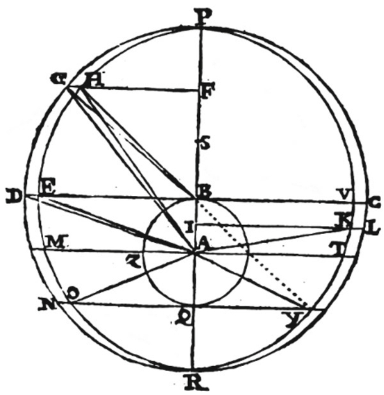

Fig. 9 Measurement of the displacement of the planet in time (Riccioli 1651, vol. 1, 531)

The Jesuit recognizes that Kepler resolved the second inequality as optical, i.e., dependent on the motion of the Earth.[24] As for the first inequality, Kepler recognized its *physica ac reale* nature (Riccioli 1651, vol. I, 526). This distinction is very important to understand Riccioli's commentary on Kepler's second law.

According to Kepler, if the observer is placed in the Sun, then he would see the planet unfold its orbit by traveling unequal arcs in equal times. Despite the goodness of the idea, it still confirms the error of dividing eccentricity in two. Riccioli claims the second law, comparing the areolas on the elliptical orbit with the areas of the sectors on the circular orbit. In fact, the measure of time in any arc of an ellipse is not taken from the arc itself, but from the area of the elliptical segment subtended by such arc.[25]

This idea impressed the epic-epicycle method. Thought not believing in the *physical*$_2$ reality of Kepler's model for planetary motions, Riccioli adopts the elliptical hypothesis and the area law as a useful computing device. The measure of the displacement of the planet in time is given by the measure of the ellipse's area-segment, not simply the arc.[26] Riccioli explains Kepler's calculation of the movement of the planet in time by comparing the relation between the circle and the ellipse. Let A be the Sun as the center of the universe, the planet be in H, A is the ellipse's focus, PR the absidal line, B the center of the ellipse and of the circle PDRC where the ellipse PERV is inscribed in (Fig. 9). Be the area we are interested in is among the segments AP and AH and the curve HP. To calculate the planetary velocity, Riccioli divides both the area of the ellipse and time into equal parts, say 360, taking advantage of the fact that the Planet has a total period in which it travels the entire ellipse. Thus, he obtains the distribution of speeds. This measure will be smallest at apsis P, largest at R, average around points E and V.

[24] Riccioli, AN, I, 526: "esto si oculus in Solem transferatur, … ita ut Planeta circa suae orbitae centrum, immò et circa solem, inaequales arcus temporibus aeqalibus conficiat"; "This is if the eye moves to the Sun, …, so that the Planet makes unequal arcs at equal times around the centre of its orbit, rather than around the Sun".

[25] Riccioli, AN, I, 531: "Mensura vero morae ac temporis in quolibet arcu Ellipsis sumitur non ab ipso arcu, sed ab area et segmento areali ellipsis"; "But the measure of delay and interval of time in each arc of an ellipse is derived not from the arc itself, but from the area and areal segment of the ellipse".

[26] See pages AN, I, 531–532.

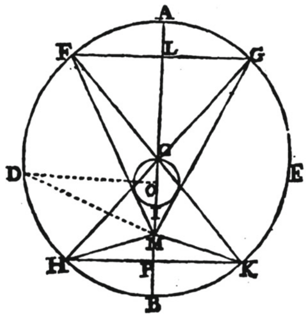

Fig. 10 Determination of different achronic planetary paths on the basis of the circle (Riccioli 1651, vol. 1, 550)

Quantifying these differences leads to the correct description of the libration: it is the difference between the average longitude of the planet, where motion is fastest, and the other longitudes, where motion is slowest. Referring to the figure, such is the difference between PAE, which is the average longitude of the planet in E, and PAH, which is the longitude in H.

