International Journal of Theoretical Physics (2023) 62:54
https://doi.org/10.1007/s10773-023-05313-z

A Logico-Epistemic Investigation of Frauchiger and Renner's Paradox

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Received: 31 December 2022 / Accepted: 13 February 2023 / Published online: 6 March 2023

Abstract

The scientific literature on Wigner's Friend extended paradox rapidly grew in the last years. A sign that Frauchiger and Renner (2018)'s argument caught an important point. Indeed, they conclude that either we must abandon the universal validity of quantum mechanics, or a certain kind of traditional objective knowledge is impossible. We investigate this contradiction through a logico-epistemic toolbox. We show that abandoning the transmissibility of knowledge, as proposed by many kinds of relational approaches to quantum mechanics, is a heavy epistemological renouncement. Perhaps, it is better to bite the bullet and accept Frauchiger and Renner's contradiction, until a new revolutionary solution will appear.

Keywords Wigner's Friend extended paradox · Transmissibility of knowledge · Universality of quantum mechanics · Interpretation of quantum mechanics · Epistemic logic

1 Introduction

In a groundbreaking paper, Frauchiger and Renner [11] – FR hereafter – originally propose a more sophisticated version of Wigner's friend paradox [29]. In the original version of the paradox, a Friend of Wigner measures a dichotomic observable in a superposed state in an isolated laboratory, finding a certain result. In contrast, if Wigner, which is out of the laboratory, would apply quantum mechanics to the whole system – composed of the laboratory, his Friend and the measured observable – Wigner would find that the system his Friend is measuring inside the laboratory is still superposed. Thus, the physical situation is different for Wigner and his Friend. The conclusion is that, to avoid the contradiction if quantum mechanics is a *universal*[1] theory, the only reason why the Friend observes the collapse of the superposition could be the intervention of his consciousness on the physical system.[2]

[1] That is, quantum mechanics holds for every *physical* system.

[2] For a more complete presentation of Wigner's Friend, see Tarozzi [25].

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Indeed, at the appearance of the first decoherent models, Wigner changed his mind endorsing this new perspective [8].

Contrary to the original Wigner's paradox, Frauchiger and Renner's version has the form of a 'no-go theorem': it assumes some principles, and it shows how these assumptions imply a contradiction. We already reviewed FR's original paper elsewhere [3]. However, Renato Renner – one of the authors of the original paper – and Nuriya Nurgalieva published a new paper. Renner and Nurgalieva [19] (NR henceforth) clarify many obscure points of FR's original paper. These obscurities did not concern the argument, but the relationship of FR's conclusion with the current interpretations of non-relativistic quantum mechanics. In the present paper, we apply our logico-epistemic framework to this new presentation of FR's argument.

Before entering the subject, a couple of papers similar to the present one must be considered. Nurgalieva and Del Rio [18], independently of us, had the idea of recasting FR's argument in a logico-epistemic framework. They conclude that standard epistemic logic is not suitable to account for (knowledge of) quantum phenomena. Therefore, epistemic logic should be generalized. Since Putnam [20], many scholars attempted to defend the thesis according to which quantum mechanics would entail a change in logic. However, there is a simple reason why we side against such a point of view. As many authors in the quantum logic literature – most notably Marisa Dalla Chiara[3] – quantum theory favours the establishment of new logical languages – as testified by the variegated panorama of existing quantum logics – but it does not imply that our notion of rationality must change. In other words, quantum logic enlarges the realm of logic, but they do not substitute classical logic. For instance, the logic of quantum states in Hilbert's space is not Boolean, but this does not mean that we must adopt a non-Boolean logic when we are speaking about, say, the philosophy of science.[4] Something similar holds for epistemic logic as well. For, the normative rules holding for scientific knowledge and belief established by Hintikka [13] could not be lighthearted modified. We will come back to this point later on. Be it as it may, our attitude concerning the relation between epistemic logic and quantum theory is quite different: we would like to use the former to critically investigate the latter.

