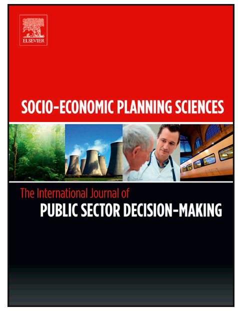

Identification of spatial regimes of the production function of Italian hospitals through spatially constrained cluster-wise regression

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Highlights

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- A methodology is developed to identify spatially constrained regimes, in which the production units are maximally homogeneous in functional terms.
- A cost function is estimated for a large sample (681) Italian hospitals.
- Spatial heterogeneity of relevant aspects, like demand, internal organization, clinical and managerial governance, etc., can be associated to the heterogeneity of the identified spatial regimes, to be used as effective information for policy implications.

Identification of spatial regimes of the production function of Italian hospitals through spatially constrained cluster-wise regression

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Abstract

Building on the idea that different hospitals may operate with different technologies, the objective of this paper is a spatial characterization of the production function of hospital services through the identification of different spatial regimes. We introduce an original methodology to identify spatially constrained regimes, namely spatially constrained portions of territory in which the production units are maximally homogeneous in functional terms. The empirical algorithm can be described as a k-means cluster-wise regression procedure in which the units are belonging to a proximity graph and where distance is assessed in regressive terms. The analysis is implemented, first, on simulated data and, then, on output and input data of Italian hospitals for the year 2010. Our results, besides their methodological value, allow to shed light on the working of the hospital sector in Italy. The heterogeneity of the identified technological regimes can be associated to spatial heterogeneity of relevant aspects, like demand, internal organization, clinical and managerial governance, etc., and consequential policy implications can be, therefore, gathered.

1. Introduction

The assessment of the efficiency of healthcare provision is a well-covered subject by now. The topic is relevant not only for its contribution to the general knowledge of the operation of healthcare systems and their components, but also for the potential use of the efficiency assessment results for important policy purposes like performance monitoring and resource allocation (Newhouse, 1994; Magnussen, 1996; Smith, 2002).

The most critical issue for the theoretical and practical relevance of the information provided by the application of the different techniques, already pointed out by Newhouse (1994), is related to the unavoidable heterogeneity of the providers of services under scrutiny. While several studies on efficiency of healthcare provision have dealt with this issue, considering the impact of the heterogeneity along different dimensions, like the nature of the outputs of provision or of providers (profit/no profit, teaching status, etc.), or the different clinical areas, we will focus on the heterogeneity characterizing hospital care at a very general level, in terms of its production function. Production functions are a representation of the general technology conditions characterizing the actual realization of a production process and can be, therefore, regarded as a sort of "primitive" (Ackerberg et al., 2007) of efficiency measurement, even if they are not an analytical tool for this latter purpose and, in a sense, provide more general information. We will, therefore, examine the potentially different technologies used by hospitals to realize their services and explore their economic and policy implications.

Even if the research effort on the production function of healthcare services is not so broad as the one on the production frontiers and on the production function of health (Cohen, 2014), still there are a few works that deal with its estimation (Reinhardt, 1972; Scheffler and Kushman, 1977; van Montfort, 1981; Jensen and Morrisey, 1986; Thurston and Libby, 2002; Grassetti et al., 2005; Reyes SantÃŋas et al., 2011; Mateus et al., 2015; Antelo et al., 2017). The most recent studies make an attempt to deal with heterogeneity of hospitals and hospital care, estimating the production function for different groups of hospitals. Reyes SantÃŋas et al. (2011) work on regional (Galicia) Spanish data and differentiate hospitals by size, in terms of number of beds, identifying three groups (small, medium and large), while Antelo et al. (2017), them too for Spanish hospitals, estimate a production function separately for different clinical services: gynaecology and obstetrics, general and digestive surgery, internal medicine, traumatology and orthopaedic surgery. The objective of our study is, instead, a spatial characterization of the production function of hospital services through the identification of different spatial regimes for the technologies of production of hospital services. In other words, we develop the idea, already at the basis of other works (Reyes SantÃŋas et al., 2011; Antelo et al., 2017), that different hospitals may operate with different technologies but considering these variations in technology as "*arising from locally-specific solutions that satisfy the environmental or social conditions within which firms operate*" (Billé et al., 2018).

The pursuit of such an objective requires, however, an appropriate and consistent methodology of analysis of data

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and of identification of the spatially homogenous production areas. Different problems need to be dealt with. The form of spatial heterogeneity is typically unknown, *i.e.* unobserved, and it can be related to heteroscedasticity, to spatially varying coefficients or both. Moreover, the heterogeneity in terms of spatially varying coefficients can be identified by clusters in space, also known in the literature as spatial regimes[1] (*e.g.* Anselin, 2010). The differences in coefficients that, strictly speaking, represent the fact that, in the different regimes, the marginal effect of the same independent variable may be different, could be interpreted, in a broader sense, as the result of the differentiated impact of omitted variables or, even better, of intangible factors - that cannot be captured by any observable variable and that are spatially characterized.