Riccioli goes on to relate Kepler's method to usual astronomical practices. The mean anomaly is the time interval calculated with respect to the aphelion and corresponds to the area of the circular sector of the elliptical orbit's section in comparison with the circular orbit. For example, with reference to the point H, the area of the elliptical sector should be calculated with reference to G. Similarly, the equated (*coaequata*) anomaly is the angle between the center of the Sun and the line passing through the planet: if it is less than two right angles then the angle itself is taken (e.g., if the planet is in H, the angle HAP); if the angle is greater than two right angles, then the explementary should be taken (therefore, not YAP, but PAY). The place of the eccentric point is determined on the Zodiac by the line joining the Sun and the planet: in the figure, it is along the line AH. The mean anomaly is calculated supposing the planet move uniformly on the auxiliary circle. Given the center B of the ellipse and the auxiliary circle, it is the angle between the straight line BP and the straight line joining B with the locus of the planet on the auxiliary circle.[27]

The difference between mean anomaly and equated anomaly gives the eccentric locus equation. For example, if PAG mean anomaly is 45° and PAH equated anomaly is 40°, the eccentric locus is 5°. The angle close to the planet gives the optical equation, AGB in the example; the physical equation corresponded instead to HAB. At this point, the law of areas is deduced by geometrically comparing the physical equation with the optical equation. The optical equation is bigger than physical one in the superior semicircle CPD, and smallest in the inferior DRC. The physical equation is maximum in D and C, the optical one in O and Y. However, the true trajectory is on the ellipse, one must consider the physical equation in E and optical equation in V.

Riccioli brings back the law of the areas to circular method insofar he supposes his measurements of the first inequality more precise than Kepler's. Concerning Fig. 10 (Riccioli 1651, vol. 1, 550), AB is the apsides' line, and M the center of the world and C the center of the circle. The aphelium and perihelium are in A and B. F and G, H

[27] It is possible to prove that, if e is the eccentricity, E the eccentric anomaly and M the mean anomaly, it is $M = E\text{-}e\sin E$. All the anomalies can be measure either with angles or with areas.

and K are the achronic places of the orbit observed and taken as reference points for the calculation. Let the planet be in F. Let F be connected with the centers C (by FCK and GCH) and M (by FM, GM, MG, and MK). Besides, be the chords FLG and HPK traced.

ACF is the eccentric's anomaly for the arch AF. The sinus FL and cosinus CL are given. The center's equation is MFC, known by comparison between the true motion, on the base of observations, and the mean motion. The angle MFC (equation of the center) is known (as the difference between the real motion given by FMC and the mean motion FCM). Thus, by a series of passages, the side ML of the right triangle FML is calculable. The difference between ML and CL gives the eccentricity. But F moves up to D and the reference point for determining the true motion becomes the point O. The libration up and down along the *circellum* CI determines the variation of the eccentricity.

Summarizing, on the one hand, while examining Kepler's law, Riccioli studies the discrepancies between the true motion along the ellipse and the mean motion along the circle. On the other hand, explaining his own method, Riccioli does not consider the ellipse, and the discrepancies between true and mean motions are attributed to the variation in eccentricity determined by the eccentric's oscillation along CI. This is the core of the epic-epicycle method. In other words, Riccioli adapts Keplerian approach and the variation in the planet's velocity is explained along an eccentric orbit.

4 Concluding remarks

Riccioli could draw on Kepler's kinematics without adopting his dynamical insights. His reading of Kepler occurred on distinct levels: an examination of kinematic interpretations and a critique of the dynamical proposal of the concept of magnetic force as the cause of motions. Riccioli proposed a cosmological model, developed on Tycho's one. Under this respect, he also searched for a causal—and so, dynamic—foundation of his model. Nevertheless, he encountered many difficulties because of the unavailability of empirical proofs. Thus, he gave it up. Therefore, he insisted on a $physical_1$ solution and devised a new method of describing and predicting planetary motions in AN: he combined variable eccentrics and epicycles with variable diameters and named this calculation as "epic-epicycle method".

Dynamics plays a limiting role in Keplerian astronomy: there are some movements which are not allowed by what Kepler thought to be dynamical causes but the four level—empirical, kinematical, cosmological and dynamical—always are intertwined. Regarding Riccioli's work, the complexity of the term "physical" remains significant. First, he approached physics as a philosopher of nature within the Jesuit context, attentive to new discoveries yet cautious and distrustful about openly embracing heliocentrism. Second, as a clever reader of Kepler, Riccioli sought deep reasons for explaining a new model for the cosmos, whether heliocentric or geo-heliocentric, looking beyond kinematics. However, his insistence on grounding astronomy in data and observations, coupled with the lack of empirical evidence for terrestrial and celestial motions, led him to abandon any fixed foundation and adopt a probabilistic approach to scientific knowledge. Thus, empirical arguments limit every foundational attempt.