After Del Rio's and Nurgalieva's paper, another interesting result on our subject has been published by Boge [1], which shows that FR's argument is a (sort of) theorem of epistemic logic. Boge proves that there is a logical contradiction between FR's assumption and epistemic logic, both for the system KT45 and BD45. The problem with these proofs is that Boge assumes 5, which is not a reasonable request concerning scientific knowledge and belief. **5** has the form "$\neg Kp \rightarrow K\neg Kp$" and the same with B in the place of K. We briefly consider, in turn, the two axioms. Sometimes, 5 is called "negative introspection", because the axiom implies that who does not know p would know that s/he does not know p. In consideration of the by now clear importance of what has been dubbed "unknown unknown" this philosophical assumption is highly controversial [9]. For instance, considering examples coming from scientific research, is it reasonable to suppose that Darwin knew that he did not know the helicoidal structure of DNA? Clearly not. A bit more complex is the question about the principle "$\neg Bp \rightarrow B\neg Bp$". Indeed, at first sight, it seems that from a normative point of view, we must be aware of our beliefs when doing scientific research. In fact, in doxastic logic – as in epistemic logic – 4 holds, that is "$Bp \rightarrow BBp$". However, this

[3] See, for instance, Dalla Chiara, Toraldo di Francia [7:88].

[4] Fortin and Lombardi's [10] solution to FR's paradox seems to us to be based on the same point of view criticized above. We will not belabour this point in what follows.

is not the same as *negative* introspection. Again, with an example like the preceding one, is it reasonable to suppose that Darwin believed that he did not believe that DNA's structure is helicoidal? Clearly not.

Even if Boge's theorems are very interesting, it seems to us that he assumes too much. In justifying the use of KT45 and BD45, Boge quotes many computer scientists. It is well-known [27:14] that epistemic logical systems for computer scientists comprehend 5 – since, generally speaking, a machine has no problem implementing negative introspection – whereas this is not a reasonable assumption from the point of view of the philosophy of science.

Let us consider 5 from the semantic point of view as well. 5 corresponds to the symmetry of the accessibility relations between possible worlds. Symmetry would mean that if Alice looks at a possible world where she acquires either new knowledge or new beliefs, the actual world must be accessible to her in the new cognitive situation. This conclusion is highly implausible since, with the new pieces of knowledge or beliefs, Alice could have precluded her access to the actual world. Again, an example can clarify the point. Before Hubble's discovered that some nebulas are galaxies, we lived in a world from which two worlds were accessible: one in which Milky Way was the only galaxy and one in which there were many galaxies. In the new cognitive world, where we know that there are millions of galaxies, the old one becomes epistemically inaccessible.

After this brief introduction, we now turn to the core of the paper. We start by presenting Deutsch' [6] version of Wigner's Friend in "Deutsch's experiment". Then, we discuss, in "The assumptions of the new experiment", FR's assumptions from a logico-epistemic perspective. Next, we present, in "The new experiment" and "The inference", a simplified form of FR's argument. A brief discussion on how the different interpretations of quantum mechanics faces FR's paradox ends the paper ("FR impact on the interpretation of quantum mechanics"). Conclusions follow suit.

2 Deutsch's experiment

NR emphasizes that FR's argument is very useful to evaluate the different interpretations of quantum mechanics. We completely agree on this issue. And indeed, they repeatedly highlighted in the paper that FR's argument shows that physics, rather than philosophy, can decide among the different interpretations of quantum mechanics. This could be a good point, but, as we will show in what follows, our unsatisfaction with quantum mechanics is not only physical but philosophical as well.[5] The gist of our argument indeed is that FR sheds light once more on the fact that quantum mechanics presents important unsolved problems. This does not mean that we must abandon the theory, only that we must not forget that no theory is the end of the road of science.

Moreover, NR note that many thought experiments are based on a sort of double role of an entity, both as an agent and as a physical system. Indeed, this double role is present both in Maxwell's Demon and in Wigner's Friend. The Demon and the Friend are both agents and physical systems. Furthermore, the solution of Maxwell's Demon is a correct physical representation of the Demon's alleged agency. NR propose a definition of an agent as something

[5] In our opinion, the distinction between physics and philosophy of science must be intended respectively as nearer or farther to an experimental test.

that could be completely physical, as a machine, which measures and register the results of measurements. We stick with this definition. For it seems not a good idea to do physics with muddled concepts such as that of mind, consciousness, agency, etc. It is better to avoid such notions, not because they are irrelevant, but because there is not yet a good scientific definition of what we are speaking about.