The standard approach in empirical studies is basically a two stage approach where the a priori exogenous information is used (Billé et al., 2018) to split the sample and then to carry out a regression on the different subsamples. Examples for the hospital production function are the ones mentioned above (Reyes SantÃŋas et al., 2011; Antelo et al., 2017; Cavalieri et al., 2020). The crucial difference with the approach proposed in this paper is that the identification of homogeneous areas and regressive estimation is carried out in a single stage to ensure that local areas are maximally homogeneous in functional terms. In other terms, if both zones and regressive function are not estimated at the same time, nothing ensures that the clusters/dummies obtained in the first phase (especially in spatial framework) are optimal - or even only correlated - with $E(Y|X)$ (Alvarez et al., 2012).

The aim of this paper is, therefore, to develop a methodology for the endogenous determination of homogenous spatial regimes, based on a spatially constrained and not overlapping cluster-wise regression algorithm that allows to identify geographically connected areas that are homogeneous in functional terms. We will, then, use this methodology for identifying the spatial regimes of the production function of hospitals, using Italian data for the year 2010. As we shall see, our results provide relevant information not only for the appropriate evaluation of each single provider's production choices, but also to put policy decisions, aiming at influencing the production process management, in the right context of homogenous production geographic areas.

The remainder of the paper is organized as follows. In section 2, the relevance of spatial heterogeneity for the health field is briefly reviewed and some approaches to the identification of spatial regimes are surveyed. Section 3 develops the methodological contribution of this paper, with the proposed procedure for the identification of spatial regimes, while section 4 explains the properties of our methodology with the help of a simulation exercise. Section 5 identifies the different spatial regimes of the production function of Italian hospital services, on the basis of the methodology developed in the previous sections. Section 6 is devoted to some concluding remarks.

[1]Please note that the term "spatial regime" should not be understood as a synonym for "cluster". More precisely, the term "cluster" does not presuppose any functional relationship between the variables considered, while the term "regime" is linked to the production function underlying the spatial process. Identifying different spatial regimes, therefore, is equivalent to estimating different functional production regimes.

2. Spatial heterogeneity in healthcare and the approaches to identify different spatial regimes

In the models for the economic analysis for spatial data the attention is typically focused on three main aspects: dependence, heterogeneity and scale (Bhattacharjee et al., 2012, 2016). The latter is not dealt with in this paper as microdata are used which, by definition, are the minimum possible analysis scale. The first two aspects, on the other hand, can have very significant effects on the quality of the estimates and can occur independently of each other and also, obviously, coexist. One of the forms in which heterogeneity is often observed concerns the local non-stationarity of the model parameters which will, therefore, be constant within aggregates of units whose borders are, however, unknown.

The residuals of a model, estimated without using this sample partition, will tend to form spatially contiguous groups of positive or negative values within the homogeneity groups of the parameters with the result that any spatial dependence test will be significant.

Heterogeneity and dependence, in addition to being able to coexist, have overlapping effects that are very difficult to distinguish if an appropriate method for identifying the boundaries of potential zones of homogeneity of the parameters is not used. This is a typical case of spatial dependence induced not by the presence of spill-overs (Glass et al., 2016; Carvalho, 2018; Fusco and Allegrini, 2020; Laureti et al., 2021), but by the misspecification of the model which would not take into account the variability of its parameters.

It often happens that, once these groups have been introduced into the model, any autocorrelation parameter reduce its effect until it becomes non-significant (see Section 5). It is not necessary or even obvious to obtain this result, as said the two aspects can exist simultaneously, but what we must expect is that introducing one aspect necessarily reduces the empirical effects of the other, perhaps not always until its extinction.

2.1. Spatial heterogeneity in the production of healthcare services

Several aspects in the health field have been examined with respect to their spatial dimension. For instance, the main risk factors for health can be geographically characterized (Baltagi et al., 2017) or, on the supply side, competition among providers can be generally regarded as localized. The instruments of spatial analysis have, therefore, found several applications in this field, both at the theoretical and at the empirical level. As far as the empirical research is specifically concerned, the applications of spatial econometrics models to health are mainly focused on the analysis of spatial dependence (for a recent survey of these applications see Baltagi et al., 2018) or on spatial clustering (Basu and Das, 2021). Examples of these works are the several studies on spatial dependence and spillovers of efficiency and quality choices of providers, whose estimation takes into account their spatial proximity (Mobley, 2003; Mobley et al., 2009; Brekke et al., 2011; Gravelle et al., 2014; Longo et al., 2017). Less attention has been devoted to the issue of spatial heterogeneity (Anselin, 1988, 2010). Baltagi et al. (2017) provides an analysis of the relationship between healthcare expenditure and income, at a global level, using a panel data model with heterogeneous slope coefficients, so as to take into account the heterogeneity in characteristics of countries across the world. An attempt to consider spatial heterogeneity is also made in Auteri et al. (2019) for the estimation of technical efficiency of Italian hospitals. Our work, to the best of our knowledge, is, therefore, one of the few to consider spatial heterogeneity in the field of spatial health econometrics.

