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Can variable elasticity of substitution explain changes in labor shares?

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ABSTRACT

In CES production functions, the magnitude of the elasticity of substitution between capital and labor (σ) is crucial to explain the evolution of the labor share. The decline in labor share observed worldwide can be explained by capital accumulation if $\sigma > 1$. However, empirical evidence on the value of σ is mixed. To shed light on this issue, we employ a Variable Elasticity of Substitution (VES) production function where σ is an *endogenous* driver of the labor share. Using macro data for six advanced OECD economies from 1980 to 2020 we provide estimates of σ under *imperfect competition*. We test the prediction of the model by means of simulations. Mainly, we find that capital deepening, markup and technological change explain a significant part of the observed decline in labor shares. The results suggest *complementarity* between labor and capital in all the countries except the United States.

1. Introduction

The constancy of the labor share is a crucial feature of macroeconomic models, "*with broad implications for the shape of the production function, inequality, and macroeconomic dynamics*" (Karabarbounis and Neiman, 2014, p.61). Recently, the labor share has been on a downward trend in many countries. Although debate remains on the extent of this phenomenon due to technical considerations such as the treatment of capital depreciation, indirect taxes, and housing (Rognlie, 2016; Bridgman, 2018; Cette et al., 2019), the apportionment of mixed income (Cette et al., 2019; Gutiérrez and Piton, 2020), and intangible capital (Koh et al., 2020), the consensus is that the fall has been significant.

Ricardo referred to the functional distribution of income as "*the principal problem of political economy*" (Ricardo, 1891). Thereafter, the topic was debated in economic theory until the work of Kaldor's (1961), when its "stylized facts" were accepted by economists. For Kaldor, in a *full employment* equilibrium, real wages end up matching productivity gains, with the result that, in the long run, the labor share can only be constant. Therefore, issues regarding the macro-division of income were marginalized and the focus of economists shifted from *distribution* to *growth*.

For more than forty years, the labor share in industrialized countries never ventured far from 68% of national income, remaining stable through expansions, recessions, high and low inflation, and the transition from production-based to service-based economies. Keynes (1939, p.48) referred to this fact as "*one of the most surprising, yet best-established, facts in the range of economic statistics*". Consequently, for a long period of time, factor shares were considered constant, which led to a reduction in the role of income

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distribution in academic discussions (Atkinson, 2009).[1]

The recent worldwide decline in labor share and growing income inequality have brought new life to this field of research. Blanchard (1997) was the first to notice a decline in the share of income earmarked for the remuneration of labor in several European countries. Karabarbounis and Neiman (2014) documented that the global labor share has declined significantly since the early 1980s, with the decline occurring within most countries and industries. The downward trend was later confirmed by Piketty and Zucman (2014), who also noted an increase in capital shares.

The CES production function, with constant returns to scale and competitive markets, predicts a stable relationship between the labor share and capital accumulation, with the value of the (constant) *elasticity of substitution* between capital and labor (σ) which determines (by being greater or lower than one) a decrease (increase) in *labor share* as the *ratio of capital to output* in the economy increases. Empirical evidence on the subject, however, is conflicting. For example, it was observed that capital-output ratios and labor shares increased together in most advanced economies before 2000, suggesting a σ lower than one (Bentolila and Saint-Paul, 2003; Saumik, 2019). But their movement had contrasting signs since then (Table A1). Similarly, Karabarbounis and Neiman (2014) and Piketty and Zucman (2014) estimate σ to be greater than one, in contrast to a large micro-literature that, exploiting time series and variation across firms, documents an average σ of less than one (Chirinko and Mallick, 2017; Oberfield and Raval, 2021).[2] In this paper we focus on this puzzle.

We employ a Variable Elasticity of Substitution (VES) production function that allows the elasticity of substitution to *change over time*. Notably, this may reconcile different estimates of σ with the observed evolution of the capital-output ratio. In fact, with a VES production function, we can observe a declining labor share even when σ is lower than one. In addition, a VES can reduce the upward bias of σ resulting from the estimation of a CES, where the latter "*includes part of the variation due to the capital-labor ratio in the average product*" (Kazi, 1980, p.169). We test the prediction of the model by means of simulations. Using macro-data for six industrialized OECD countries over a forty-year period we estimate the structural parameters of the VES production function and calibrate the model on the actual level of the variables. We find that changes in wages and capital intensity, profit margins and technological change explain a significant part of the decline in labor share over the past forty years. Notably, the results suggest *complementarity* between labor and capital in all the countries considered except the United States (U.S.). Considering these results, we also draw some important *policy implications*. Whether policymakers should pursue the goal of capital deepening depends on how they evaluate the benefits of a higher σ in the form of *efficiency* and *economic growth* against the loss inflicted on the economy due to the exacerbation of *inequality*.

2. Literature review

Several attempts have been made by the literature to explain the non-constancy of the labor share in the medium run. However, a consensus has yet to be reached. Recently, two assumptions of neoclassical economics have been questioned, namely the production technology assuming the Cobb-Douglas (CD) functional form and the perfect competitiveness of markets. These assumptions underlying the Solow-Swan growth model imply that the *equilibrium labor share* is *constant* over time. Since this is not the case and a declining trend is observed, new models shift away from the CD and employ a general Constant Elasticity of Substitution production function (CES) that allows for an elasticity of substitution between capital and labor different from one (Arrow et al., 1961). When firms produce with CES technology, even if markets are competitive, the *labor share* can be expressed as a function of the *capital-output ratio (K/Y)* and eventually, evolve over time. The labor share is an increasing or decreasing function of K/Y depending on the elasticity of substitution between factors of production (σ). In other words, the parameter σ links K/Y to the labor (capital) share. It is this latter which determines how much the rate of return on capital falls when K/Y rises. The resulting "*share-capital schedule*" (Bentolila and Saint-Paul, 2003, p.5) is not altered by changes in real wages, capital accumulation, or labor-augmenting technological progress. This means that shocks in capital-augmenting technological progress and factors that generate a gap between the marginal product of labor and real wages are the main candidates to explain shifts in the relationship (Bentolila and Saint-Paul, 2003). When everything is constant, labor share dynamics can only occur if the economy is off its balanced growth path, meaning that capital and output are not growing at the same rate. In this case, the positive correlation between the capital-output ratio and the labor share is the other side of a negative correlation with capital productivity (Sala and Trivín, 2014). The role of *capital accumulation* has received much attention from economists, and has been studied, among others, by Piketty (2014) and Piketty and Zucman (2014) where the latter is the engine of income inequality, and Karabarbounis and Neiman (2014) who focused on the role of relative prices.[3] A fall in the relative price of investment goods, has induced firms to replace workers with machines. New capital goods have become cheaper and increasingly able to replace workers in their routine activities. Therefore, in response to capital accumulation, and because of low diminishing returns, the return on capital did not adjust sufficiently downward and this led to an increase in the capital share of income.

However, a consistent body of empirical literature estimates σ to be actually below one, implying a greater degree of complementarity between capital and labor. Thus, from this perspective, another group of studies has emphasized the role of *market imperfections* as explanatory factors for the decline in labor shares.

[1] For instance, most business cycle research is performed assuming *constant factor shares* (Choi and Rios-Rull, 2009).

[2] "It is natural to imagine that [the elasticity of substitution between labor and capital in a two-factor, one-commodity neoclassical model] was less than 1 in the 18th-19th centuries and became larger than 1 in the 20th-21st centuries. One expects a higher elasticity in diversified economies where capital takes many forms" (Piketty and Zucman 2014).

[3] Note that since Bentolila and SaintPaul (2003) control for the capital-output ratio they implicitly control for investment and consumption prices, the main findings of Karabarbounis and Neiman (2014) are hence enclosed.

Even when the production technology is CD, movements in factor shares can be the result of changes in the bargaining power of workers induced by changes in government policy, labor market institutions and/or the monopoly power of firms. If markets are not competitive, i.e., there is market power in the labor market and potentially the goods market, bargaining power between capital and labor determines factor income distribution besides capital intensity and capital augmenting technological change (Blanchard and Giavazzi, 2003). The literature has emphasized the role of product market competition (Azmat et al., 2012; Barkai, 2020; Autor et al., 2020) and labor market institutions (Bentolila and Saint-Paul, 2003; Bental and Demougin, 2010). Azmat et al. (2012) showed how privatization exerted strong downward pressure on the labor share in European industries, only partially offset by increased competition in the product market. Barkai (2020), focusing on the U.S., found a negative relationship between changes in labor share and changes in market concentration. Finally, Autor et al. (2020), using data from U.S. firms, showed that market concentration increases when industries become dominated by "superstar firms" with higher profit margins and lower labor shares.

In *marginal productivity theories*, a crucial role is played by the *elasticity of substitution between capital and labor*. The elasticity represents the set of technically feasible combinations of inputs to produce a certain amount of output. Intuitively, it can be regarded "*as a measure of the efficiency of the productive system*" (La Grandville, 1989, p.479). Macroeconomic models deliver different implications depending on the value of this parameter. The degree of substitutability influences the response of investments to variation in the interest rate (Chirinko, 2008), the relation between technology shocks and hours worked (Cantore et al., 2017), and notably, income distribution (Piketty, 2014). Despite awareness of the importance of σ for a wide range of theoretical and empirical issues, there is no agreement on its value in the short run or over longer time periods (Chirinko, 2008; McAdam, 2016; Federici and Saltari, 2018; Gechert et al., 2021).

The estimation of σ has challenged many researchers. Arrow et al. (1961), and a voluminous literature exploiting time-series and cross-firm variation (Chirinko, 2008; Klump and Papageorgiou, 2008; Oberfield and Raval, 2021) found an elasticity of substitution below unity (≈ 0.6), thus documenting *gross complementarity* between labor and capital. However, recently, empirical studies exploiting cross-country variation in factor shares (Duffy and Papageorgiou, 2000; Karabarbounis and Neiman, 2014; Piketty and Zucman, 2014) provided evidence for a σ of about 1.25 which imply *gross substitutability*. This leaves a puzzle and probably means that σ is *non-unitary* and *changes over time* (Pereira, 2003; Palivos, 2008).

Issues concerning σ used to be relevant in the 1960s, when, following Arrow et al. (1961), there was an increase in research devoted to the study of production functions. At the time a whole new class of production functions characterized by *variable elasticity of substitution* was developed by Lu and Fletcher (1968), Sato and Hoffman (1968), Revankar (1971) and Kazi (1980). The approach of these studies was to consider the elasticity of substitution as a function of the input ratio and then integrate the differential equation to recover the implied production function. Surprisingly, the most pressing problem at the time was that of explaining an apparent increase in labor share. Unfortunately, the idea of a variable elasticity of substitution has been lost in subsequent decades, characterized by the *constancy* of *factor shares* in many developed economies (Kaldor, 1961; Knoblach and Stöckl, 2020). The purpose of this paper is to reassess the importance of this neglected piece of economic theory.

We study the technological determinants of income distribution in a context where the elasticity of substitution is *endogenous* and *changes over time*. This is achieved by employing a specific VES in the form developed by Lu and Fletcher (1968). Further, we calculate an aggregate price markup measure to detach from the assumption of perfect competition in the product market (Raurich et al., 2012). A VES is an appealing alternative to standard production functions considered by the literature, as it is more general and nests different technological structures as special cases.[4] In addition, it can account for the observed decline in labor share with *complementary factor inputs*.

The rest of the paper is organized as follows. In the next Section we introduce the theoretical model and discuss its properties. In Section 4 we present the data (4.1), calculate the aggregate price markup (4.2), conduct unit root and cointegration tests (4.3) and estimate the structural parameters of the VES (4.4). In Section 5 we study the determinants of labor share. Section 6 concludes and derives some policy implications.

3. The VES production function

In this section we derive the labor share with a production function characterized by *variable elasticity of substitution*. The model draws on a specific VES developed by Lu and Fletcher (1968) which is eventually extended to the case of *imperfect competition*. We employ the standard notation to denote a general production technology $Y = F(K, L)$:

$$Y_t = A\left[\delta K_t^{-\rho} + (1-\delta)\eta L_t^{-\rho} k_t^{-\ c(1+\rho)}\right]^{-\frac{1}{\rho}} \tag{1}$$

where Y, K, and L are respectively *output* and the *two inputs of production* (i.e., capital and labor), k is the *capital-labor ratio*, $\rho = (1/b) - 1$, is the *substitution parameter*, δ is the *distribution parameter*, $\eta = (1-b)/(1-b-c)$ is a *composite parameter*, and A, δ, b, c, are parameters

[4] Theoretical studies which rely on VES production functions, such as Karagiannis et al. (2005) or Sharmila and Lahiri (2018) consider Cobb-Douglas generalization (where the CD can be obtained from the VES as a special case). CES generalizations of the VES (where the CES can be obtained from the VES as a special case) do exist (e.g., Lu and Fletcher, 1968; Kadiyala, 1972), although they have remained little used in both theoretical and empirical applications due to their complicated nature. As will be clear in the next section, the variant of the VES production function we consider (i.e., the Lu and Fletcher's one) is a CES generalization. Specifically, if the parameter c is equal to zero, $k_t^{-c(1+\rho)}$ and η tends to unity and (1) collapse into the standard CES production function.