Now, it is useful to dwell a little on NR reformulation of Deutsch's form of Wigner's Friend paradox. Suppose that Alice measures, in the so-called computational basis $|1\rangle$ and $|0\rangle$, the state:

$$|+\rangle_R = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \tag{1}$$

Leaving apart the environment for the sake of simplicity, let us call "Z" the considered observable, and then Alice will find that either $Z=0$ or $Z=1$. On the contrary, Wigner, applying quantum mechanics to Alice's isolated lab, will find, with an obvious notation:

$$|+\rangle_{lab} = \frac{1}{\sqrt{2}}(|1\rangle\,|\,registered\,Z=1\rangle + |0\rangle\,|\,registered\,Z=0\rangle) \tag{2}$$

Wigner's Friend paradox is simply the diversity between (2) and the determinate result observed by Alice. Deutsch adds another registration instrument shared by Wigner and Alice: let us call it the "*book*". Alice, after having measured, writes in the *book* that she gets an outcome – without specifying which. Therefore, the *book* could be either empty or filled. After Wigner received the information that Alice has measured, he switches on a control, which undoes the measurement in the lab. In principle, even if not in practice, this is possible. After making this inversion, the possible results are two: either in the lab – after Alice's measurement – there was still an actual superposition, then the undoing brings back the *lab* in the state:

$$|+\rangle_R\,|\,ready\rangle \tag{3}$$

Or, if the state is no longer superposed, Wigner will find either

$$|1\rangle\,|\,registered\,Z=1\rangle \text{ or } |0\rangle\,|\,registered\,Z=0\rangle \tag{4}$$

This in principle possible experiment would discriminate physically between different interpretations of quantum mechanics. Indeed, if Wigner will find one of the states (4), we must find a good physical explanation of the collapse. On the contrary, if Wigner finds (3'), the many-worlds interpretation, that is quantum mechanics without collapse, is favoured.

From this last statement, it is possible to catch a glimpse of how the contradiction is built in FR's argument. Indeed, if one finds a method to bring outside the information that Alice achieved an outcome, maintaining that of the superposition of the whole system, the conceptual tension between Alice's and Wigner's point of view – typical of Wigner's Friend experiment – would be wholly ascribed to Wigner. In this way, the tension becomes a contradiction in Wigner's knowledge. The experimental set proposed by FR reaches exactly this aim.

3 The assumptions of the new experiment

To reach the result of having both the cake – superposition – and eating it – Alice's outcome – we must double Wigner's experiment. Therefore, we have two isolated labs – Alice's and Bob's – and two external experimenters – Wigner and Bell. To reach the

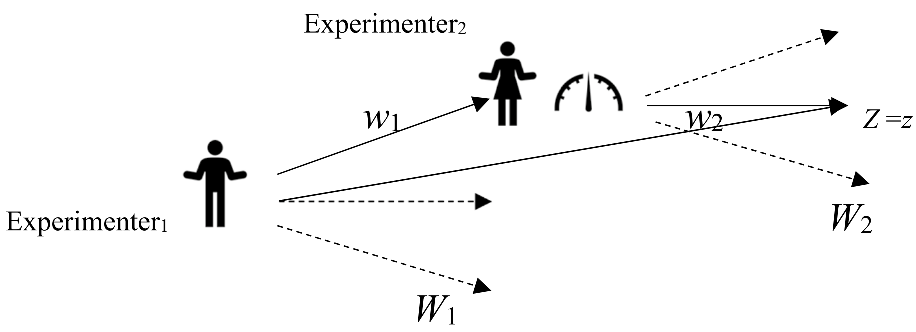

Fig. 1 Pictorial of principle **C**

situation in which Bell eat the cake and has it – the contradiction – there must be partial communication among the four experimenters. Not only, but they must also make some inferences. These inferences, as we will see, could be performed also by a machine. Therefore, no kind of human direct intervention is involved. Moreover, we will also see that these inferences are logically indisputable.

The whole argument is based on three (main) assumptions. The first is:

Q. The agents implement quantum mechanical rules to describe *each* possible system and they predict and retrodict outcomes using Born's rule.

Note that in the original FR's paper, Q was not assumed in this universal form. This point is highlighted by Nurgalieva and Del Rio [18], that criticize FR, emphasizing that the argument works only on this universal basis. Q means that if the outcome of a given observable Z assumes with certainty the value z – based on quantum mechanics – then $Z=z$ is certain.

At this point, it is useful to establish which interpretations of quantum theory violate Q. Copenhagen interpretation clearly does,[6] since it distinguishes between what is quantum and what is classical. On the contrary Q – as already said – must be universal. Moreover, Bohmian mechanics is, for different reasons, also against Q, since at the micro level many unknown facts happen, which are not registered by the Born rule.[7] Finally, all objective collapse theories, such as GRW, do not accept Q since they maintain that at the macroscopic level quantum mechanics does not hold.[8] On the contrary, Q is satisfied by the Many-worlds interpretation.