In his geo-heliocentric model, celestial mechanisms combine regularity and irregularity. For the irregular aspects, Riccioli does not attribute cosmic motions to angelic intelligence. Instead, he believes that God maintains cosmic order, even if phenomena can change locally. Astronomy remains a divine science, but its data and methods must be guided by tireless research and experimentation.

Riccioli was the last Tychonian before Newton and it is crucial how it could use Keplerian discoveries in a geocentric system. Riccioli engaged with Kepler on multiple levels, utilizing his kinematic interpretations while critiquing his dynamical ideas and mathematical solutions. Building on Tycho's cosmological model, Riccioli sought a physical foundation and partially rejected Aristotelian physics for his own model but ultimately faced too many difficulties and abandoned this attempt. Instead, he opted for a kinematic solution, devising a new method for describing and predicting planetary motions. This method, the epic-epicycle method, combined variable eccentrics and epicycles with variable diameters: it involves ellipses but maintaining the Earth at rest. Compared with some Kepler's forcings (e.g., Bussotti, Marcacci, *in preparation*, see Fig. 6, about the first steps by Kepler to deduce Mars' orbit), this method succeeds in giving a more articulated geometric justification of ellipses. What it lacks, however, is the possibility of generalization: the variation of the radius of the epicycle and the oscillation of the eccentric are continuous, but the way of adding or subtracting them changes on an empirical rather than deductive basis. Besides, as Kepler, but more than him, Riccioli is often confused: on the one hand, he wants to establish thorough geometrical demonstrations; on the other hand, he amasses extensive collections of data and comparisons with other astronomers, often risking a loss of coherence and struggling to achieve synthesis. In addition, he modifies some of his techniques between his first (Riccioli 1651) and second works (Riccioli 1665). He moves closer to Kepler by increasingly adopting the use of ellipses but complicates matters by incorporating them within a geo-heliocentric model. Lastly, the study of the areas subtended by planetary paths is useful for calculating planetary motion. However, in a geo-heliocentric model, this calculation approximates continuously circles and ellipses, with a fixed focus (the Earth) and a blind "focus" (the center of the oscillating eccentric). This approach "hides" the variability of the planet–Sun distance behind the oscillation of the eccentric. While the mathematics may still work, the method is far from intuitive and perspicuous.

As a consequence of the above, the distinction between kinematics and dynamics highlights the complexity of the transitional period from Galileo to Newton. The seventeenth century marked the era of the scientific revolution, with numerous groundbreaking discoveries in the physical sciences, far too many to cover in this context. However, if one were to single out the most crucial shift that influenced the development of physics, it would be the transition from a kinematic to a dynamic approach. This shift signified that scientists started seeking the underlying causes of phenomena rather than merely creating kinematic models that could describe and predict events without offering a causal explanation. An intriguing epistemological question arises: how far back can we trace the chain of efficient causes? Let us offer an example to clarify this situation: Newton proved that gravity is the cause of the rotation of the planets around the Sun. Following the Cartesian tradition and considering Newton's characterization of gravity, Leibniz tried to determine the mechanical cause of gravity through the development of vortex theory. Huygens behaved similarly.[28] Both did not accept the action at a distance which they considered inextricably tied to Newton's theory. To overcome this difficulty, they tried to establish a second level of causality: gravity causes the motions of planets and comets; the movements of the particles composing the vortices surrounding celestial bodies explain gravity. One could go on and wonder what causes the motion of the vortices' particles. From an epistemological standpoint, when is it appropriate to break the causal chain? No a priori answer exists. Gravity could be inserted within a mathematized, working theory of physical actions. The cause of gravity could not be found. It remained a qualitative model (apart from some considerations by Huygens) which added nothing to the theory, which was independent from the theory itself and from which theory was independent. Hence, Newton's "Hypotheses non fingo" is fully justified.