There are several reasons to suppose that there may be different sources of spatially characterized heterogeneity (observed and not observed), affecting demand and supply of healthcare services (Bhattacharjee et al., 2014), which interact each other so as to require providers of services to adjust production technology to the different spatial contexts.

On the demand side, one of the main sources of spatial heterogeneity is related to differences in health needs. Baltagi et al. (2017), for instance, recently show how health needs and their determinants are heterogeneous across different countries, but there is plenty of evidence that they can also vary within the same country. Phillips II et al. (2020) emphasizes the same advice on a single city level highlighting the close link between geographic location and health campaign efficacy. This is, at least, true when considering a country like Italy: there are still large differences across the regions and the main macro geographic areas (North, Centre and South) in morbidity rates for several illnesses as well as in the demographic, social and economic determinants of health needs (Osservatorio Nazionale sulla Salute nelle Regioni Italiane, 2019; Ministero della Salute, 2014). The potential implications of the differences in needs are in terms not only of volumes of services, but also of composition of services and input requirements for the production of these services. In other words, hospitals and other providers located in different geographic areas may have different behaviours (Berta et al., 2021) and accomplish their main mission of satisfying the health needs of their patients by using different medical technologies.

On the supply side, providers may operate in different institutional contexts, which may create different sets of constraints for the production of their services. In several countries, for instance, the institutional and organizational responsibility for provision of services are decentralized. In Italy, regional governments are responsible for the organization of the provision of services while local health authorities as well as hospitals are held accountable for the material provision at the local level. The macro-allocation of resources realized by each regional government across the different areas of care (public health, ambulatory care, specialist care, hospital care, etc.) will likely affect the demand for the different services, because of the complementarity/substitutability relations among them. Each type of provider, therefore, may end up facing a demand (in terms of volume and composition of services) conditioned on the specific political priorities characterizing the allocation of resources in the regional area in which it operates and, consequently, it will adjust its production technologies to the local overall supply of care. The spatial institutional differences may also impact on production technologies because of the differences in inputs prices they may create. Cavalieri et al. (2017, 2018) show how environmental corruption in the different Italian geographic areas (as measured by an index computed at the provincial level) negatively influences the efficiency of execution of healthcare infrastructures, thus raising the cost of capital goods in the production of services.

As for the supply, we can also consider spatial heterogeneity characterizing clinical choices, which have a relevant impact on the selection of the quantity and the mix of the different inputs as well as on their transformation into medical services. Starting with the seminal work of Wennberg and Gittelsohn (1973), based on small area variations in healthcare delivery, there is now a wide theoretical research and empirical evidence about what is called “medical practice variation”, an expression used to designate the different treatments that similar patients receive because of the different clinical choices of providers, even despite clinical evidence about the best practice (for a recent survey of these studies, see Corallo et al., 2014). The sources of this heterogeneity of medical practice styles, and its related impact on production choices, may be of different nature, and some of them can have a spatial characterization. Lippi Bruni and Mammi (2017), for instance, examine the influence arising from spatial differences in primary care organization on per-patient hospital expenditures, by means of spatial econometrics methods. Lay-Yee et al. (2013) show how clinical activity of family doctors varies across different practices in New Zealand according to the socio-economic context characterizing each practice. There are also unobservable factors that can affect clinical decisions. If we consider that variations in medical choices are also a reflection of the information advantage of physicians and, therefore, they are related to the extent of transaction costs and opportunism in the healthcare sector, Preker et al. (2000) argue that they vary from one cultural setting to another. Again, cultural settings may have a spatial characterization.

Moreover, in the same way as we consider different medical practices, we can also think of different management practices. One of the potential reasons of these differences may lie in what Bloom et al. (2014) refer to as the "design" perspective: "*all practices are designed to be adapted to the idiosyncratic local environment and do not systematically reflect any better or worse management quality*".