The second assumption of FR's paradox is:

C. If an experimenter_1 is certain that another experimenter_2 has found "$Z=z$", then experimenter_1 is certain that "$Z=z$".

[6] NR correctly distinguishes the different forms of "Copenhagenism": the subjective Copenhagenist puts Heisenberg's cut where it is most convenient, so perhaps, one can maintain that s/he respects Q. On the contrary, the objective Copenhagenist puts the cut in a determinate position, therefore violating Q.

[7] In fact, there are possible Bohmian answers to FR's argument. See Sudbery [23], Tausk [24], Lazarovici and Hubert, [15].

[8] Also at the microscopic level, but it is not statistically relevant.

Let us consider a justification of **C** based on epistemic logic (see Fig. 1). Suppose that W_2 is the set of worlds epistemically accessible to experimenter$_2$. If experimenter$_2$ knows that "$Z=z$", then in each w belonging to W_2 "$Z=z$" holds. Let us consider the set of worlds epistemically accessible to experimenter$_1$, that is, W_1. If experimenter$_1$ knows that experimenter$_2$ knows that "$Z=z$", then in each w belonging to W_2 holds that "experimenter$_2$ knows '$Z=z$'". Now, take any w belonging to W_1; in it "experimenter$_2$ knows '$Z=z$'" holds, therefore if w belongs to W_2, then in w "$Z=z$" holds, therefore experimenter$_1$ knows that "$Z=z$".

This argument shows that **C** could be violated only if:

1. There is no world accessible to experimenter$_1$.
2. 2 there is no world accessible to experimenter$_2$.
3. $W_1 \cap W_2 = \varnothing$.

1. and 2. trivialize the notion of knowledge, since they hold if and only if the truth of a sentence is equivalent to the knowledge of it; that is, iff "$p \leftrightarrow Kp$". For this reason, 1. and 2. must be discarded, as possible cognitive situation. On the other hand, 3. is less radical, but it seems like Gorgia's strongly sceptical third *dictum* attributed to him: "He says that nothing is; and if [scil. something] is, it is unknowable; and if [scil. something] both is and is knowable, *it cannot be indicated to other people*." (our italics, LM, D26). Indeed, even 3. seems untenable for a non-sceptical theory of knowledge. Therefore, we conclude that **C** – sometimes called "transmissibility of knowledge" – is a very solid principle.[9]

Despite this, **C** is explicitly denied by QBist [5]. Furthermore, according to NR, all relational approaches in a certain sense reject **C**. Indeed, Rovelli's relational quantum mechanics (RQM) and Copenhagen's subjective interpretations seem of this kind.[10]

Considering the above argument we proposed in support of **C**, it seems that abandoning **C** boils down to renouncing to do science in a certain intersubjective way. For this reason, we conclude that this kind of interpretation after FR must be accepted only after a long investigation. We will come back to this point in what follows.

The last central assumption of FR's paradox is:

S. An experimenter registers only one value of a measured observable.

In FR, it appeared that **S** could be eliminated in the Many-worlds interpretation. Moreover, since **Q** and **C** seem not negotiable, the conclusion could be that FR favoured Many-worlds. On the contrary, NR maintain that **S** is accepted by all standard interpretations of quantum mechanics. For this reason, we will no longer be concerned with this premise.

We conclude this part with a general consideration. All three premises of FR's argument could be physically implemented. That is, one can imagine machines able to realize the tasks they represent. Therefore, assuming **Q**, **C** and **S** does not mean exiting from the realm of physics. It is relevant to note that all three assumptions have also a normative character, in the sense that they could be thought of as valid principles. And their

9 A classical investigation – and defence – of the transmissibility of knowledge is 12, Ch. 15).

10 Indeed, Yang [30] and Waaijer and Neerven [28] defend RQM exactly on this argumentative line. The latter proposes to substitute **C** with a much weaker form of knowledge transmission, which considers it impossible to transmit knowledge concerning what is no longer actual. According to this principle, it would be impossible to do history as a scientific enterprise.