Kepler's approach was remarkably modern in several respects: (a) he sought the causes of planetary motion, (b) he refrained from claiming to define a deeper causal level, and (c) he attempted to construct a comprehensive theory where kinematical aspects would be derived mathematically from dynamical ones. In this final attempt, he did not succeed; in fact, the opposite holds true: dynamics is built upon observed kinematical effects rather than deriving kinematics from dynamical causes. Nonetheless, no scientist before Newton had such a clear understanding of the relationship between kinematics and dynamics as Kepler. Riccioli, for his part, embraced both Keplerian laws and the quest for deeper explanations beyond mere kinematics. The rapid advancements in celestial discoveries demanded both mathematical and philosophical interpretations, making it increasingly difficult to maintain the traditional cosmic views. Riccioli carefully examined empirical data, corrected planetary geometry, and investigated the universal laws governing falling bodies on Earth and the heavens, hoping to uncover new causal insights into the cosmos. Ultimately, he concluded that the Earth is at rest and that no angelic influence could sufficiently explain celestial mechanics. In summary: (a) he gathered data on terrestrial and celestial motions, (b) endeavored to rationalize them through well-founded kinematics, and (c) searched for new explanations that could support his geo-heliocentric system—or potentially even the heliocentric one—but eventually abandoned this pursuit. Although Riccioli did not fully grasp the emerging principles of modern science, his effort to combine celestial advancements with ancient methodologies represents the most significant opposition to heliocentrism before Newton.

Data availability The authors declare that their manuscript has no associated data or the data will not be deposited.

Declarations

Conflict of interest The authors have no conflict of interests to state.

[28] Leibniz's ideas on gravity are examined in Bussotti 2015, Chapter 5. Bussotti-Lotti 2022 offers a wide panorama of the topics here presented in an outlined form regarding the origin of gravity and the relation between kinematics and dynamics in Descartes', Huygens', Newton's and Leibniz's theories.

References

Angeli, S. 1667. *Considerationi sopra la forza di alcune ragioni fisicomattematiche, addotte dal m.r.p. Gio. Battista Riccioli.* Venetia: appresso Bortolo Bruni.

Angeli, S. degli. 1668a. *Seconde considerationi sopra la forza dell'argomento fisicomattematico del m. reu. p. Gio. Battista Riccioli*. Padova: per Mattio Bolzetta de Cadorini.

Angeli, S. degli. 1668b. *Terze considerazioni sopra vna Lettera del ... signor Gio. Alfonso Borelli*. Venetia: appresso li heredi Leni.

Angeli, S. degli. 1669: *Quarte considerationi sopra la confermatione d'vna sentenza del sig. Gio. Alfonso Borelli...e sopra l'Apologia del m.r.p. Gio. Battista Riccioli...a fauore d'un suo argomento detto fisico-matematico contro il sistema copernicano*. Padova: per Mattio Cadorin detto Bolzetta.

Bertoloni Meli, D. (1988). Leibniz on the Censorship of the Copernican System. *Studia Leibnitiana*, *20*(1), 19–42. http://www.jstor.org/stable/40694091.

Bònoli, F., and A. Braccesi. 2001. Les recherches astronomiques de Gio. Domenico Cassini à Bologne: 1649–1669. In *Sur les traces des Cassini: Astronomes et observatoires du sud de la France*, ed. P. Brouzeng and S. Débarbat, 101–127. Paris: Éditions du CTHS.

Borelli, G.A. 1667. *De vi percussionis*. Bononiae: ex Typographia Iacobi Montij.

Borgato, M. T., Fiocca, A. 1994. *Giambattista Guglielmini. Carteggio* De Diurno Terrae Motu. *Canterzani, Isolani, Matteucci, Bonfioli Malvezzi, Caldani,* Calandrelli, Bonati. Firenze: Olschki, 1994.

Borgato, M.T. Pepe, L. 1999. *Giambattista Guglielmini, la biblioteca di uno scienziato nell'Italia napoleonica.* Ferrara: Corbo Editore.

Borgato, M. T. eds. 2002. *Giambattista Riccioli e il merito scientifico dei Gesuiti nell'età barocca*, Firenze: Leo S. Olschki.

Boulliau, I. *Astronomiae Philolaicae fundamenta clarius explicata*. Paris, 1657.

Bussotti, P. 2022. Kepler's Astronomy: An Interplay between Kinematics and Dynamics. In V. Zanini, A. Naddeo, F. Bonomi (Eds.), *Atti del XLI Convegno Annuale SISF* A – *Proceedings of the 41st Annual Conference SISFA - Arezzo 2021*, pp. 343–349. Pisa: Pisa University Press.

Bussotti, P., and B. Lotti. 2022. *Cosmology in the early modern Age. A Web of Ideas*. Cham: Springer.