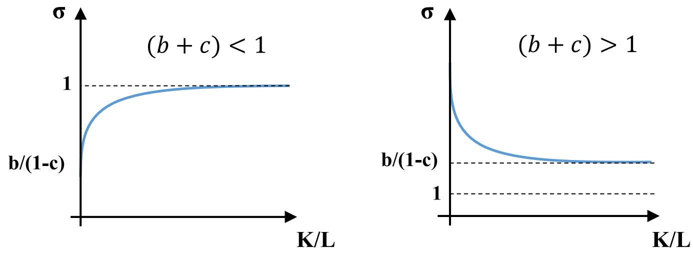

Fig. 1. Elasticity of substitution as a function of the K/L ratio.
Source: Authors' own elaboration.

related to *different forms of technological change*.[5] The production function in question is characterized by homogeneity of the first degree in its inputs, that is, it displays constant returns to scale (CRS). In addition, when c is equal to zero, $k_t^{-c(1+\rho)}$ and η approach unity and the VES in (1) collapse into the standard CES, which is thus included as a special case.

Technological change, defined by Solow as a shift in the aggregate production function (Solow, 1956), can be seen as a change in the parameters of the function itself. There are four parameters that can change the VES specification in (1) and not all of them affect the marginal rate of technical substitution between labor and capital ($MRTS_{K,L}$) in the same way (see the Online Appendix A – II). $A > 0$ is an *efficiency parameter* which captures *Hicks-neutral* technological change so that the larger the value of A the greater the level of output for a given K/L ratio. By its very definition, an increase in A has no effect on the $MRTS_{K,L}$. On the other hand, for a given K/L ratio, the $MRTS_{K,L}$ varies with changes in δ, b and c. Indeed, a change in the value of these parameters modify the shape of the isoquants for each input bundle, affects the marginal productivity of labor (MP_L) and capital (MP_K) and is a source of *biased technological change* (Acemoglu, 2002). Specifically, a change in δ causes MP_K to increase relative to MP_L; therefore, the change is *capital-biased*. Similarly, a higher value of b and c causes the $MRTS_{K,L}$ shifts downward if the capital stock grows faster than the labor force and b is lower than one. Notably, b and c also affect $\sigma_{K,L}$. This movement of the isoquant resulting from a higher σ is a common feature of both the VES and CES production functions (Gamlath, 2018). However, unlike CES, in VES the elasticity of substitution is constant only "*along a ray*" that connects points of different isoquants (Revankar, 1971, p.67). Given that these parameters are key determinants of several characteristics of the economy they are interpreted as important structural variables. In particular, the increase in b and c represents a technology that enables greater factor substitution. But note that it is precisely the value of c that determines, being equal to zero, whether the production function behaves like a standard CES or not. When the production function takes the form of a VES (i.e., when $c \neq 0$), capital intensity can directly influence labor productivity through the term $k_t^{-c(1+\rho)}$. The idea that labor productivity increases with capital intensity is not new in economics. In fact, it was Kaldor who pioneered the idea of *endogenous technical change* and argued that innovation depends, at least in part, on the rate of investment per person employed (Kaldor, 1961; Kaldor and Mirrlees, 1962; Bellocchi et al., 2023).[6]

In VES production functions, the effect of k on σ runs through the $MRTS_{K,L}$. As shown by Lu and Fletcher (1968), the elasticity of substitution of the VES in (1) can be derived using Allen (1938) formula as the inverse of the cross second-order partial derivative of the (linear and homogeneous) production function:

$$\sigma(k) = \frac{b}{1 - c\left(1 + \frac{MRTS_{K,L}}{k}\right)} \tag{2}$$

It is clear from equation (2) that, in order to ensure a positive value for the elasticity of substitution (i.e., $\sigma \geq 0$), the condition $c < (1/(1 + MRTS_{K,L}/k))$ must be satisfied. Note that since $MRTS_{K,L}$ is a function of k, moving along an isoquant, the elasticity of substitution varies non-linearly with capital intensity. Indeed, in the VES production function, the value of σ changes along a given isoquant. From (2) it also is evident that σ will be equal to b if c is zero. In this case, σ estimated by the VES will be equal to the *constant elasticity of substitution* of the CES production function.

The limiting properties of (2) are such that (Lu and Fletcher, 1968): (i) If $(b + c) > 1$, $MRTS_{K,L}$ decreases to zero and σ eventually decreases to $b/(1 - c)$; (ii) If, on the other hand, $(b + c) < 1$, then $MRTS_{K,L}$ increases to $(1 - b - c)/c$ and σ increases from $b/(1 - c)$ to one. Thus, the value of σ rises with the capital-labor ratio if $(b + c) < 1$ and falls if $(b + c) > 1$ (Fig. 1). However, as is common for VES production functions, the elasticity of substitution cannot cross the unity (Revankar, 1971).

Under *imperfect competition*, the price of products may differ from their marginal cost, that is, firms may charge a *markup*. Recent

[5] The parameter b of the VES determines the parameter ρ which in the standard CES production function determines the value of the (constant) elasticity of substitution and must lie in the range $-\infty < \rho < 1$.

[6] "*Most, though not all, technical innovations which are capable of raising the productivity of labor require the use of more capital per man*" (Kaldor, 1957, p. 595). Improved knowledge is "*infused into the economy through the introduction of new equipment*" (Kaldor, 1961, p.207).

empirical research suggests that there has been an increasing concentration of economic activity and a decrease in the intensity of competition among industries. Therefore, markups have increased since 1980, and the increase has been most pronounced in the U.S. (Van Reenen, 2018).[7] Reasons for a decrease in the intensity of intersectoral competition have not yet been identified. Some of the explanations include an increasing role of scale economies, increasing entry barriers through selective technological progress (Grullon et al., 2018), external growth strategies of firms and the lack of public competition law enforcement (Gutierrez and Philippon, 2018; De Loecker et al., 2020).

In the context of our setup, changes in markups have an immediate implication for the determination of the labor share. Under imperfect competition in the product market, the profit-maximizing condition becomes:

$$mw = MP_L \tag{3}$$

where $w = W/P$ is the real *wage per unit of labor*, MP_L is the *marginal product of labor* and m measures the *price markup*.[8] Equation (3) results in a slightly different expression for the labor share and suggests pro-cyclical fluctuations of the latter if movements in markups are counter-cyclical (Galì, 2015). Therefore, as the markup increases, we expect to observe a decrease in the labor share. That is, when markets are not perfectly competitive, the extent to which rents are allocated to labor or capital becomes crucial to explain income distribution (Kaplan and Zoch, 2020).

The relationship between the labor share, the parameters of the VES production function, and the capital-labor ratio can be derived by substituting in (3) the marginal productivity of labor derived from the VES in (1). Solving the resulting equation for the labor share (i.e., WY/L) and taking logs, under the usual assumption that labor efficiency increases at a *constant geometric rate* ($A = A_0 e^{\lambda t}$) we obtain:[9]

$$ln\frac{w_t L_t}{Y_t} = [b\ln(1-\delta) + (b-1)\ln A_0] + (1-b)\ln w_t + \lambda(b-1)t - c\ ln\ k_t - b\ln m_t \tag{4}$$

Or, by totally differentiating (4) with respect to time:

$$\dot{LS} = (1-b)\dot{w} - c\dot{k} + \lambda(b-1) - b\dot{m} \tag{5}$$

Equation (5) is an extension of the labor share equation derived by Lu and Fletcher (1968), where the additional factor $-b\dot{m}$ accounts for the price markup. As in its original form, the LS refers to the share of labor in value added and a dot on the variable denotes its growth rate. It is evident from (5) that changes in labor share depend on the parameters of the production function b and c, the growth rate of real wages ($\dot{w}$), the growth rate of the capital-labor ratio ($\dot{k}$), the rate of technological change (λ), and the growth rate of the price markup ($\dot{m}$). When technological progress is constant and markets are competitive (i.e., $\lambda = 0$ and $m = 0$), an increase in real wages and the capital-labor ratio will raise or reduce the labor share, depending on the interaction of two conflicting forces and hence the fact that $(1-b)\dot{w} \gtrless c\dot{k}$. When technical change occurs, the increase in output due to the technical change can be allocated to capital or labor, depending on the value of b. If $b > 1$, the increment of output due to technical change will be allocated entirely to labor; on the other hand, if $b < 1$, technical change will favor the capital share. Finally, a growth in markup always depresses labor share ($-b\dot{m}$).

4. Empirical analysis

Previous empirical studies using VES production functions can be classified into two different groups, depending on whether they relied on time-series or cross-sectional data (Table A2).[10] Sato and Hoffman (1968) employed data from the non-farm business sector in the U.S. and Japan, concluding that the VES is a more realistic model with respect to the CES. Similarly, Revankar (1971) rejected the CD form in favor of the VES for the U.S. While Lovell (1973) failed to get the same result with a CES for the manufacturing sector, he rejected both the CD and the CES in favor of the VES for some manufacturing industries. Notably, *time series studies* usually estimate a σ of less than one.

The remaining studies are based on cross-sections. Lu and Fletcher (1968) rejected the CES in favor of a VES specification in half of the manufacturing industries included in the analysis. The same outcome is obtained by Revankar (1971) against a CD when analyzing U.S. manufacturing. Similarly, in most of the cases Kazi (1980) rejected the CES as the VES turned out to be more realistic. Finally, Diwan (1970), using micro data for U.S. firms, rejected both the CD and the CES in favor of the VES. *Cross-sectional studies* estimate a σ of less than one, although in some cases not different from unity.

One of the most recent attempts to estimate a VES production function was made by Karagiannis et al. (2005), who conducted their analysis on a panel of eighty-two countries over the period 1960-1987 by means of nonlinear least squares (NLLS). In our analysis, we

[7] Grullon et al. (2018) examined publicly traded companies in the U.S. and found that concentration increased in 75% of all industries since 2000. De Loecker et al. (2020) used the inverse of the cost of goods sold to revenue and found that markup in the U.S. increase from 1.2 in 1980 to 1.6 now. Finally, Barkai (2020) employed industry level data and showed that markups have grown over time, lowering both the labor and capital shares.

[8] This equation is presented in Galí (1995) and corresponds to the first order condition for a symmetric equilibrium. It is derived by assuming monopolistic competition with n symmetric sectors with $m = \varepsilon/(\varepsilon - 1)$. Here ε is the elasticity of substitution of consumers' and firms' demand, that we assume to be equal.

[9] See the Online Appendix A - III for the derivation of equation (5).

[10] Among these studies, here we mention only those with a significant impact on later developments of the literature.

follow their approach closely, but we employ modern time-series techniques to estimate the parameters of the production function as suggested by Antràs (2004). This means addressing several issues related to the non-sphericality of the disturbances, potential endogeneity of the regressors and the non-stationarity of the series involved. All these technical aspects, the treatment of raw data and the construction of the variables, as well as the preliminary tests necessary to avoid spurious correlations are discussed in detail in the next subsections.

4.1. Model specification

Our estimation of the VES relies on macro data for six advanced OECD countries over a period of 40 years (1980-2020). All the raw data are obtained from the Macro-economic database of the European Commission (AMECO), the OECD Economic Outlook database and the Penn World Table of the Groningen Growth and Development Centre. Time series for aggregate output, capital, and labor inputs were converted to constant 2015 prices. The stationarity of the series in levels and first differences was evaluated using several unit-root tests, namely the ADF (Dickey and Fuller, 1979), the PP (Phillips-Perron, 1988), the DF-GLS (Elliott et al., 1996) and the KPSS tests (Kwiatkowski et al., 1992) (Section 4.3).

Estimation requires data on the flow of labor services L_t, the nominal price of these services w_t, the flow of capital services K_t, the rental price of capital r_t, nominal output Y_t, as well as its associated price P_t. To illustrate the effect of data quality on the estimates of parameters, we experiment with different methods in the construction of these variables. However, we report the results of our preferred configuration in the paper.

We assume that labor services are proportional to employment and proxy the flow of these services by total employment, denoted as the sum of the number of employees in domestic industries and the number of self-employed workers. Jorgenson and Ho (2000) have argued that total employment is not an appropriate measure of the flow of labor services because it ignores significant differences in the quality of labor services provided by different workers. Therefore, in addition to considering unadjusted labor, L_t, as an input of the VES specification, in our baseline model, we adjust the labor input for human capital accumulation (Tallman and Wang, 1994). Specifically, we follow Karagiannis et al. (2005) and adopt a simple proxy for human capital adjusted labor. First, we define the stock of human capital in country i at time t H_{it}, as $H_{it} = E_{it}$, where E_{it} is a human capital index based on the average years of schooling in country i at time t.[11] Then we define the *human capital adjusted labor supply* as $HL_{it} = H_{it}L_{it} = E_{it}L_{it}$. In estimating the VES specification for aggregate production, we use both L_t and HL_t as measures of labor input. Further details concerning the construction of the dataset, along with summary statistics for the main variables are available in the Online Appendix C. Compensation of employees includes wages and salary accruals, as well as supplements to wages and salaries (e.g., employer contributions for social insurance). Following the approach in Gollin (2002), we correct this measure by scaling it by a factor that considers the self-employed, who are given a wage equal to the average wage of regular employees.[12] Finally, as is standard in the literature, we assume that the flow of capital services is proportional to the net capital stock present in the economy.

Like CES production functions, VES functions are nonlinear in their parameters. This means that simple least squares cannot be used to obtain the value of the latter. Further, the level of inputs is jointly determined with the level of output, which may result in a simultaneity bias. Finally, the unlikely independence of inputs raises the possibility of multicollinearity. In fact, the low popularity of nonlinear production functions in applied economics work is related to the difficulties associated with their econometric estimation.[13] Therefore, parameters of the CES are usually calibrated, rather than estimated. In contrast to the CES, which is also nonlinear in parameters, the VES production function (because of the additional parameter c) cannot be linearized by logarithmic transformation or by the Taylor series approximation of Kmenta (1967).[14]Alternatively, the parameters of the function can be estimated by imposing additional conditions on side relations or by means of non-linear least-squares (Karagiannis et al., 2005). However, as is well known, the estimation of nonlinear production functions usually has poor results because the estimation algorithms are essentially iterative and sensitive to the initial values of the parameters (Kemal, 1981).