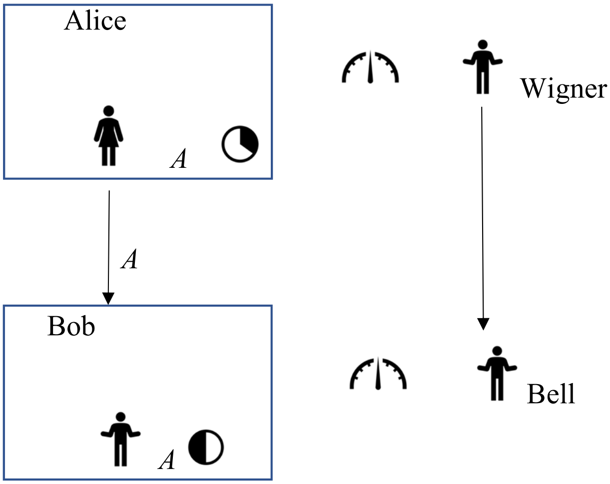

Fig. 2 Pictorial of FR's paradox

normative character is an important part of the thought experiment since from them it is possible to deduce a contradiction. This means that – as emphasized by NR – physics proposes to us that there is a problem. Physics, not only philosophy, because the three principles could be physically implemented. Therefore, it seems that our theory makes possible something physically impossible.

4 The new experiment

In Fig. 2, the reader can see a pictorial of the experimental setting. Alice and Bob stay in their respective *isolated* labs. They interact only when Alice sends Bob the state A. Out of Alice's lab there is Wigner with a hypothetic instrument to measure Alice's lab. Out of Bob's lab, there is Bell with a hypothetic instrument to measure Bob's lab in the basis $|ok\rangle_B$, $|fail\rangle_B$. Wigner can communicate with Bell.

First step, $t=1$.

Alice measures the state A:

$$\frac{|0\rangle}{\sqrt{3}} + \frac{2|1\rangle}{\sqrt{3}} \tag{5}$$

in the computational basis $|0\rangle$, $|1\rangle$. Alice registers the result (application of **S**). Then, Alice prepares the new state A according to this rule:

If she found $|0\rangle$ she prepares $|0\rangle$.
If she found $|1\rangle$ she prepares the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

Finally, Alice sends A to Bob in the other lab.

Second step,$t=2$.

Bob measures A in the computational basis. Bob will find $|\,0\rangle$ with probability 2/3 and $|\,1\rangle$ with probability 1/3. Bob registers the result obtained.

Third step, $t=3$.

Wigner measures Alice's lab in the basis:

$$\begin{aligned} |\,ok\rangle_A &= \tfrac{1}{\sqrt{2}}\big(|\,lab_{A0}\rangle - |\,lab_{A1}\rangle\big) \\ |\,fail\rangle_A &= \tfrac{1}{\sqrt{2}}\big(|\,lab_{A0}\rangle + |\,lab_{A1}\rangle\big) \end{aligned} \tag{6}$$

where $|\,lab_{A0}\rangle$ and $|\,lab_{A1}\rangle$ are the states of Alice's lab in the respective cases in which she finds $|\,0\rangle$ and $|\,1\rangle$. Wigner registers the result (application of **S**).

Fourth Step, $t=4$.

Bell measures Bob's lab in the basis:

$$\begin{aligned} |\,ok\rangle_B &= \tfrac{1}{\sqrt{2}}\big(|\,lab_{B0}\rangle - |\,lab_{B1}\rangle\big) \\ |\,fail\rangle_B &= \tfrac{1}{\sqrt{2}}\big(|\,lab_{B0}\rangle + |\,lab_{B1}\rangle\big) \end{aligned} \tag{7}$$

where $|\,lab_{B0}\rangle$ and $|\,lab_{B1}\rangle$ are the states of Bob's lab respectively in the cases Bob finds either $|\,0\rangle$ or $|\,1\rangle$. Bell registers the result of his measurement (application of **S**).

5 The inference

Let us consider the case in which Alice finds $|\,1\rangle$. Therefore, she prepares L in the state:

$$\frac{1}{\sqrt{2}}(|\,0\rangle + |\,1\rangle) \tag{8}$$

This means that Alice can deduce that Bob's state must be (application of Q):

$$|\,fail\,\rangle_B = \frac{1}{\sqrt{2}}\big(|\,lab_{B0}\rangle + |\,lab_{B1}\rangle\big) \tag{9}$$

Hence, Alice concludes that Bell will find $|\,fail\rangle_B$ (application of S).

Therefore, Alice can register that she found $|\,1\rangle$ and Bell certainly will find$|\,fail\rangle_B$.