Bussotti, P., Marcacci, F. (2024). *Kepler's laws with and without dynamics*.

Delambre, J.B.J. 1821. *Histoire de l'Astronomie Moderne*. Paris: Courcier.

De Pace, A. 2009. *Niccolò Copernico e la fondazione del cosmo eliocentrico*. Milano: Mondadori.

Dijksterhuis, E.J. 1986. *The Mechanization of the World Picture*. Princeton: Princeton University Press Clarendon Press. (1st Dutch ed. 1950. 1st English translation 1961).

Di Teodoro, E.M., R. Bedogni, and F. Bònoli. 2010. I primi esperimenti sulla caduta dei gravi: Galileo e Riccioli. *Giornale Di Astronomia* 36 (3): 32–40.

Dinis, A. 2017. *A Jesuit against Galileo? The strange case of Giovanni Battista Riccioli Cosmology*. Braga: Axioma, Publicaçoes da Faculdade de filosofia.

Evans, J. 1988. The division of the Martian eccentricity from Hipparchos to Kepler: A history of the approximations to Kepler motion. *American Journal of Physics* 56: 1009–1024.

Evans, J. 1998. *The History and Practice of Ancient Astronomy*. New York: Oxford University Press.

Feingold, M., ed. 2003. *The New Science and Jesuit Science: Seventeenth Century Perspectives*. Dordrecht: Springer Science + Business Media.

Galluzzi, P. 1977. *Galileo contro Copernico. Il dibattito sulla prova 'galileiana' di G. B. Riccioli contro il moto della Terra*, «Annali dell'Istituto e Museo di Storia della Scienza di Firenze» 2, 87–148.

Gambaro, I. 2021. Geo-heliocentric models and the Society of Jesus: From Clavius's resistance to Dechales's Mathesis Regia. *Annals of Science* 78 (3): 265–294.

Giard, L. 1995. *Les Jésuites à la Renaissance: Système éducatif et production du savoir*. Year: Presses universitaires de France.

Gingerich, O. 1993. The Mercury Theory from Antiquity to Kepler. In Gingerich O. (ed.) *The Eye of Heaven: Ptolemy, Copernicus, Kepler*. New York: American Institute of Physics, 379-387 (reprinted from *Actes du XIIe Congrès International d'Histoire des Sciences*, Paris, 1968, vol. 3A. Paris: Librairie Scientifique et Technique Albert Blanchard, 1971, 57-64)

Gingerich, O. 2011. Kepler's Trinitarian Cosmology. *Science and Theology* 9 (1): 45–51.

Granada, M.A. 2002. *Sfere solide e cielo fluido: Momenti del dibattito cosmologico nella seconda metá del Cinquecento*. Milan: Guerini.

Granada, M.A. 2009. Novelties in the heavens between 1572 and 1604 and Kepler's unified view of nature. *Journal for the History of Astronomy* 40 (4): 393–402.

Granada, M.A. 2010. "A quo moventur planetae?". Kepler et la question de l'agent du mouvement planétaire après la disparition des orbes solides. *Galilaeana* VII, 111–141.

Graney, C.M. 2015. *Setting Aside All Authority. Giovanni Battista Riccioli and the Science against Copernicus in the Age of Galileo*. University of Notre Dame Press: Notre Dame.

Grant, E. 1971. *Nicole Oresme and the Kinematics of Circular Motion: Tractatus de Commensurabilitate vel Incommensurabilitate Motuum Coeli*. Madison (Wis.): University of Wisconsin Press.

Grant, E. 1984. In Defense of the Earth's Centrality and Immobility: Scholastic Reaction to Copernicanism in the Seventeenth Century. *Transactions of the American Philosophical Society* 74 (4): 1–69. https://doi.org/10.2307/1006444.

Koyré, A. 1955. A Documentary History of the Problem of Fall from Kepler to Newton: De Motu Gravium Naturaliter Cadentium in Hypothesi Terrae Motae. *Transactions of the American Philosophical Society* 45 (4): 329–395. https://doi.org/10.2307/1005755.

Koyré, A. 1973. *The astronomical revolution: Copernicus – Kepler – Borelli*. London. Methuen; Ithaca (NY): Cornell University Press. (1st French ed., 1961).