Thus, we recover the parameters of the VES from the empirical relationship from which (1) was integrated (see the Online Appendix A - I). Using the marginal productivity condition under imperfect competition in (3), with the standard assumption of neutral technological change which proceeds at a constant geometric rate and making use of logs, the marginal product of labor can be

[11] Following a common approach in the literature (e.g., Caselli, 2005), the Penn World Table (PWT) calculates a human capital index based on the average years of schooling from Barro and Lee (2013) and a rate of return on education, based on Mincer equation estimates around the world (Psacharopoulos, 1994).

[12] Kravis (1959) suggests treating all proprietor's income as labor income. This alternative adjustment turns out to have only a marginal effect on the estimates. Results are available upon request.

[13] In recent years, because of advances in econometric techniques and the availability of detailed sets of microdata, CES production functions have gained importance in macroeconomic analyses (Bentolila and Saint-Paul, 2003; Antràs, 2004) and growth theory (Klump and Papageorgiou, 2008), where they replaced the traditional CD.

[14] The applicability of Kmenta approximation is limited because it returns reliable results only if ρ is close to its point of approximation (i.e., zero) (Thursby and Lovell, 1978). In contrast, non-linear optimization algorithms usually face convergence problems or return unreliable (i.e., meaningless) parameters' estimates (Henningsen et al., 2012).

rewritten in the following linear form:[15]

$$ln\ y_t = \beta_0 + \beta_1 t + \beta_2\ ln\ m_t w_t + \beta_3\ ln\ k_t + \varepsilon_t \tag{6}$$

where β_0 is a constant, $\beta_2 = \widehat{b}$, $\beta_3 = \widehat{c}$, $\beta_1/(1-\beta_2) = \widehat{\lambda}$, and ε_t is a random error term.[16] Then, we use the estimated value of the parameters b and c and the mean value of $(m_t w_t / r_t \cdot 1/k_t)$ over the period to calculate the (average) elasticity of substitution for each country considered, as implied by equation (2). Note that if the partial regression coefficient of $ln\ k_t$ (β_3) is equal to zero, equation (6) would become the wage equation of a standard CES production function and β_2 would represent the (constant) *elasticity of substitution*. If, however, this assumption does not hold, the estimated σ from the CES function is likely to be biased, since it "*includes part of the variation connected with the capital-labor ratio in the average product*" (Kazi, 1980). Under the assumption that the error term has a normal distribution, the null hypothesis that the regression coefficient of *log k* equals zero can be tested using the t-statistic *(*see Online Appendix A – IV*)*.

In reviewing previous estimates of equations (1) and hence (6), two different procedures emerge, which, along with micro panels and nonlinear estimates have been widely employed by the literature. The literature review with which we opened this section warns of the possibility that slight variations in the period or in the specification considered tend to produce slightly different estimates of σ. Therefore, we proceed with several specifications based on both classical regression methods and modern time series analysis to strengthen our results. Indeed, while there is little reason for the differences in σ estimates from the literature, there are several sources of bias that can explain these discrepancies (Gechert et al., 2020).

Since direct estimation of the parameters of the VES requires the simultaneous estimation of a system of nonlinear equations, much of the literature followed the indirect method of Arrow et al. (1961) – exploiting the marginal productivity conditions implied by firm's profit-maximization behavior. Hence, the elasticity of substitution is estimated from the regression of *real wages* on *labor productivity* or a related form. However, the procedure rests on several assumptions: (i) constant returns to scale; (ii) the equality of the wage rate and the marginal product of labor; and (iii) the exogeneity of the wage rate. If there are IRS at the firm or industry level, the equality of the wage rate and the marginal product of labor is invalid. Indeed, factors of production cannot be paid at the marginal product without over-exhausting the total value added. Therefore, IRS are compatible with profit maximization only when there are imperfections in product or factor markets. Concerning the first assumption, we detach from the condition of perfect competition by employing a price markup measure to account for markets imperfections (Section 4.2). Alongside this problem, there is the one raised by assumption (iii). Indeed, the regression of log *y* on log *w* will be biased unless *y* and *w* are uncorrelated, or returns to scale are constant (Feldstein, 1967). Thus, if the wage rate is not exogenous, this approach will produce biased and inconsistent estimate of σ even if returns to scale are constant and the wage rate is equal to labor's marginal product. In addition, without information on the behavior of the firms, it is difficult to account for the simultaneity bias resulting from the estimation of (6).[17]

The econometric literature of the last half century has been devoted to solving this endogeneity problem. Berndt (1976) acknowledged the simultaneous equation bias and proposed a two-stage least squares (2SLS) procedure to solve it. IV estimation requires finding variables that are correlated with observed input choices, but uncorrelated with the unobservables determining production. In his first stage regressions, Berndt (1976) introduced several instruments. However, as noted by Antràs (2004), the use of large sets of instruments can be detrimental in two respects. On the one hand, if any of these instruments is endogenous, 2SLS estimates will be inconsistent. On the other hand, if some of the instruments are weakly correlated with the regressors, even if the exogeneity requirement is met, small sample biases may occur (Bound et al., 1995). For these reasons, we rely on IV to reinforce our basic estimates (which are conducted with a simple ARDL model) but focus on a narrower set of instruments (Antràs, 2004).

Specifically, we take the following three variables as exogenous to the model but correlated with the regressors: (1) the *total population*, (2) *wages in the government sector*, and (3) the *real capital stock owned by the government.*[18] We interpret these variables as different types of supply shifters. The size of the population is likely to have a significant effect on the supply of both capital and labor services. Government wages are also likely to affect the supply of labor in the private sector, with government capital formation having a similar effect on the supply of capital in the private sector. The exogeneity of the instruments seems plausible and has already been used (Antràs, 2004). Finally, as is standard in macro literature, we take the fertility choice as exogenous to the model, while the

[15] As shown by Antras (2004), *biased technological change* may affect the estimated value of σ. Assuming neutral technological change is a limitation of our study. Note, however, that part of biased technological progress is captured by the VES through the parameter *b*, which, when lower (higher) than one, causes technological change to have a negative (positive) impact on the labor share. We thank an anonymous referee for bringing this point to our attention.

[16] The derivation of (6) and the mapping of the parameters to the VES in (1), is given in the Online Appendix A(I).

[17] In a detailed survey Lucas (1963) concluded that *time series* are more appropriate than *cross-sections* to estimate a production functions. His argument is based on the existence of important sources of bias in cross-sectional data which tend to push the estimated elasticities towards unity. On the other hand, he considers the two sources of bias in the time series context - *simultaneity* and *misspecification of the lag structure* - and concludes that these *do not bias the time series estimates in some direction*. The latter conclusion is supported by his finding that trying different lag schemes and a simultaneous equation model does not change his estimates by much.

[18] Wages in the government sector are computed as labor income accruing to government employees divided by their total number and deflated by the aggregate input price index. To construct the real stock of capital owned by the government we divide the nominal capital stock by the price of capital index. We rely on a standard OECD dataset that was already used in related studies by Lane (2003), Lamo et al. (2013) and Marzinotto and Turrini, (2017), among others. Specifically, we use the OECD Economic Outlook database. Missing variables have been completed with successive reviews and interpolations (Online Appendix C - Table OA8).

government variables are assumed not to respond simultaneously to market prices and quantities.

Having discussed the choice of instruments, we now turn to the selection of an appropriate estimation technique. One alternative would be to run equation (6) using a standard 2SLS regression. Nevertheless, there is no reason to believe that instrumenting would solve the autocorrelation problems discussed above. Following Antràs (2004), we choose instead to implement the generalized instrumental variable (GIV) model. In a widely cited result, Fair (1970) shows that if the model is estimated using an iterative Cochrane-Orcutt procedure, all the lagged left and right-hand side variables must be included in the instrument list to obtain consistent estimates. Therefore, our 2SLS estimates are adjusted by adding AR terms to account for serial correlation.

4.2. The price markup

The main difficulty related to our empirical analysis lies in the calculation of the time-varying price markup. Measuring the degree of competition intensity in the product market is notoriously a challenging issue. Depending on the nature of the data available and the size of the market considered, alternative measures can be employed. In fact, markups can be measured directly, based on microdata, via the elasticity of profits to marginal costs or indirectly, based on national accounts series, via a range of alternative methods (Ciapanna et al., 2022).

Most of the macro literature follows Rotemberg and Woodford (1999) and obtain the growth rate of the price markup from the Solow residual. However, the problem of this approach lies in the unobservable nature of the aggregate price markup and the shape of the production function to obtain the elasticity of substitution. This issue can be resolved by means of some specific assumptions on the value of σ. Unfortunately, this unknown is precisely the variable we try to address in this paper.

Beyond the methodological questions, cross-country studies must deal with the additional issue of data harmonization and comparability. To analyze the evolution of markups across countries, several works have considered national accounts (NA) data. National accounts data are harmonized internationally and ensure a high degree of representativeness (Giordano and Zollino, 2017). The simplest method by which accounting data can be used to measure profit margins is by calculating the ratio of revenues to total variable costs. This ratio, when both measures are divided by the quantity produced, is equal to the ratio of price to average variable cost. However, the average variable cost is not generally equal to the marginal cost since this is only the case when the marginal cost is constant at all quantity levels (Syverson, 2019).

To calculate the price markup, we pursue the best trade-off between time extension of the series, cross-national representativeness of the data and precision of the estimate. Specifically, we follow the procedure developed by Raurich et al. (2012), who extend the dual approach by Roeger (1995) to compute a non-constant markup using Euler's Theorem and firms' first order condition. Roeger exploits the two ways to measure the Solow residual, the one in terms of quantities (from profit maximization) and the one in terms of prices (from cost minimization) (Hall, 1988). The dual approach to growth accounting can be readily derived from the dual cost function of any production function. However, it can also be derived from the basic national accounting identity that aggregate output is equal to factor incomes. The advantage of using the national identity instead of a cost function to derive the dual growth accounting methodology is that it makes explicit that the equality of the dual and primary measures does not depend on any assumptions about "*the form of the production function, bias of technological change, or relationship between factor prices and their social [...] products*" (Hsieh, 1999).

$$Y_t = (w_t L_t + r_t K_t) m_t \tag{7}$$

In our setting with CRS, m_t may be greater than one on account, exclusively, of price markup. By differentiating both sides of equation (7) with respect to time, we obtain the primal estimate of the Solow residual as the growth rate of output after subtracting the share-weighted growth in factor quantities. Similarly, the dual measure of the Solow residual is equal to the share-weighted growth in factor prices. This results in an equation exclusively in terms of observable variables:

$$\dot{m}_t = \dot{Y}_t - m_t LS_t(\dot{w}_t + \dot{L}_t) - (1 - m_t LS_t)(\dot{r}_t + \dot{K}_t) \tag{8}$$

where $\dot{w}_t$ and $\dot{r}_t$ are the growth rate of real wages and the rental price of capital, respectively. Equation (8) characterize the dual approach to calculate markup.[19] Since this is the amount of income that it is not labor nor capital income, it should be interpreted along the lines of the Solow residual. Note that the time path of the markup can be outlined as a recursive backwards solution without any specific assumption on the underlying production function.[20] The only requirement is the availability of data on GDP, capital stock, employment, wages, and the rental price of capital.

The first three variables (i.e., quantities), are directly available through the AMECO database. The latter two - i.e., prices - require some extra work. Wages need to be computed because the total compensation of dependent employees must be adjusted for the share

[19] See Appendix A – Section V) for the derivation of equation (9). Note that (9) holds if all variables adjust instantaneously and are measured without error. However, if there is a measurement error in labor, this would affect efficiency but not consistency. It remains the case, that any measurement error in nominal output growth or capital cost growth could lead to another bias. Regarding nominal output growth, we think that significant measurement error is unlikely. For capital costs, this possibility is higher. In any case, it is useful to discuss the direction of the bias these two types of measurement errors would induce (see Christopoulou and Vermeulen, 2008).

[20] As a first approximation, for the last point in the time series, we compute the markup as the *ratio of price to average variable cost*. Intermediate consumption data were recovered from the National Accounts.