Bob received (7), therefore she can conclude from the setting of the experiment that Alice measured $|\,1\rangle$. Bob, applying Q and S can establish that Alice knows that Bell will find$|\,fail\rangle_B$. Hence, by applying C, Bob can conclude that Bell will find $|\,fail\rangle_B$.

Let us now consider the case in which Wigner observes$|\,ok\rangle_A$. It is possible – although not trivial – to show that in those cases, Wigner, on the basis of Q, can show that Bob measured $|\,1\rangle$. Indeed, by applying Q, S and C, Wigner can be certain that Bell will find$|\,fail\rangle_B$. Wigner *communicates* this result to Bell, which is now sure of finding $|\,fail\rangle_B$ (by applying C).

Let us consider the cases in which Wigner finds $| ok\rangle_A$ and Bell finds$| ok\rangle_B$. It is not difficult to show that these couple of results happen with probability 1/12, that is, 1 time every 12 runs of the experiment on average. This means that sometimes Bell must accept a contradiction, that is he knows that the result of his measurement must be$| fail\rangle_B$, but he finds$| ok\rangle_B$. On the other hand, if he finds $| ok\rangle_B$ he knows that the outcome is $| ok\rangle_B$ (application of **S**). Therefore, Bell knows that on average 1 time out of 12 the result of his experiment is both $| fail\rangle_B$ and$| ok\rangle_B$, against **S**. Contradiction!

6 FR impact on the interpretation of quantum mechanics

As already emphasized in "The assumptions of the new experiment", many approaches deny Q: viz. Objective Copenhagen interpretation, Bohmian mechanics and Collapse theories. The problem with these perspectives is that in more than half a century of research we did not find a trace of evidence favouring the limit of applicability of quantum mechanics. Recently, also the case raised by Hawking against quantum mechanics in black hole physics seems deflated [4].

As emphasized also by NR (p. 193) "However, with the development of quantum technologies, more and more complex systems are investigated, and so far no indications have been found that quantum theory could be inaccurate on larger scales." This fact compels us to take into serious consideration that our capacity of knowing cannot respect C.

On the contrary, many approaches do not pose limits to the applicability of quantum mechanics: Relational quantum mechanics, Many-worlds interpretation, Subjective Copenhagen interpretation and QBism. These interpretations of quantum mechanics reject C, albeit they do so in different ways. As shown in "The assumptions of the new experiment", these approaches endorse a sceptical point of view. Scepticism, like several interesting philosophical theses, could be tested empirically [26]. Indeed, it could be that certain features we believe knowledge must have – as C, for instance – are not achievable. In other words, it could be that the large empirical confirmation of quantum mechanics is saying to us that it is not possible to reach the form of shared knowledge we aimed at. Nevertheless, before accepting this sceptical conclusion one must investigate every alternative. As shown by Don 14, pp. 245 ff.), the most important reason why Einstein did not trust quantum mechanics was exactly the relational character of their states, which makes objective knowledge problematic. That Einstein was not persuaded of the possibility of making physics in this relational context is not an argument. Indeed, Howard himself in the following section (*ibidem*), and Muller [17], advocate a structural metaphysics for quantum mechanics. Nevertheless, it must be emphasized that if quantum entities would be relations, this seems to pose serious limitations to our objective knowledge, as shown by the influence of this fact on the validity of C. Perhaps an example can clarify all the scepticism implicit in the physical negation of C.

Let us consider three different physical systems S_1, S_2 and S_3. Between S_1 and S_2 the *external* and asymmetrical relation "*knows*" holds: "S_2 *knows* S_1." "External" means that *knows* is not reducible in any sense to the properties of S_1 and S_2. Between S_1 and S_3 *knows* holds as well; that is "S_3 knows S_1." This set makes it possible that S_2 and S_3 know different things about S_2. Here it is the scepticism favoured by the universality of quantum mechanics.

QBism and RQM deny explicitly C, but other relational approaches are in the same boat. For instance, as emphasized by NR, also Many-worlds must deny C, in the sense that what is known in one branch cannot be known in another.[11] In other terms, if it is not possible to ascribe a state to something independently of the measurement apparatus, at the end of the day, objective knowledge as we conceived it before the advent of quantum mechanics is not an achievable goal.