Lerner, M.-P. 1980. L'Achille des Coperniciens. *Bibliothèque D'humanisme Et Renaissance* 42 (2): 313–327.

Marcacci, F. 2016. Stile argomentativo e dimostrazioni probabili. Considerazioni intorno all'epistemologia di Giovanni Battista Riccioli. *Physis* LI, Nuova Serie Fasc. 1–2, 2016, 357–368

Marcacci, F. 2018. *Cieli in contraddizione. Giovanni Battista Riccioli e il terzo sistema del mondo*. Modena-Perugia: Accademia Nazionale di Scienze Lettere e Arti di Modena - Aguaplano L'Officina del Libro.

Marcacci, F. 2019a. Lo statuto dell'astronomia e il metodo delle ipotesi secondo Giovanni Battista Riccioli. *Syzethesis* 1 (2019): 9–24.

Marcacci, F. 2019b. All the planets are related to the Sun. Riccioli and his "spiralized" skies. In B. Campanile, L. De Frenza, A. Garuccio (eds.), *Atti del XXXVII Convegno annuale (Proceedings of the 37th Annual Conference) SISFA – Bari 2017* (101–106). Pavia: Pavia University Press, 101–106.

Marcacci, F. 2021a. La scienza e l'ipotesi assoluta. Metodologia e logica della ricerca in Giovanni Battista Riccioli. *Les Archives Internationales D'histoire Des Sciences* 70: 72–107.

Marcacci, F. 2021b. Seeing at a Glance. The World-System Debate and the Role of the Comparative Tables in Giovanni Battista Riccioli's *Almagestum novum. Nuncius* (36), 2021, 119–142.

Marcacci, F. 2022. Understanding, Disseminating, and Interpreting Kepler: The Role of Giovanni Battista Riccioli and the Law of Orbits. In *Società italiana degli storici della fisica e dell'astronomia: Atti del XLI Convegno annuale (Proceedings of the 41st Annual Conference: Arezzo, 6–9 settembre 2021)*. Pisa: Pisa University Press, 32–39.

Marcacci, F. 2023. G.B. Riccioli's Geo-heliocentric Use of Epicepicycles, Ellipses and Spirals. *Journal for the History of Astronomy*, May 2023, 1–22

Marcacci, F. 2024. "La caduta degli angeli tra teologia e nuova Scienza." In P. Capitanucci, C. Coletti, S. Petrillo, A. Serra, *La natura e i suoi regni tra idea e rappresentazione (secoli XVI-XX)*. Napoli: Editori Guida, forthcoming.

Poulle, E., Gingerich, O. 1967. Les positions des planets au moyen âge : application du calcol electronique aux tables alphonsines. *Académie des inscriptions et belles-lettres comptes rendus des séances*, 531–548.

Riccioli, G.B. 1651. *Almagestum novum astronomiam veterem novamque complectens*, 2 voll. Bononiæ: Ex Typographia Hæredis Victorij Benatij.

Riccioli, G.B. 1665. *Astronomiae reformatae tomi duo*. Bononiæ: ex typographia hæredis Victorij Benatij, 1665.

Riccioli, G.B. 1668. *Argomento fisicomattematico... contro il moto diurno della Terra* (per la cura di M. Manfredi). Bologna: per Emilio Maria, e fratelli de' Manolessi.
Riccioli, G.B. 1669. *Apologia..pro argumento physicomathematico contra systema Copernicanum*. Venetia: apud Franciscum Salerni & Ioannem Cagnolini.
Romano, A. 1999. *La Contre-Réforme Mathématique. Constitution et diffusion d'une culture mathématique jésuite à la Renaissance (1540-1640)*. Rome-Paris : École française de Rome.
Schofield, C. 1981. *Tychonic and semi-Tychonic World Systems*. New York: Abaris Books.
Small, R. 1804. *An Account of the Astronomical Discoveries of Kepler*. London: Mawman.
Swerdlow, N.M., and O. Neugebauer. 1984. *Mathematical astronomy in Copernicus's De Revolutionibus. In two parts*. New York: Springer.
Thijssen, J.M.M.H., Luthy, C. (eds.) 2007. *Nicole Oresme's* De visione Stellarum *(On Seeing the Stars)*. Brill: Leiden-Boston.
Thordnike, L. 1949. *The Sphere of Sacrobosco and its Commentators*. Chicago: The University of Chicago Press.