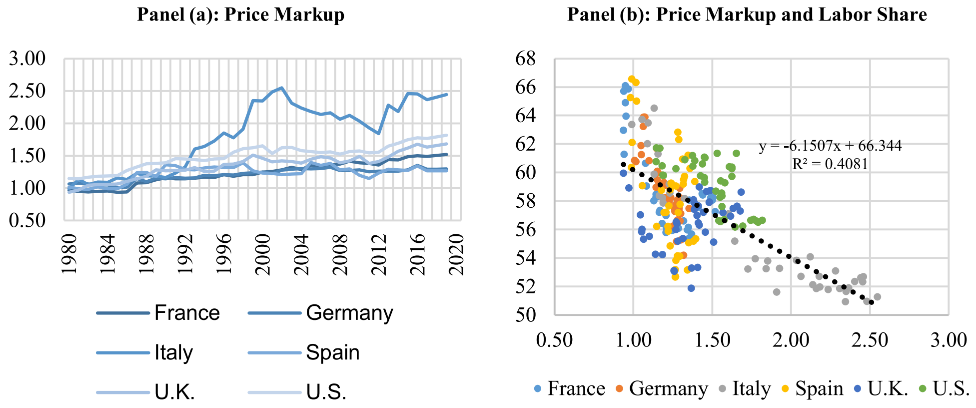

Fig. 2. Price markups and labor shares (1980-2020).
Source: Author's own calculation.

of self-employment (Gollin, 2002). To this end, we employ the GDP at market prices and compute self-employment income as actual labor income. On this basis, w_t is defined as $LS_t Y_t / L_t$, where LS_t is the (adjusted) labor share of income at market prices. On the other hand, we employ the rental price of capital, r_t, as a price-based measure exploiting the non-arbitrage condition on factor markets. The advantage of price-based measures "*is that they are based on market prices paid by agents who are incentivized to get the prices right*" (Hsieh, 1999). The rental price of capital is given in terms of the Hall-Jorgenson formula, namely, the real interest rate plus the depreciation rate multiplied by the relative price of capital (Jorgenson, 1963; Hall and Jorgenson, 1967).

$$r_t = \frac{R_t}{p_t} = \frac{p_t^k}{p_t}\left(i_t - \dot{p}_t^k + \delta_t\right) \tag{9}$$

where R_t is the nominal rental price of capital, p_t^k is the nominal price of one unit of capital, δ_i is the depreciation rate, i_t is the nominal interest rate, and p_t is a price index. Equation (9) has a straightforward interpretation: when assets are owner utilized, the equilibrium value of the implicit rental must cover the real opportunity cost of an investment of value $(i_t - \dot{p}_t^k)$, as well as the loss in asset value as the asset ages (δ_t). As a measure of the annual interest rate, we use the nominal long-term interest rate on government bonds.[21] Finally, for p_t^k and p_t we use the price deflator of gross fixed capital formation and GDP, respectively.

The term δ_t deserves further attention since it represents the rate of economic depreciation (Kopits, 1982). Hicks (1946) defines income as the maximum amount that can be spent in a period while keeping the capital value intact; hence *economic depreciation* is the amount of money that must be set aside to maintain the value of capital in real terms. When capital stock and gross fixed capital formation (i.e., investment) data are available, the depreciation rates are not directly observable but can be estimated using the capital accumulation equation (ECB, 2006).[22]

Based on these data, we obtain homogeneous time series of marginal price-cost ratios (or *markup ratios*), for all six countries in our sample. The markup, estimated at the economy-wide level over the period 1980-2020 is shown in Fig. 2 (Panel a), while its correlation with the labor share (by country) is shown in Panel b.[23]

It is worth noting that the price markup derived from (8) has some noteworthy characteristics. First, the cumulative effect of the series has been positive in all the countries analyzed, increasing steadily between 1985-2008 and between 2010-2020. Moreover, we observe a stable behavior during the years of economic integration and an increase in recent years, following the persistence of economic crisis.[24] The second characteristic is its countercyclical pattern. As stressed by Rotemberg and Woodford (1999), this countercyclical pattern is necessary to reconcile theory and empirical evidence on the procyclical behavior of wages. Indeed, in New Keynesian models, the labor share is equal to the inverse of the price markup (Galì, 2015). This makes the labor share pro-cyclical if the price markup is counter-cyclical (Nekarda and Ramey, 2013). As Fig. 2 clearly shows, since the 1980s markups have grown by an

[21] The capital rental price was calculated for both a short-term and long-term interest rate. The nominal long-term interest rate in AMECO has different definitions depending on the country considered. For most of the countries, it is equal to the central government benchmark bond of 10 years. We experimented with various other series, including a short-run interest rates consisting of the 3-month interbank rate. All these series provide very similar pictures in terms of both capital rental price of capital and price markup.

[22] The capital accumulation equation links the capital stock (K) to investment (I): $K_t = (1\text{-}\delta_t)\ K_{t\text{-}1}+I_t$. In other words, the gross capital stock each year equals that of the previous year minus that part of the stock that has reached the end of its service life plus the gross fixed capital formation in the current year.

[23] For more details on the markup estimation procedure and the data used, see Section V of the Online Appendix A.

[24] In the European Union, access to national markets was simplified with the development of the single market. Similarly, there have been product market reforms aimed at intensifying competition by taking down barriers to entry. This was also accompanied by the expectation that the scope of firms' market power would be reduced.

Table A7
Johansen-Juselius Cointegration Test.

Maximum rank	r=0 vs r=1			r=1 vs r=2			r=2 vs r=3		
Num. of lag	1	2	3	1	2	3	1	2	3
France	47.277	31.967	35.554	13.779*	14.911*	17.429	3.829	5.250	4.549
Germany	32.697	26.922*	19.552*	14.214*	5.987	6.136	0.460	0.032	0.162
Italy	50.025	64.027	39.712	10.711*	8.371*	11.235*	0.236	1.814	1.392
Spain	33.257	19.871*	33.464	5.286*	8.090	13.779*	0.102	0.207	0.165
United Kingdom	44.968	36.902	32.776	12.775*	14.542*	14.343*	5.446	4.183	5.490
United States	55.666	33.992	55.842	20.307	16.031	19.698	2.591*	3.624*	5.685
95% Critical Value	29.68			15.41			3.76		

Note: *Null Hypothesis Rejection (r=r*<k), 5% critical threshold.

Table 1
Estimates of the VES production function (1980-2020).

Country	Panel (a): ARDL model (1 lag) - EC			Panel (b): IV model			Panel (c): GIV model		
	b	c	b+c	b	c	b+c	b	c	b+c
France	0.55	0.61	1.16	0.06	0.49	0.55	0.11	0.48	0.59
	-0.289	(0.275)*		-0.136	(0.092)***		(0.114)*	(0.475)***	
Germany	0.15	0.21	0.36	0.22	0.10	0.32	0.19	0.17	0.36
	-0.108	(0.092)*		(0.096)*	-0.166		(0.193)**	(0.169)**	
Italy	0.05	0.93	0.98	0.27	0.48	0.75	0.23	0.49	0.72
	-0.105	(0.338)**		(0.06)***	(0.131)***		(0.228)***	(0.49)***	
Spain	0.07	0.77	0.85	0.19	0.07	0.26	0.16	0.19	0.36
	-0.131	-0.476		(0.078)*	-0.19		(0.163)***	(0.193)***	
United Kingdom	0.08	0.28	0.36	0.28	0.18	0.45	0.07	0.26	0.33
	-0.067	(0.091)**		(0.117)*	-0.107		(0.073)	(0.258)***	
United States	0.43	0.85	1.28	0.31	0.87	1.18	0.30	0.83	1.13
	(0.127)**	(0.155)***		-0.16	(0.074)***		(0.3)***	(0.826)***	
Mean	0.22	0.61	0.83	0.22	0.37	0.59	0.18	0.40	0.58
St. Dev.	0.214	0.301	0.394	0.090	0.308	0.340	0.081	0.250	0.309

Note: Generalized instrumental variable (GIV) model. Dependent variable in the underlying regression: ln(y). Instruments: (1) Total population, (2) Wages in the government sector, and (3) Real capital stock owned by the government. All the lagged left and right-hand side variables are included in the instrument list (Fair, 1970). In addition, given the presence of a structural break in the data (Tables OA6 and OA7 in the Online Appendix B) a significant time dummy was included in the model for the period 2008-2020. P-values in parenthesis. *** Significantly different from 0 at the 1% level. ** Significantly different from 0 at the 5% level. * Significantly different from 0 at the 10% level. Equations pass the standard misspecification and structural stability tests (serial correlation, linearity, normality, and heteroscedasticity). Labor input adjusted for human capital.

average of 0.80 percentage points per year. This means that today's markets are more imperfect than they were 40 years ago, and this result is consistent with *recent studies*. For instance, Raurich et al. (2012) used macro data to show that price markups have increased since 1980 in the U.S. and Spain. De Loecker et al. (2020), Autor et al. (2020), and Barkai (2020) with a firm-level approach document the same patterns for many U.S. industries and find that the increase in markups is consistent with several trends, including declining labor shares.

4.3. Unit root and cointegration tests

Before estimating the model, we must pay attention to some typical issues that arise when considering the potential non-stationary nature of the time series involved. Fig. A2 shows the four series that form the basis of our estimates, where the logarithm of the variables was normalized to zero in the first year (i.e., 1980). Two facts emerge from the figure. First, the graphs uncover the potential non-stationarities in the time series considered: the natural logarithm of y_t, w_t, k_t, and m_t all clearly trend upwards. Second, at least three variables follow *similar trends*. This suggests that the correlations captured by a standard regression might be spurious, unless the time series are *cointegrated* (Granger and Newbold, 1974; Phillips, 1986).

Tables OA2-OA5 (in the Online Appendix B) provide a summary of the unit root tests performed on each series and for each country. The first column of the table presents the results of an Augmented Dickey-Fuller (1979) test, the second column reports the t-statistics for the Phillips-Perron (1988) test, and the third column the tau of the DF-GLS test by Elliott et al. (1996). Finally, in the last column we report the results for the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test (Kwiatkowski et al., 1992).

It is evident that the combined evidence from the unit root tests allows us to accept the unit root hypothesis in all the countries considered. At the bottom of each table, we report the results of the same test performed on each of the four series, expressed in first differences (*dly*, *dlw*, *dlk*, *dlm*). In this case, all the results point to a rejection of the null hypothesis of the series being integrated of order two at 1% in most of the cases. We therefore conclude that all four series are *nonstationary and integrated of order one*.

Therefore, a cointegration analysis is performed to assess the possibility of long-run convergence of our variables. Tables A6 and A7 present the results from two cointegration tests. Specifically, the first table (i.e., A6) considers Engle and Granger's (1987)

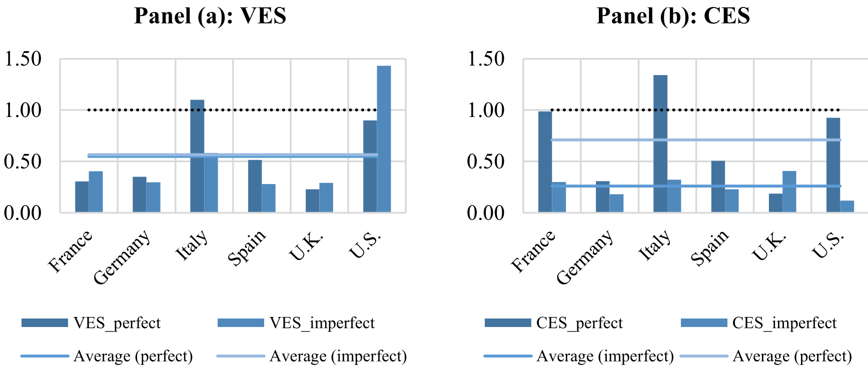

Fig. 3. The elasticity of substitution (VES vs CES production function) (1980-2020).
Note: Sample-wide estimates. In the case of the VES (Panel a) we report the values of the average elasticity of substitution, while in the case of CES (Panel b) we show the estimated constant elasticity. This latter is obtained by estimating the wage equation (6) with $\beta_3 = 0$. Period of analysis: 1980-2020. Estimates of equation (8) under perfect and imperfect competition with GIV are given in Tables A5-A6.

residual-based two-step ADF test, which hinges on testing the stationarity of the residuals from the OLS regression of equation (6). Table A7 complement the analysis with the maximum likelihood cointegration test suggested by Johansen and Juselius (1990), which tests the null hypothesis of the existence of r cointegrating vectors against the alternative of the existence of (r+1) cointegrating vectors. When the estimation includes a lag, the *null hypothesis of no cointegration is rejected* for all the countries considered. To sum up, the results of the cointegration tests indicate that estimates of equation (6) can be interpreted with relative confidence with respect to spurious correlation problems.

As discussed by Hamilton (1994), a natural cure for spurious regressions would be to difference the data before estimating the equations. The disadvantage of this approach is that important long-run information would be lost, and the interpretation of our estimates in connection to the structural parameters of the production function would become less transparent. Given the satisfactory results of cointegration tests, we rely on the GIV model as our main estimator, which is used to reinforce the basic estimates obtained with the ARDL model of Pesaran and Shin (1999) and Pesaran et al. (2001).

4.4. Estimation

In this section we present our estimates of the VES production function. We exploit the empirical relationship in (6) to fit the VES and obtain the value of its coefficients under imperfect competition. In Table 1 (Panel c) we report the results obtained with the generalized instrumental variable (GIV) procedure for our preferred data configuration, which constitutes our *baseline model*. However, ARDL (Panel a) and simple 2SLS (Panel b) estimates are available for comparison. All the specifications pass the standard misspecification and structural stability tests. The time trend displays opposite signs and is usually significant at conventional critical values.

The key finding regarding our testable hypothesis is that the sign for the coefficients b and c is positive and significant, thus providing first evidence of the existence of a VES production function. All the estimated coefficients are significantly different from zero at the one percent level in all the countries considered and economically plausible, regardless of whether unadjusted or adjusted labor is employed. Since our estimated coefficients for b and c are positive and significantly different from zero, it follows that *factor shares vary with a country's capital-labor ratio*.

The summary statistics reported in the last two rows of Table 1 highlight some interesting facts. The average estimated value obtained through the generalized instrumental variable model for coefficients b and c is 0.18 and 0.40, respectively. The significance of the coefficients is satisfactory. Of the 12 coefficients estimated for individual countries, 8 of them are significant at 1%, 2 at 5%, 1 at 10%, and only one falls below the conventional significance threshold. A quick comparison with the coefficients obtained with our preferred data configuration of the ARDL model (Table 1 - Panel a), reveals that, compared with ARDL estimates, IV and GIV estimates (Table 1 - Panels b and c) are generally lower (the former are in the range of 0.33-0.90 for b; 0.15-0.40 for c). The modified t-statistics are also lower than the ARDL ones. We interpret this finding as an indication that the instruments are correctly dealing with the simultaneous equation bias.