There is a peculiar many-worlds interpretation, which deserves our attention. Zurek [31] attempts ante litteram to give body to the idea that C must be violated. More precisely, in his perspective, the "worlds" are only the different aspects of a physical system an experimenter with her apparatus can register. In other words, an apparatus applied to a certain system establishes a division between what is apparent and what must be dispersed in the other "worlds", that is in the environment. In this perspective, the same physical system would differently appear to different observers for precise physical reasons. Nevertheless, as emphasized also by Schlosshauer [22], decoherence is not a general solution to the measurement problem in quantum mechanics, but an ongoing program of opening the black box orthodox perspective had built on the collapse of the wave function.

7 Conclusion

Then, what we must do? Are we sure that we must accept the scepticism implicit in the negation of **C**? The history of physics presents plenty of cases in which our knowledge of physical phenomena radically changed. For instance, Aristotle thought the violent movement must be necessarily connected with a mover – with the theological exception of the unmoved mover. This position probably depended on the fact that he did not have the notion of inertial mass. Therefore, the discovery of inertial mass by Galilei, Descartes and Newton considered movement a non-relational fact. Why something similar cannot happen in the case of the relational character of quantum entities?

Perhaps we can learn a lesson from EPR story. Indeed, the problem about nonlocality raised by EPR has been resolved in favour of quantum non-separability, else the ascription of the Nobel Prize to Aspect, Clauser and Zeilinger for having confirmed the violation of Bell's inequality would have not occurred. Nevertheless, Maldacena [16] presents the ER-EPR conjecture as a sort of explanation of Bell's correlations. Indeed, it could be that where there are correlations, there is a spacetime deformation, which connects (apparently) distant spacetime points. This perspective is highly speculative, but it shows that the progress of physics often follows unexpected roads. The same happened with the action at distance introduced by Newton in 1686 and explained by Einstein in 1915. If Maldacena is right, exactly as in the case of Newton's gravity, the weirdness of Bell's correlation will be explained in an altogether surprising framework. Why the same could not happen in the case of the so-called measurement problem?

FR show once more that there is a conflict between the universality of quantum mechanics and the determinateness of our knowledge of measurement outcomes. We have no serious experimental reason to abandon Q, and neither to abandon C seems a mild epistemological renounce. There is a third possibility on the table: *we must learn to live with the contradiction*. FR show us that there is a contradiction between the universality of quantum mechanics and a certain form of objective knowledge. It is not necessary to hurry up in abandoning one of the two horns of the dilemma. Science is always *in fieri*. And this

[11] The relationship between Many-worlds and **C** is quite more complex than how we put it here. For a throughout discussion on this point, see Corti et al. [3].

becoming is characterized by long periods during which contradictions are in the agenda. Another example is the one emphasized by [21], cap. 5): today's student of physics in the first lecture of quantum field theory learns that spacetime is flat, whereas in the second lecture of general relativity s/he realizes that spacetime is curved.[12] This does not mean that we must accept contradiction; we must accept only that for a certain period of research we live with a contradiction. We are not endorsing dialetheism (Priest et al., 2022), that is the thesis that reality is contradictory, but only that the representation of our knowledge at a certain time could be [2].

Author contributions Vincenzo Fano wrote the first draft of the paper, and prepared the figures. Preliminary work on Wigner's paradox has been carried out by Gino Tarozzi; the formal aspect of epistemic logic and its application to the quantum domain has been done by Vincenzo Fano. The implications for the different interpretations of quantum mechanics is the result of the work of Alberto Corti. The main thesis of the paper and the analysis of the paradox is the result of the shared work of the three authors, as the paper has been revised several times by each co-author.

Funding Open access funding provided by Università degli Studi di Urbino Carlo Bo within the CRUI-CARE Agreement.

Declarations

Competing interests The authors declare no competing interests.

References

1. Boge, F.J.: Quantum information versus epistemic logic: An analysis of the frauchiger–renner theorem. Found. Phys. **49**(10), 1143–1165 (2019). https://doi.org/10.1007/s10701-019-00298-4
2. Brown, B. (1992): "Old quantum theory: A paraconsistent approach", in PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association (Vol. 1992, No. 2, pp. 397–411), Philos. Sci. Assoc.
3. Corti A., Fano V., Tarozzi G.: "Wigner's friend extended thought experiment. A philosophical analysis", Isonomia. https://isonomia.uniurb.it/wp-content/uploads/2022/05/Corti_Fano_Tarozzi_Wigner_Friend_Isonomia_Epistemologica_Maggio_2022.pdf. (2022)
4. Cinti, E., Sanchioni, M.: The Devil in the (Implicit) Details. Int. J. Theor. Phys. **60**(9), 3234–3253 (2021)
5. DeBrota, J.B., Fuchs, C.A., Schack, R.: Respecting one's fellow: Qbism's analysis of Wigner's friend. Found. Phys. **50**(12), 1859–1874 (2020)
6. Deutsch, D.: Quantum theory as a universal physical theory. Int. J. Theor. Phys. **24**(1), 1–41 (1985). https://doi.org/10.1007/BF00670071
7. Dalla Chiara, M. L., & di Francia, G. T.: Le teorie fisiche: un'analisi formale. Bollati Boringhieri (1981)