A second interesting fact concerns the heterogeneity of values among countries. While in European countries the sum of b and c is estimated to be lower than unity - meaning that an increase in the capital-labor ratio has a positive impact on the elasticity of substitution - in the U.S. the value of the same sum turns out to be higher than one (and robust across different models), meaning that an increase in the capital-labor ratio has a negative impact on σ. In the former case the elasticity of substitution starts from above one and decreases to a threshold value that is also greater than one (with the *speed of adjustment* set by b and c), while in the latter case it starts from below one and converge to unity as capital intensity increases. This relates to the fact that in the VES the marginal rate of technical substitution varies with k along a given isoquant.

A $\sigma < 1$ is consistent with most empirical estimates of the elasticity in the literature, yet, the results is surprising in that, as stated by Glover and Short (2020), in this situation capital accumulation cannot explain the observed decline in labor share with a CES-type

Table 2
Starting/Convergence values of the Elasticity of Substitution (σ) (1980-2020).

Country	b	c	b+c	b/(1-c)
France	0.11	0.48	0.59	0.22
Germany	0.19	0.17	0.36	0.23
Italy	0.23	0.49	0.72	0.45
Spain	0.16	0.19	0.36	0.20
United Kingdom	0.07	0.39	0.46	0.12
United States	0.30	0.83	1.13	1.73
Mean	0.18	0.42	0.60	0.49
St. Dev.	0.081	0.240	0.292	0.617

Source: Authors' own estimation.

Table 3
Average (annual) growth rates of real wages, capital-labor ratios and labor shares (1980-2020).

Country	y=Y/L	w	k=K/L	m	ls_cp	ls_fc
France	1.25%	0.83%	1.83%	1.39%	-0.18%	-0.17%
Germany	0.96%	0.79%	0.86%	0.53%	-0.08%	-0.08%
Italy	1.03%	0.49%	2.10%	3.57%	-0.25%	-0.17%
Spain	1.32%	0.72%	3.02%	0.60%	-0.25%	-0.23%
United Kingdom	1.55%	1.72%	1.03%	1.94%	0.06%	0.02%
United States	2.20%	1.93%	1.99%	1.43%	-0.09%	-0.14%
Mean	1.39%	1.08%	1.80%	1.57%	-0.13%	-0.13%
St. Dev.	0.0045	0.0059	0.0079	0.0112	0.0012	0.0009

Note: Labor productivity (y) = Gross domestic product at constant market prices per person employed; Real wages (w) = Real compensation per employee, deflator GDP; Capital intensity (k) = Net capital stock at constant prices per person employed; Price markup (m); Labor share (ls_cp) = Adjusted wage share total economy (% GDP at market prices); Labor share (ls_fc) = Adjusted wage share total economy (% GDP at factor cost). Note that when K/L (capital intensity) increases more than Y/L (labor productivity), K/Y (capital accumulation) is also increasing.

production function. In contrast, in the VES, impact of capital intensity on the labor share is jointly determined by the parameters b and c. Notably, the value of b has important implications for both the direction of technological change and the determination of factor shares, since it governs the proportion of output due to technological change allocated to capital or labor. This point will be discussed in the next section on labor share dynamics.

Using the relationship in (2) and exploiting the first-order condition for cost minimization, we derive the (average) elasticity of substitution (σ) over the period of analysis. We estimate (2) with GIV both under perfect competition, as in Antràs (2004), and under imperfect competition by considering the time series of the price markup, as in Raurich et al., 2012. Estimates of σ, as well as a comparison of the latter and the one calculated in a CES framework (which is obtained by estimating (6) with $\beta_3 = 0$) are shown in Fig. 3 (Panels a and b). While the time path of the estimated σ during the period 1980-2020 for the six countries considered is reported in Fig. A1.

From the empirical analysis conducted so far, we can draw some preliminary conclusions. The first remarkable fact concerns σ, which is likely to be lower than one in the countries considered (the average value with the VES is 0.55). The only exception is the U.S., where, in contrast elasticity is estimated to be slightly above unity under imperfect competition. This may be related to the idea that, at economy-wide level, aggregate elasticity of substitution also accounts for *non-technological determinants* (e.g., consumption preferences) or other *institutional factors* (Knoblach and Stöckl, 2020). In contrast, when markup is not considered, the estimated value of σ is very close to the average value found in the literature (Knoblach et al., 2020). In addition, our results are generally lower than those estimated with macro data and similar methodologies using CES production functions (Gecherta et al. 2020). As argued by Kazi (1980) this may be connected with the fact that the CES tends to overestimate the elasticity parameter, since it "*includes part of the variation connected with the capital-labor ratio in the average product*". Further, confirming the main finding of Raurich et al. (2012), the consideration of price markup causes estimates of the elasticity to deviate from unity, because of a misspecification of the output elasticity of labor. This result goes in opposite directions and can produce an upward or downward bias. Finally, the large dispersion of individual country estimates in both models means that σ is statistically different from the CD hypothesis in most of the cases (Antràs, 2004).

A second important fact is related to the properties of the VES production function outlined in Section 3. In fact, an increase in the capital/labor ratio will increase, hold constant, or decrease the elasticity of substitution over time, depending on the parameters b and c. If $(b + c) > 1$, the value of σ declines with an increased K/L and approaches $b/(1 - c)$, which is greater than unity, as K/L increases to infinity. On the other hand, if $(b + c)$ is less than one, the value of σ increases from $b/(1 - c)$, which is less than unity, to unity as K/L goes from zero to infinity. This means that for a given structural configuration of the economy, the effect of capital intensity on the labor share is different depending on the country considered. The starting and convergence values of the (variable) elasticity for each country of the sample are reported in Table 2.

More importantly, the VES can explain the observed decrease in labor share, along with the observed increase in capital accumulation over the period 1980-2020 (Table 3 below) with an average σ less than unity, whereas the CES achieves the same result with

Table 4
Average annual growth rate of technological progress (λ) (1980-2020).

Country	Panel (a): ARDL model (1 lag) - EC			Panel (b): IV			Panel (c): GIV		
	b	t	λ	b	t	λ	b	t	λ
France	0.554	-0.002	-0.55%	0.06	0.001	0.15%	0.115	0.001	0.04%
	-0.289	(0.001)*		-0.44	-0.44		(0.058)*	-0.001	
Germany	0.146	0.002	0.22%	0.221	0.003	0.40%	0.193	0.003	0.36%
	-0.108	-0.001		(2.3)*	-1.02		(0.068)**	-0.001	
Italy	0.049	-0.004	-0.39%	0.275	-0.015	-2.05%	0.228	-0.013	-1.75%
	-0.105	(0.001)*		(4.57)***	(-8.6)***		(0.046)***	(0.001)***	
Spain	0.071	-0.002	-0.20%	0.186	-0.002	-0.21%	0.164	-0.003	-0.37%
	-0.131	(0)***		(2.39)*	(-0.74)		(0.035)***	(0)***	
U.K.	0.08	0.003	0.28%	0.277	0	-0.06%	0.074	0.006	0.61%
	-0.067	-0.001		(2.37)*	(-0.13)		-0.046	(0.001)***	
U.S.	0.43	-0.003	-0.57%	0.312	-0.006	-0.90%	0.301	-0.005	-0.76%
	(0.127)**	-0.001		-1.95	(-1.33)		(0.047)***	(0.001)**	
Mean	0.222	-0.001	-0.20%	0.222	-0.003	-0.45%	0.179	-0.002	-0.31%
St. Dev.	0.197	0.002	0.003	0.083	0.006	0.008	0.074	0.006	0.008

Note: Generalized instrumental variable (GIV) model. Dependent variable in the underlying regression: ln(y). Instruments: (1) Total population, (2) Wages in the government sector, and (3) Real capital stock owned by the government. All the lagged left and right-hand side variables are included in the instrument list (Fair, 1970). In addition, given the presence of a structural break in the data (Tables OA6 and OA7 in the Online Appendix B) a significant time dummy was included in the model for the period 2008-2020. P-values in parenthesis. *** Significantly different from 0 at the 1% level. ** Significantly different from 0 at the 5% level. * Significantly different from 0 at the 10% level. Equations pass the standard misspecification and structural stability tests (serial correlation, linearity, normality, and heteroscedasticity). Labor input adjusted for human capital.

an average σ greater than unity. Unfortunately, the latter fact goes against the empirical evidence, so that *additional factors* (capital-biased technological change or market institutions) are necessary to make CES compatible with observed labor share dynamics.

As a final exercise, we examine the implications that the value of the parameters b and c and the degree of imperfect competition have for capital deepening and the determination of the labor share. Specifically, we make use of the computed price markups and estimated parameters of the CES/VES to simulate the labor share in three *different scenarios*. These are eventually used to decompose the labor share and examine to what extent the price markup, capital intensity and technological progress account for significant portions of its actual trajectory.

5. The labor share

In what follows we rationalize the above results by means of an accounting exercise. This is achieved by performing a simulation on the evolution of the labor share. We examine how much of the variation in labor share can be explained by real wage growth, capital deepening and how much by the trajectory of technological change and price markup. In addition, we compare the overall performance of CES and VES models. To address these questions, we exploit equation (5). As inputs of the simulation, we use the time series of the capital-labor ratio (capital stock at 2015 prices per person employed), real wages (real compensation per employee), the estimated value of the structural parameters b and c, estimates of the growth rate of technological progress, and the price markup. The value of b and c determines the relationship between gross domestic product (in efficiency units) and capital intensity, while the value of the markup relates these two variables with the labor share.

To define the impact of technology we assume, as we did in the previous section, that technological progress advances at a constant rate. As already pointed out, this simplification represents a limitation of our study, since biased technological change may affect the estimated value of σ (Antras, 2004) Note, however, that part of the capital-biased technological progress is captured by the VES through the parameter b, which, when lower than one, causes a negative impact of technological change on the labor share. When technological progress is assumed to be equal to zero and there is no markup (i.e., $\lambda = 0$ and $m = 0$), an increase in real wages and capital deepening will raise or reduce the labor share, depending on the result of two contrasting forces: $(1 - b)\dot{w}$, which favors labor and $c\dot{k}$, which favors capital. On the other hand, when technological progress occurs, $(\lambda > 0)$, the increment of output attributable to it may be allocated to labor or capital, depending on the parameters of the production function. Finally, growth in markup ($\dot{m} > 0$) tends to depress the labor share. Specifically, equation (5) states that the increment of output due to technological progress will be allocated to labor if b is greater than unity. On the other hand, if b is less than unity, gains from technological change will be distributed to capital.

From this perspective, it is worth noting the strong assumptions introduced in the literature to explain the dynamics of the labor share. On the one hand, Galí (1995) assume $\sigma = 1$ (Cobb-Douglas production function) so that labor share dynamics can only arise from *biased technological change* or the evolution of price markups; On the other hand, under the assumption that technological change is neutral and $m = 1$ (i.e., perfect competitive markets), the price markup effect vanishes, and the dynamics of the labor share is explained by *capital accumulation* (CES production function) (Antràs, 2004). However, the CES framework is consistent with an increasing intensity and an increasing labor productivity, along with declining labor share only with a $\sigma_{K,L}$ greater than one. Notably, this latter fact is frequently against empirical evidence, so that other factors are necessary to capture the downward trend in labor share.

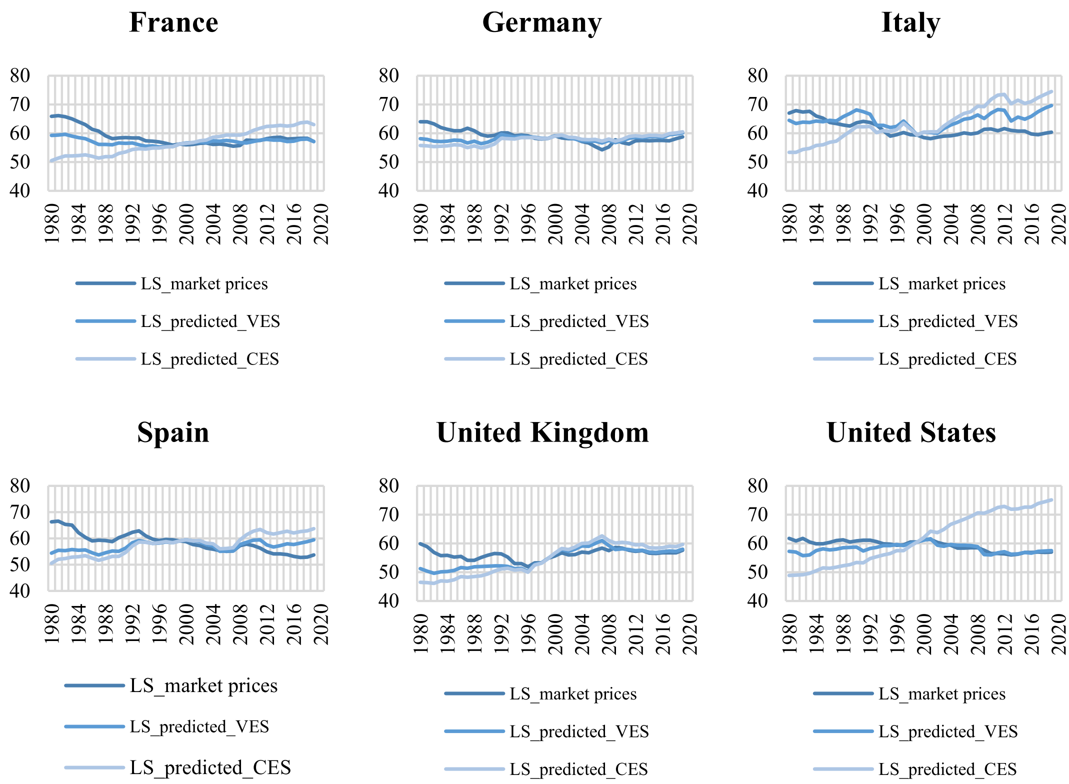

Fig. 4. Simulated labor income shares with the CES/VES production function (1980-2020).
Note: Labor share estimated with CES and VES production functions and observed labor share at market prices. The CES simulation was obtained by setting the parameter $c = 0$ in equation (5). Source: Authors' own elaboration.

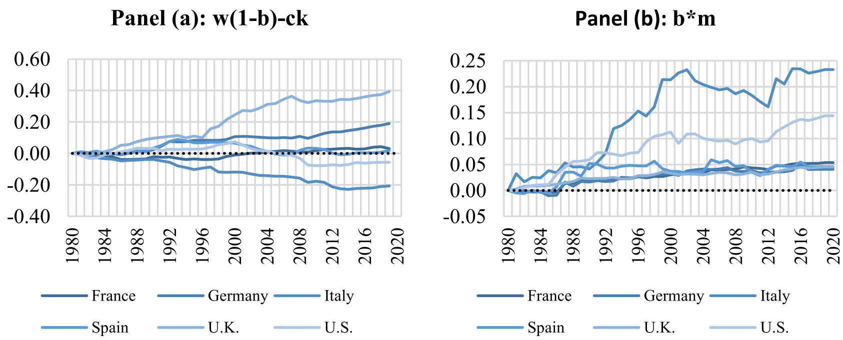

Fig. 5. Real wages, capital-labor ratios and price markups (1980-2020).
Note: Cumulative changes, 1980=0. Source: Authors' own elaboration.

Changes in the labor share at current prices and factor cost, along with the growth rates of wages and capital-labor ratios for the 6 countries investigated are shown in Table 3, while the estimated growth rate of technical progress (i.e., λ) is reported in Table 4.

In all the countries considered, observed wage rates, capital-labor ratios and labor productivity have increased over time in the last forty years. However, from 1980 onwards, capital deepening has been on average 1.7 times higher than that of real wages. The largest gap between the two variables is in continental Europe, where real wages growth was, on average, 1.3 percentage points per year lower than capital intensity. Spain is the country where wages have risen the least relative to capital (0.72% vs. 3.02% percentage points of annual growth), followed by Italy (0.49% 2.10%) and France (0.83% 1.83%). In contrast, wages and capital intensity have moved hand in hand in Anglo-Saxon countries. This is consistent with the observation of the OECD (2018) that European countries are struggling

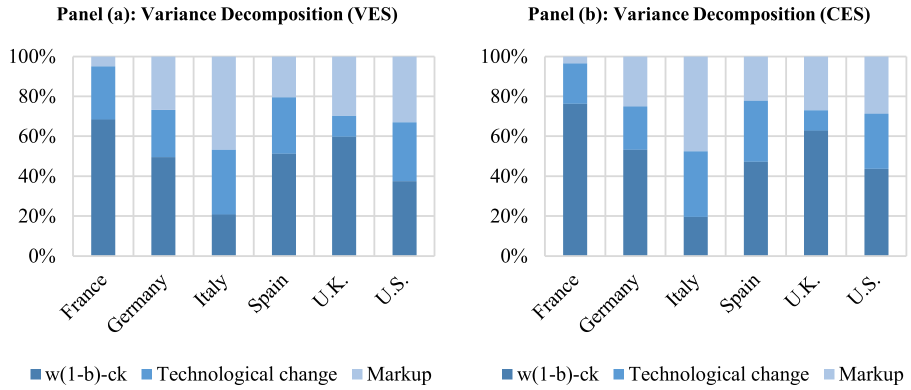

Fig. 6. Labor share variance decomposition with the CES/VES production function (1980-2020).
Source: Authors' own simulation.

Table 5
Simulated labor shares' fit (1980-2020).

Country	Panel (a): ARDL				Panel (b): IV				Panel (c): GIV			
	VES		CES		VES		CES		VES		CES	
	RSS	**R2**	**RSS**	**R2**	**RSS**	**R2**	**RSS**	**R2**	**RSS**	**R2**	**RSS**	**R2**
France	0.30	0.09	0.29	0.09	0.60	0.36	0.62	0.38	0.61	0.37	0.63	0.39
Germany	0.53	0.28	0.55	0.30	0.65	0.42	0.62	0.38	0.70	0.65	0.49	0.42
Italy	0.53	0.28	0.57	0.32	0.51	0.26	0.48	0.23	0.53	0.29	0.49	0.24
Spain	0.60	0.37	0.59	0.35	0.74	0.55	0.73	0.53	0.78	0.61	0.74	0.54
U.K.	0.71	0.51	0.73	0.53	0.64	0.41	0.65	0.42	0.83	0.69	0.81	0.65
U.S.	0.30	0.09	0.29	0.09	0.56	0.31	0.26	0.07	0.60	0.36	0.33	0.11
Mean	0.495	0.269	0.504	0.280	0.618	0.387	0.560	0.336	0.676	0.493	0.579	0.391

Note: RSS = Residual sum of squares; the R^2 and the RSS are obtained from regressing the actual trend component of the LS on a constant and the simulated LS in each scenario. Outliers (i.e., the observations greater than 2.5 the interquartile difference) were excluded from the simulation.

not only with slow productivity growth, but also with a slowdown in real wage growth relative to productivity growth. From a cross-country perspective, real wage growth rates were stronger in the United Kingdom and the United States, while growth in the capital stock per worker has remained in line across different countries.

The annual rate of *technological change* estimated with the VES is obtained by taking the derivative of log a with respect to time (*i.e.* λ).[25] Consistent with other studies using similar data, the time trend coefficient estimated from equation (6) is negative and significant, indicating that for the six countries of our sample the growth of the log of real GDP has, on average, decreased over time between 1980 and 2020 (λ=-0.31%) (Karagiannis et al., 2005; Heshmati and Rashidghalam, 2020; Bellocchi et al., 2021).

The value of the labor share is set so that the predicted labor share coincides with actual labor shares midway through the simulated period. Since the growth rate of technological progress is constant, we abstain from business cycle considerations. Instead, we distinguish three different scenarios that combine: (i) the net effect of wage and the capital-labor ratio; (ii) the presence of price markups; and (iii) the presence of price markups and technological progress. In this way we can infer the extent to which wage growth and capital deepening, technological change or price markup play a dominant role in explaining the actual trajectories of labor share. The model's predictions with either the CES and the VES are then compared with observed value of the labor share at market prices and the variance is analyzed. The results of the simulations are shown in Fig. 4.

Since the parameter b is estimated (on average) to be lower than c, the labor' share raising force, which is given by the net effect of the real wage $(1-b)\dot{w}$ and the capital-labor ratio $(-c\dot{k})$ growth rates, has been (on average) positive over the period (Fig. 5, Panel a). Thus, labor shares would have increased at a rate between 0.08 percent (per year) in France and 1 percent in the United Kingdom if no technical changes had occurred and if markets had been competitive. In contrast, in the United States and Italy, the increase in capital deepening was not offset by an equally strong wage growth, and the negative impact on the labor share was 0.14 and 0.53 percent (per year), respectively.

However, the labor share declined between 1980 and 2020, at an average rate of 0.18 percentage points per year in Europe (-0.33 in Spain, -0.27 in Italy, -0.22 in France, -0.12 in Germany), while it has remained virtually constant in the United Kingdom.

[25] We assume that technological progress is neutral and proceeds at a geometric rate: $A = A_0e^{\lambda t}$. The annual growth rate of technological progress is obtained by taking the derivative of *log y* with respect to t $(\dot{A}/A)$, i.e., λ.

Table A1
Capital-output ratios and labor shares (1980-2020).

Country	Capital output ratio (K/Y)				Labor share (LS)			
	1980-1990	1990-2000	2000-2010	2010-2020	1980-1990	1990-2000	2000-2010	2010-2020
France	-0.45%	0.33%	0.02%	-0.13%	-1.20%	-0.42%	0.28%	0.21%
Germany	0.08%	0.74%	1.98%	1.09%	-0.79%	0.02%	-0.39%	0.68%
Italy	-0.06%	-0.20%	0.73%	1.05%	-0.81%	-1.34%	0.56%	-0.16%
Spain	0.48%	0.13%	1.34%	0.82%	-0.93%	-0.24%	-0.26%	-0.20%
United Kingdom	-1.01%	-0.80%	-0.04%	0.57%	-0.67%	-0.01%	0.52%	0.63%
United States	-0.72%	-0.73%	0.85%	0.19%	-0.14%	0.09%	-0.78%	0.27%
Average	-0.28%	-0.09%	0.81%	0.60%	-0.76%	-0.32%	-0.01%	0.24%
St. Dev	0.005	0.006	0.008	0.005	0.004	0.005	0.005	0.004

Note: Labor share at market prices. Source: Authors' calculation on AMECO data.

Table A2
Empirical literature on VES production function.

Study	*Country*	*Period*	*Data*	*Sector*
Lu and Fletcher (1968)	U.S.	1957	Time Series	Manufacturing
Sato and Hoffman (1968)	U.S. and Japan	1909-60	Time Series	Non-farm business sector
Lovell (1968)	U.S.	1949-63	Time Series	Manufacturing
Diwan (1970)	U.S.	1955-57	Cross Section	Manufacturing
Revankar (1971a)	U.S.	1957	Cross Section	Manufacturing
Revankar (1971b)	U.S.	1929-53	Time Series	Non-farm business sector
Lovell (1973a)	U.S.	1958	Cross Section	Manufacturing
Lovell (1973b)	U.S.	1947-68	Time Series	Manufacturing
Meyer and Kadiyala (1974)	U.S.	-	Cross Section	Agriculture
Tsang (1976)	U.S.	1957	Cross Section	Food-processing Industries
Roskamp (1977)	West Germany	1950-60	Time Series	Manufacturing
Kazi (1980)	India	1973-75	Cross Section	Manufacturing
Battese and Malik (1988)	Pakistan	1976-1981	Cross Section	Food-processing Industries
Bairam (1989)	Japan	1878-1939	Time Series	Economy
Bairam (1990)	U.S.S.R.	1950-75	Time Series	Economy
Zellner and Ryu (1998)	U.S.	1957	Cross Section	Transportation equipment
Karagiannis et al. (2005)	82 countries	1960-1987	Panel (NLLS)*	Economy

Note: *Nonlinear least squares (NLLS). Among the studies which employed linearly homogeneous VES, we mention here only those with a significant impact on the later developments of the literature.

This can be attributed mainly to two additional factors, namely price markups and technology. Based on the results in Section 4.2, we found that aggregate markups were stable before 1985. Yet, in most countries there has been a steady increase since then. Our simulation allows us to decompose the impact of markups on the labor share ($b\dot{m}$). This latter is shown in Fig. 5, Panel b.

The remaining part of the variation in the labor share can instead be traced back to the nature of technological progress. When technical change occurs, the increment of output due to technical change may be allocated to capital or labor, depending on the value of b. We estimated b to be significantly less than unity for the entire sample of countries. Specifically, our estimate of the parameter ranges from 0.07 in the United Kingdom to 0.30 in the United States. Between 1980 and 2020 technical change has taken place at an average rate of 0.31 percent a year. Since b was less than unity, changes in technique have on average depressed labor's share by an annual rate of $\lambda = -0.22$ percent. From this perspective, Italy has been most affected by the negative effect of technology, with a 1.35-percent reduction in its labor share each year since 1980, followed by the U.S. (0.53) and Spain (0.31). On the other hand, this factor has played a positive role in the Germany and United Kingdom, where the annual contribution to the evolution of the labor share has been respectively of 0.29 and 0.57%.

The net effect of the three forces is negative. Therefore, we expect the labor share of total output to decrease slightly over time. This forecast is close to the observed rate of change in the labor share at market prices.

Finally, to quantify how much each of the different forces analyzed contributed to the total observed movement in labor shares, we decompose its total variance. The averages for each period are shown in Fig. 6.

Using the VES (Fig. 6, Panel a), we found that the combined dynamics of wages and capital intensity accounted on average for 37 percent of the labor share movement observed in the United States, with the remaining 61 percent due to price markup (32 percent) and technological progress (29 percent). Similarly, when we perform the same exercise on European economies, we find respectively shares of 46%, 27% and 24%. In the United Kingdom, on the other hand, the role of technology has been limited (10%). These results are not overturned by CES, but as expected, there is a redistribution of weight among the various explanatory factors (Fig. 6, Panel b), as capital intensity L no longer exerts direct influence on σ. Both the models perform reasonably well in explaining the evolution of the labor share. However, the resulting time series obtained with the VES provides the closest approximation to the actual labor share trajectories in both economies, with an average residual sum of squares (RSS) and R2 of, respectively, 0.68 and 0.49 (against 0.58 and 0.39 for the CES). The overall fit is evaluated in Table 5.

Table A3
Estimation of the CES and VES production function parameters (ARDL).