[12] Indeed, this contradiction is milder than that proposed by FR, since it concerns the relation between two different theories.

8. Esfeld, M.: Essay review Wigner's view of physical reality. Stud. Hist. Philos. Mod. Phys. **30**, 145–154 (1999)
9. Fano V., Graziani P.: "A Working Hypothesis for the Logic of Radical Ignorance", first on line, Synthese, https://doi.org/10.1007/s11229-020-02681-5 (2020)
10. Fortin, S., Lombardi, O.: The Frauchiger-Renner argument: A new no-go result? Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **70**, 1–7 (2020)
11. Frauchiger, D., Renner, R.: Quantum theory cannot consistently describe the use of itself. Nature Commun. **9**(1), 3711–3721 (2018). https://doi.org/10.1038/s41467-018-05739-8
12. Hendricks, V. F.: The convergence of scientific knowledge: a view from the limit (Vol. 9), Springer Science & Business Media (2001)
13. Hintikka, J.: Knowledge and Belief. Cornell University Press, Ithaca (1962)
14. Howard, D.: Anche Einstein gioca a dadi. Carocci, Roma (2015)
15. Lazarovici, D., Hubert, M.: How quantum mechanics can consistently describe the use of itself. Sci. Rep. **470**(9), 1–8 (2019)
16. Maldacena, J.: Black holes, wormholes and the secrets of quantum spacetime. Sci. Am. **315**(5), 26–31 (2016)
17. Muller, F.A.: The rise of relationals. Mind **124**(493), 201–237 (2015)
18. Nurgalieva, N., del Rio, L.: "Inadequacy of modal logic in quantum settings", *Electronic Proceedings in Theoretical Computer*. Sci. **287**, 267–297 (2019). https://doi.org/10.4204/EPTCS.287.16
19. Nurgalieva, N., Renner, R.: Testing quantum theory with thought experiments. Contemp. Phys. **61**(3), 193–216 (2021)
20. Putnam, H.: Is logic empirical? In: Boston studies in the philosophy of science, pp. 216–241. Springer, Dordrecht (1969)
21. Rovelli, C.: Reality is not what it seems: The journey to quantum gravity. Penguin (2018)
22. Schlosshauer, M.: Quantum decoherence. Phys. Rep. **831**, 1–57 (2019)
23. Sudbery, A.: Single-world theory of the extended Wigner's Friend experiment. Found. Phys. **47**(5), 658–669 (2017). https://doi.org/10.1007/s10701-017-0082-7
24. Tausk, D. V.: "A brief introduction to the foundations of quantum theory and an analysis of the Frauchiger-Renner paradox", arXiv preprint arXiv:1812.11140 (2018)
25. Tarozzi, G.: The theory of observations, Wigner' paradox and the mind-body problem. Epistemologia **4**, 37–52 (1981)
26. Tarozzi, G.: Science, Metaphysics and Meaningful Philosophical Principles. Epistemologia **11**, 97-104–229-23 (1988)
27. Van Ditmarsch, H., van der Hoek, W., Halpern, J. Y., & Kooi, B.: Handbook of epistemic logic. College Publications (2015)
28. Waaijer, M., Neerven, J.V.: Relational analysis of the Frauchiger-Renner paradox and interaction-free detection of records from the past. Found. Phys. **51**(2), 1–18 (2021)
29. Wigner, E. P.: Remarks on the mind-body question. In I. J. Good. (ed.) The Scientist Speculates. pp. 284–302. Heineman (1961)
30. Yang, J.M.: Consistent descriptions of quantum measurement. Found. Phys. **49**(11), 1306–1324 (2019)
31. Zurek H. W.: "Decoherence and the Transition from Quantum to Classical—Revisited", https://arxiv.org/abs/quant-ph/0306072 (2003)