ARDL model (1 lag) - EC

Country	VES				CES		
	ln(w)	ln(k)	t	const	ln(w)	t	const
France	0.55	0.61	-0.0025	3.26	0.61	-0.0006	0.64
	(0.289)	(0.275)*	(0.001)*	(2.01)	(0.455)	(0)	(1.855)
Germany	0.15	0.21	0.0019	-6.99	0.28	0.0015	-5.52
	(0.108)	(0.092)*	(0.001)	(2.198)**	(0.109)*	(0)	(2.058)*
Italy	0.05	0.93	-0.0037	5.37	0.15	-0.0006	0.93
	(0.105)	(0.338)**	(0.001)*	(2.421)*	(0.28)	(0)	(1.738)
Spain	0.07	0.77	-0.0019	3.02	0.20	-0.0011	1.67
	(0.131)	(0.476)	(0)***	(0.694)***	(0.129)	(0)	(0.582)**
United Kingdom	0.08	0.28	0.0026	-8.28	0.05	0.0025	-6.33
	(0.067)	(0.091)**	(0.001)	(3.027)*	(0.129)	(0.001)	(3.292)
United States	0.43	0.85	-0.0033	2.64	0.66	-0.0002	-0.11
	(0.127)**	(0.155)***	(0.001)	(3.12)	(0.917)	(0.001)	(3.362)

ARDL model (2 lag) - EC

Country	VES				CES		
	ln(w)	ln(k)	t	const	ln(w)	t	const
France	0.70	0.23	-0.0014	2.00	0.50	-0.0005	0.48
	(0.552)	(0.515)	(0.001)	(2.184)	(0.449)	(0.001)	(1.9)
Germany	0.18	0.24	0.0015	-7.14	0.35	0.0010	-5.02
	(0.116)	(0.099)*	(0.001)	(2.579)*	(0.109)**	(0.001)	(2.277)*
Italy	0.10	0.50	-0.0022	2.75	0.11	-0.0006	0.68
	(0.114)	(0.394)	(0.002)	(3.221)	(0.24)	(0.001)	(1.876)
Spain	0.10	0.51	-0.0024	3.20	0.20	-0.0008	0.57
	(0.054)	(0.167)**	(0)***	(0.935)**	(0.067)**	(0)*	(0.717)
United Kingdom	-0.01	0.32	0.0034	-9.47	-0.03	0.0031	-7.40
	(0.133)	(0.145)*	(0.001)*	(3.33)**	(0.19)	(0.001)	(3.573)*
United States	0.15	0.39	0.0013	-4.03	0.32	0.0006	-1.63
	(0.26)	(0.413)	(0.002)	(3.775)	(0.625)	(0.001)	(3.711)

ARDL model (3 lag) - EC

Country	VES				CES		
	ln(w)	ln(k)	t	const	ln(w)	t	const
France	-0.13	0.70	-0.0001	-0.29	0.07	0.0001	-0.56
	(1.068)	(0.869)	(0.001)	(2.554)	(0.764)	(0.001)	(1.89)
Germany	0.41	0.02	0.0002	-4.91	0.45	-0.0001	-4.65
	(0.151)*	(0.151)	(0.001)	(3.89)	(0.065)***	(0)	(2.481)
Italy	0.10	0.24	-0.0013	1.39	-0.04	-0.0003	0.32
	(0.15)	(0.639)	(0.002)	(3.919)	(0.426)	(0.001)	(2.013)
Spain	0.10	0.43	-0.0029	3.57	0.17	-0.0007	0.11
	(0.039)*	(0.125)**	(0)**	(1.246)**	(0.05)**	(0)	(0.825)
United Kingdom	0.04	0.19	0.0044	-12.20	-0.11	0.0025	-5.70
	(0.121)	(0.106)	(0.002)*	(4.417)*	(0.362)	(0.002)	(4.437)
United States	0.11	0.66	0.0009	-5.02	0.68	-0.0003	0.29
	(0.176)	(0.175)**	(0.002)	(4.11)	(1.25)	(0.001)	(3.501)

Note: Equation (9), using labor productivity (y) as explanatory variable, is estimated the six advanced OECD countries of the sample, by means of an Autoregressive distributed lag (ARDL) model. Labor input adjusted for human capital (see section 4.1). The dynamic specification of the model follows an automatic selection, with a maximum of 3 lags (three years) for both the dependent variable and dynamic regressors. Long-run coefficients are derived from the reparameterization in EC form. P-values in parenthesis. *** Significantly different from 0 at the 1% level. ** Significantly different from 0 at the 5% level. * Significantly different from 0 at the 10% level. Equations passes the standard misspecification and structural stability tests (serial correlation, linearity, normality, and heteroscedasticity).

To sum up, we can draw *several conclusions* from this empirical exercise. First, the VES model can proxy the path of the labor share in last decades in the six economies considered more accurately than the CES one. Second, real wage growth, capital deepening, technological progress and the price markup contribute to explaining this path, although the former factors are clearly more determinant. Third, we found that the explanatory power of capital intensity is lower in Europe and higher in the United States. This is consistent with the larger ratio of capital stock per employee in the U.S. (Raurich et al., 2012). Finally, the VES model provides strongest empirical support to the joint increase in capital deepening and labor productivity along with a declining labor share when σ is estimated to be lower than unity. This is because capital intensity contributes to determining the labor share endogenously through its impact on the elasticity of substitution via the b and c parameters, and the CES model is unable to capture this effect.

Table A4
Estimation of the CES and VES production function parameters (2SLS).

IV model							
Country	VES				CES		
	ln(w)	ln(k)	t	const	ln(w)	t	const
France	0.06	0.49	0.0014	-6.68	0.06	0.0080	-4.17
	(0.136)	(0.092)***	(0.003)	(0.589)***	(0.177)	(0.004)	(0.745)
Germany	0.22	0.10	0.0031	-5.68	0.19	0.0048	-5.04
	(0.096)*	(0.166)	(0.003)	(1.044)***	(0.096)***	(0.002)***	(0.418)***
Italy	0.27	0.48	-0.0149	-7.40	0.36	-0.0104	-5.40
	(0.06)***	(0.131)***	(0.002)***	(0.52)***	(0.075)***	(0.003)***	(0.341)***
Spain	0.19	0.07	-0.0017	-5.15	0.20	-0.0008	-4.89
	(0.078)*	(0.19)	(0.002)	(0.667)***	(0.071)***	(0.001)	(0.322)***
United Kingdom	0.28	0.18	-0.0004	-6.33	0.48	-0.0054	-6.34
	(0.117)*	(0.107)	(0.003)	(0.448)***	(0.162)***	(0.005)***	(0.661)***
Dummy 09-13	0.37	0.19	-0.0037	-6.75	0.46	-0.0048	-6.26
	(0.085)***	(0.162)	(0.001)*	(0.499)***	(0.057)***	(0.002)**	(0.237)***
United States	0.31	0.87	-0.0062	-9.70	0.87	-0.0129	-7.65
	(0.16)	(0.074)***	(0.005)	(0.598)***	(0.497)**	(0.015)***	(2.095)***
Dummy 08-09	0.43	0.88	-0.0103	-10.26	0.26	0.0075	-5.07
	(0.062)***	(0.131)***	(0.003)***	(0.805)***	(0.091)**	(0.002)**	(0.39)***

Notes: Equation (9), using labor productivity (y) as explanatory variable, is estimated for the six advanced OECD countries of the sample. Labor input adjusted for human capital (see section 4.1). Regressions are estimated by instrumented (2SLS) regression with robust standard errors; Real wages and the capital-labor ratios are instrumented using: (1) the total population, (2) wages in the government sector, and (3) the real capital stock owned by the government; In addition, given the presence of a structural break in the data (Tables OA6 and OA7 in the Online Appendix B) a significant time dummy was included in the model for the period 2008-2020. In alternative model specifications we also include country dummies for the United Kingdom (2009-2013) and the United States (2008-2009) because of the structural break tests and their significance. The first-stage regressions are not reported in the table. Robust standard errors appear in parentheses. *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively. Figures in parentheses in the F- row are p- values.

Table A5
Estimation of the CES and VES production function parameters (GIV) - Imperfect Competition

GIV model							
Country	VES				CES		
	ln(w)	ln(k)	t	const	ln(w)	t	const
France	0.11	0.48	0.00	-6.81	0.30	0.00	-5.16
	(0.058)*	(0.058)***	(0.001)	(0.364)***	(0.092)**	(0.002)	(0.387)***
Germany	0.19	0.17	0.00	-5.88	0.18	0.01	-4.98
	(0.068)**	(0.057)**	(0.001)	(0.426)***	(0.078)*	(0.001)**	(0.34)***
Italy	0.23	0.49	-0.01	-7.25	0.32	-0.01	-5.22
	(0.046)***	(0.116)***	(0.001)***	(0.472)***	(0.052)***	(0.001)	(0.241)***
Spain	0.16	0.19	0.00	-5.62	0.23	0.00	-5.00
	(0.035)***	(0.051)***	(0)***	(0.21)***	(0.033)	(0)*	(0.152)***
United Kingdom	0.07	0.26	0.01	-5.88	0.14	0.01	-4.96
	(0.046)	(0.052)***	(0.001)***	(0.294)***	(0.082)	(0.002)	(0.335)
Dummy 09-13	0.28	0.29	0.00	-6.85	0.41	0.00	-6.05
	(0.054)***	(0.099)**	(0.001)	(0.349)***	(0.049)***	(0.001)*	(0.207)***
United States	0.30	0.83	-0.01	-9.42	-0.10	0.02	-3.57
	(0.047)***	(0.05)***	(0.001)**	(0.412)***	(0.071)	(0.002)***	(0.301)***
Dummy 08-09	0.39	0.80	-0.01	-9.66	0.12	0.01	-4.47
	(0.049)***	(0.092)***	(0.002)**	(0.629)***	(0.056)*	(0.001)***	(0.244)***

Notes: Equation (9), using labor productivity (y) as explanatory variable, is estimated the six advanced OECD countries of the sample. Labor input adjusted for human capital (see section 4.1). Regressions are estimated using the generalized instrumental variable (GIV) procedure developed by Fair (1970); Real wages and the capital-labor ratios are instrumented using: (1) the total population, (2) wages in the government sector, and (3) the real capital stock owned by the government; In addition, given the presence of a structural break in the data (Tables OA6 and OA7 in the Online Appendix B) a significant time dummy was included in the model for the period 2008-2020. In alternative model specifications we also include country dummies for the United Kingdom (2009-2013) and the United States (2008-2009) because of the structural break tests and their significance. The first-stage regressions are not reported in the table. Robust standard errors appear in parentheses. *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively. Figures in parentheses in the F- row are p- values.

6. Concluding remarks

Income distribution theories are based on Cobb-Douglas or CES production functions. Unlike functions with unit elasticity, the CES allows the labor share to be influenced by endogenous variables. However, it is still subject to the restrictive assumption that σ is a

Table A6
Estimation of the CES and VES production function parameters (GIV) – Perfect Competition.
Residual-Based Augmented Dickey-Fuller Test.

GIV model Country	VES ln(w)	ln(k)	t	const	CES ln(w)	t	const
France	-0.04	0.49	0.00	-6.27	-0.99	0.02	-0.05
	(0.258)	(0.088)***	(0.003)	(1.404)***	(0.244)***	(0.002)***	(1.003)
Germany	0.27	0.12	0.00	-6.07	0.31	0.01	-5.64
	(0.095)**	(0.058)*	(0.001)***	(0.441)***	(0.116)**	(0.001)***	(0.482)***
Italy	0.74	0.34	0.00	-8.76	1.34	0.00	-9.86
	(0.146)***	(0.074)***	(0)***	(0.41)***	(0.122)***	(0)	(0.55)***
Spain	0.39	0.16	0.00	-6.44	0.51	0.00	-6.20
	(0.075)***	(0.044)***	(0)***	(0.303)***	(0.066)***	(0)*	(0.302)***
United Kingdom	0.09	0.25	0.01	-6.03	0.19	0.01	-5.27
	(0.031)**	(0.042)***	(0)***	(0.161)***	(0.061)**	(0.001)***	(0.22)***
Dummy 09-13	0.38	0.32	0.00	-7.39	0.56	0.00	-6.64
	(0.072)***	(0.09)***	(0)	(0.305)***	(0.065)***	(0.001)	(0.247)***
United States	0.57	0.35	0.00	-8.07	0.93	0.00	-7.72
	(0.066)***	(0.05)***	(0)	(0.145)***	(0.089)***	(0.001)	(0.334)***
Dummy 08-09	0.63	0.30	0.00	-8.03	0.62	0.00	-6.61
	(0.043)***	(0.038)***	(0)	(0.246)***	(0.067)***	(0.001)***	(0.256)***

Country/Lags	Panel (a): No trend 1 lag	2 lags	3 lags	Panel (b): Trend 1 lag	2 lags	3 lags
France	-1.599	-1.597	-1.47	-1.718	-1.7	-1.631
Germany	-5.259*	-3.451	-3.554	-5.475*	-3.68	-3.676
Italy	-2.377	-2.206	-2.237	-2.304	-2.252	-2.445
Spain	-3.748*	-3.683*	-3.906*	-3.961	-3.868	-4.103*
United Kingdom	-3.745*	-3.294	-2.637	-3.749	-3.306	-2.643
United States	-4.18*	-5.803*	-4.383*	-4.183*	-5.484*	-4.145*
Critical Values	1%	5%	10%	1%	5%	10%
	-4.685	-3.967	-3.614	-5.175	-4.435	-4.073

Notes: Equation (9), using labor productivity (y) as explanatory variable, is estimated the six advanced OECD countries of the sample. Labor input adjusted for human capital (see section 4.1). Regressions are estimated using the generalized instrumental variable (GIV) procedure developed by Fair (1970); Real wages and the capital-labor ratios are instrumented using: (1) the total population, (2) wages in the government sector, and (3) the real capital stock owned by the government; In addition, given the presence of a structural break in the data (Tables OA6 and OA7 in the Online Appendix B) a significant time dummy was included in the model for the period 2008-2020. In alternative model specifications we also include country dummies for the United Kingdom (2009-2013) and the United States (2008-2009) because of the structural break tests and their significance. The first-stage regressions are not reported in the table. Robust standard errors appear in parentheses. *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively. Figures in parentheses in the F- row are p- values.
Note: *Null Hypothesis Rejection (no cointegration), 10% critical threshold.

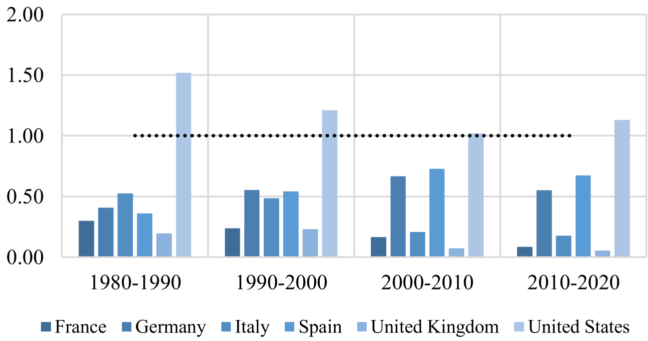

Fig. A1. Elasticity of substitution (σ), average over 10-year periods (1980-2020).
Note. Elasticity of substitution estimated with the VES production function under *imperfect competition*. After a peak in the 2000s, the average value of σ has decreased (on average) over time (however without major falls). Due to the high sensitivity to movements in real wage (w), real interest rate (r), capital-labor ratio (k) and price markup (m), we report average values over 10-year time periods between 1980 and 2020. The black dotted line represents an elasticity of substitution equal to one. Outliers in K/L values (defined as deviations greater than 1.5 times the interquartile range from q1 and q3) were excluded from the calculation.

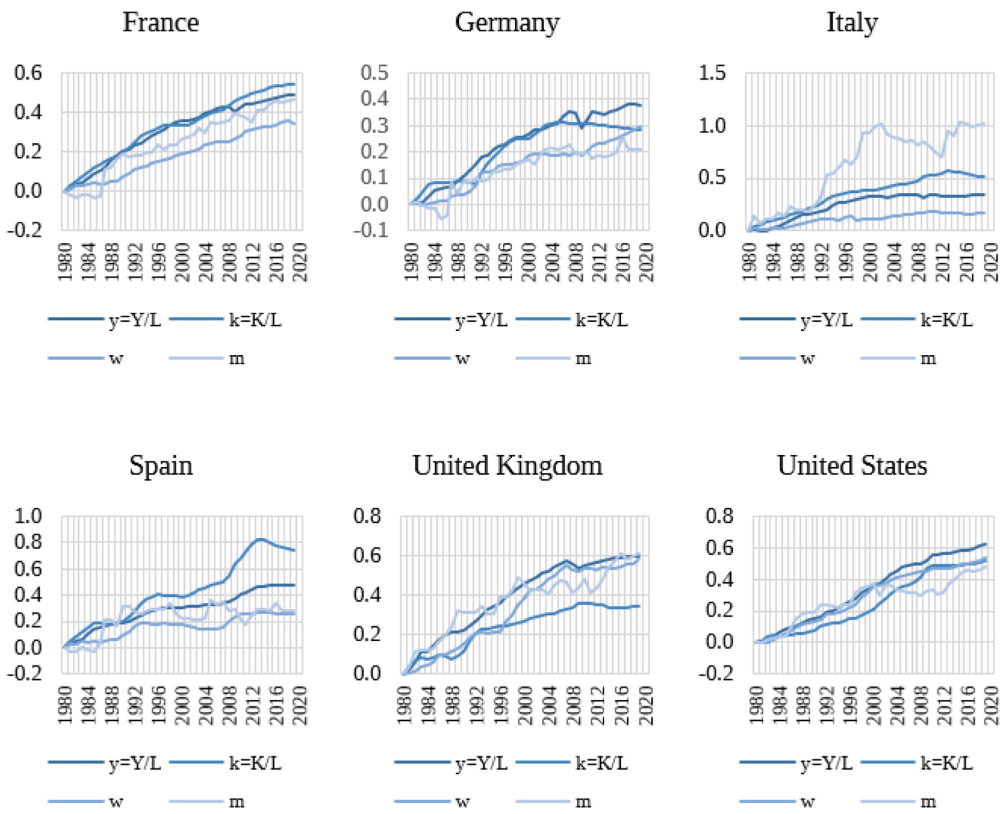

Fig. A2. Non-stationarity of the time series (cumulative d_log) (1980-2020).
Note: Index, 1980=0. Source: Authors' calculation on AMECO data.

technological constant, unchanged by factor accumulation and factor-augmenting technological change (Growiec and Mućk, 2020).

In this paper, we use a VES production function to study the evolution of the labor share in six advanced economies and provide an explanation based on the possibility that σ is endogenous to capital intensity. Notably, the VES, developed by Lu and Fletcher (1968), *extended here under imperfect competition*, includes the CES as a special case.

As a first important result, our analysis suggests that elasticity varies significantly with the capital-labor ratio. Therefore, we conclude that σ estimated by the CES and the VES is different, and the former is likely to be biased. We also conclude that a VES production function better captures the observed change in labor shares over time.

In the VES model, changes in labor share depend on the parameters of the production function, the relative supply of capital and the labor force, the growth of wage rates, and technical change. All the economies studied have experienced specific trajectories in the evolution of these variables, which traditional models struggle to explain. The VES model reconciles these dynamics by revealing the connection between capital intensity, the elasticity of substitution and technological progress. We believe that the most important contribution of the paper lies in the rationale behind these values.

By shifting the focus from the elasticity of substitution to the structural parameters b and c, the VES can explain capital deepening and declining labor shares when σ is lower than one. In accordance with recent empirical works, we found evidence for the elasticity to be currently lower than one in all the countries analyzed, with the only exception of the U.S. (when imperfect competition is considered). The paper also uncovers the bias that the assumption of perfect competition introduces in the estimates of σ. This is relevant if we consider that in macroeconomic models, small differences in the value of this parameter have strong implications for the relationship between aggregate variables.

Finally, we provide a quantitative evaluation of the extent to which the trajectory of the labor share is explained by each of the factors identified by theory. This is obtained through a *simulation exercise* which examines its basic predictions. Overall, the theory can explain much of the behavior of factor shares over a 40-year period, and the model provides predictions that are consistent with empirical data. The results confirm some previous findings and reveal new ones. Variance decompositions suggest that the joint dynamics of real wages and capital intensity are a key driver of labor share and account for about 47 percent of the total movement in the labor share. However, other factors, such as price markup (26 percent) and technological change (25 percent), played a non-negligible role in explaining its aggregate decline.

This paper has some important *policy implications* as it highlights the link between growth and distribution policies. For advanced economies, innovation is an important driver of sustainable productivity growth. National governments have ample scope to set incentives for productive investment that determine long-term productivity growth. In a CES world, capital accumulation influences the labor share through the elasticity of substitution, which is fixed and *exogenous*. In a VES world, the effect of a stronger capital-labor ratio on the labor share depends on its starting level and can be negative *even if* the elasticity of substitution is less than one. To sum up, while a higher value of σ enables the economy to reach a higher value of output per capita, this may imply a higher inequality. Therefore, a government concerned about the worsening inequality created by a higher value of capital-deepening may opt for a set of policies that keep σ at a comparatively lower level. This can be done, for instance, by strengthening labor unions, which can put pressure on governments and the private sector to keep the labor share at a high level, thus preventing a decline in wages created by a decline in the labor share and the marginal product of labor.

Author Statement

Author contributions. All the authors contributed to the *study conception* and *design*. All authors contributed to the *methodological choices* and *empirical investigation*. All the authors read and approved the final manuscript.

Declarations of Competing Interest

None.

Data availability

Data will be made available on request.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmacro.2023.103518.

Appendix

A1. Figures and tables

A2. Discussion of unit root and cointegration test results

Before estimating the model, we must pay attention to some typical issues that arise when considering the potential non-stationary nature of the time series involved. Figure A1 shows the four series that form the basis of our estimates, where the logarithm of the variables was normalized to zero in the first year (i.e., 1980). Two facts emerge from the figure. First, the graphs uncover the potential non-stationarities in the time series considered: the *natural logarithm* of y_t, w_t, k_t, and m_t all clearly trend upwards. Second, at least three variables follow similar trends. This suggests that the correlations captured by a standard regression might be spurious, unless the time series are *cointegrated* (Granger and Newbold, 1974; Phillips, 1986).

Tables OA2-OA5 (in the Online Appendix) provide a summary of the unit root tests performed on each series and for each country. The first column of the table presents the results of an Augmented Dickey-Fuller (1979) test, the second column reports the t-statistics for the Phillips-Perron (1988) test, and the third column the tau of the DF-GLS test by Elliott et al. (1996). Finally, in the last column we report the results for the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test (Kwiatkowski et al., 1992). While the first three tests have as a null hypothesis the non-stationarity of the series being tested, while the latter has the opposite null, i.e., the series being tested is trend stationary.

The ADF and PP tests are asymptotically equivalent but may differ substantially in finite samples due to the different ways in which they correct for serial correlation in the test regression. In general, Schwert (2002) asserts that ADF and PP tests have low power against I(0) alternatives that are close to being I(1). That is, unit root tests cannot distinguish highly persistent stationary processes from nonstationary processes. Further, the PP test is more size distorted than the ADF test. Therefore, the literature agrees that to have maximum power against very persistent alternatives, the tests proposed by Elliot, et al. (1996) and Kwiatkowski et al. (1992) should be preferred (Jafari et al., 2012; Katircioglu et al. 2014).

It is evident from Tables OA2-OA5 that the combined evidence from the unit root tests allows us to accept the unit root hypothesis in all the countries considered with a high degree of confidence. At the bottom of each table, we report the results of the same test performed on each of the four series, expressed in first differences (*dly*, *dlw*, *dlk*, *dlm*). In this case, all the results point to a rejection of the null hypothesis of the series being integrated of order two at 1% in most of the cases. We therefore conclude that all four series are *nonstationary and integrated of order one*, which implies that OLS estimates in level are potentially subject to a spurious regression bias. In fact, as shown by Phillips (1986), in this situation, OLS estimates will not be consistent unless a linear combination of the dependent and independent variables is stationary, that is, the variables are *cointegrated*. As discussed by Antràs (2004), this is not necessarily true for the GIV estimates. In fact, in the latter case the existence of a unit root in the OLS residuals implies that the GIV estimates are asymptotically equivalent to the estimates that would be obtained with the differenced data (Blough, 1992; Hamilton, 1994). In other words, estimates are consistent but may overlook some important long-term information.

Therefore, a cointegration analysis is performed to assess the possibility of long-run convergence of our variables. Tables A6/7 present the results from two co-integration tests. Specifically, the first table considers Engle and Granger's (1987) residual-based two-step ADF test, which hinges on testing the stationarity of the residuals from the OLS regression of equation (6). As pointed out by Engel and Granger (1987), the critical values of standard unit root tests are not appropriate when applied to the OLS residuals because they lead to too many rejections of the null hypothesis of no cointegration. MacKinnon (1991) has linked the appropriate critical values to the sample size and to a set of parameters that only vary with the specification of the cointegration equation, the number of variables and the significance level. These critical values are reported below the table.

As is apparent from Table A6, the results are not conclusive. When only a constant is included in the model (Panel a), the test suggests that the series in the estimation may in fact be *cointegrated* and move along a common trend with a reasonable level of confidence. However, when a linear trend is considered in the first-step regression (Panel b), the null hypothesis of non-stationarity of the residuals is not always rejected. In addition, the results remain stable to the inclusion of more lags.

Therefore, Table A7 complement the analysis with the maximum likelihood cointegration test suggested by Johansen and Juselius (1990), which tests the null hypothesis of the existence of r cointegrating vectors against the alternative of the existence of (r+1) cointegrating vectors. Implementing the test requires the definition of a specific model for the cointegration equation as well as choosing the number of lags of the first difference of the variables to be included in the estimation. We choose a model with a constant and compute the trace statistics with one, two and three lagged first differences of the data. The results are more robust than those in Table A6.

When the estimation includes a lag, the *null hypothesis of no cointegration is rejected* for all the countries considered. To sum up, the results of the cointegration tests indicate that estimates of equation (6) can be interpreted with relative confidence with respect to spurious correlation problems.

In fact, in those cases where we can reject the null hypothesis of no cointegration, estimates are super-consistent, since they converge to their true value at a higher speed than in the absence of non-stationarity (Phillips and Durlauf, 1986; Stock, 1987). Further, estimates are consistent even in the presence of autocorrelation in the disturbances and endogeneity of the regressors. Conversely, when the results are conflicting, the estimation may suffer from a spurious regression bias. As discussed by Hamilton (1994), a natural cure for spurious regressions would be to difference the data before estimating the equations. The disadvantage of this approach is that important long-run information would be lost, and the interpretation of our estimates in connection to the structural parameters of the production function would become less transparent.

Given the satisfactory results of cointegration tests, we rely on the GIV model as our main estimator, which is used to reinforce the basic estimates obtained with the cointegration ARDL model of Pesaran and Shin (1999) and Pesaran et al. (2001).

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