The variation of management practices in healthcare in connection with the local context has been, for instance, examined in a work by Bloom et al. (2019), who show that hospitals closer to universities providing both medical and business education have higher management quality (as well as more MBA trained managers and lower mortality rates). It is to be noted that the spatial characterization of the different factors examined in these studies are not necessarily related to the administrative boundaries of the local decision-makers. The potential spatial differentiation of (decentralized) clinical and organizational decisions, then, may contribute to the heterogeneity of production functions for healthcare services. Altogether, therefore, there are sound reasons for the empirical analysis of the production choices of hospitals to be founded on a hypothesis of spatial heterogeneity in the production function and to identify its potential spatial regimes.

2.2. Some approaches to the identification of spatial regimes

The identification of different spatial regimes has been approached from several perspectives; a non-exhaustive discussion about three different approaches - Latent Class Analysis (LCA), Spatial combinatorial optimization and Analytical regionalization - is here reported.

LCA models, first proposed by Green (1951) and Lazarsfeld (1950a,b) and widely applied in many areas of sociology, economics, and environmental studies (Everitt, 1984; van Rees et al., 1999), can be defined as a way to estimate a latent and unobserved multinomial variable (Lazarsfeld and Henry, 1968) whose goal is to categorize a population into classes (*i.e.* regimes) using the observed items, and to identify items that best distinguish between classes. LCA was, therefore, appropriately used to introduce the unobserved heterogeneity in a population and to find statistically significant groups of units that are similar in their responses to the measured variables or, by extension, in the parameters of a statistical model (McCutcheon, 1987). Formally speaking, in LCA each observation i of a population with size N is included in one of the K underlying latent classes C_j that constitute a complete partition of the population. These subgroups form the categories of a latent variable. LCA can also be effectively applied to spatial data analysis. In the classic LCA model the statistical units within each class are considered independent and identically distributed ($i.i.d.$). However, when the units are geo-referenced, this hypothesis is no longer plausible and we must take into account the spatial correlation between observations at different sites, adding a spatial structure that underlies the latent categorical classes (Wall and Liu, 2009). This problem can occur when spatial data is tested to determine if it belongs to one or more possible regimes. In literature many applications of latent model methods to economic data can be found (Paap et al., 2005; AlfÃš et al., 2008; Davis et al., 2009, among others).

Unfortunately, there are only a limited number of latent class studies that properly consider the dependence highlighted by geographically distributed data (Oud and Folmer, 2008; Papalia and Ciavolino, 2011; Papalia and Bertarelli, 2013, among others) as substantially equivalent to partition the study area into zones not necessarily conterminous that are similar according to the model parameters.

Another spatial regimes research stream can be named as "spatial combinatorial optimization"; this family includes methods that take advantage of recursive algorithms that use parametric or non-parametric methods in order to optimize the prediction of a regressive form in space: with a good degree of approximation, the papers of Postiglione et al. (2010) (Classification Analysis Regression Tree, CART), Postiglione et al. (2013) (Simulated Annealing, SA), and Andreano et al. (2010); Billé et al. (2017); Billé et al. (2018) (Adaptive Geographically Weighted Regression, AGWR) can be mentioned.

Analytical regionalization (or spatially constrained clustering) algorithms, finally, may be considered as improperly listed in this review: the aim of such methods (please see Murtagh, 1985; Gordon, 1996; Duque et al., 2007) is actually to group areas or points into a smaller number of regions based on similarities in one or more variables without, however, taking into account any regressive form.

The proposed approach aims to combine the properties of the analytical regionalization methods, namely the identification of the spatially constrained areas, with those of latent and spatial combinatorial optimization methods, namely the identification of a regressive model; the proposed algorithm solves this twofold requirement in a single stage so as to ensure consistency between areas and functional estimates.

3. The methodology for the estimation of spatial regimes

The procedure[2] proposed in this paper is borrowed from the Assuncao et al. (2006) work. They introduced a procedure called *Skater* (Spatial K'luster Analysis by TreeEdgeRemoval) for the estimation of not overlapping clusters of units that are geographically close.

The basic Assuncao et al. (2006) hierarchical k-means type algorithm[3] and the regressive function generalization (namely *SkaterF*) are presented. The *Skater* procedure can be described as a k-means clustering procedure in which the units are belonging to a proximity graph: each observation thereby belongs to the cluster with the nearest mean (measured in terms of distance), so as to partition n neighbouring observations into k clusters. The crucial difference between the algorithm proposed by Assuncao et al. (2006) and the generalization presented in this paper lies in the objective function to be maximized. In the original algorithm, the different sub-graphs are compared in terms of intra-cluster square deviation, as a measure of dispersion of attributes for the objects in a specific region. In this implementation, the evaluation statistic is the residual sum of squares (RSS) of the estimated regression model. Therefore, given a functional form $f(\cdot)$ describing the dependent variable y in terms

[2] The relative SkaterF function and the R *SpatialRegimes* package - derived from the *spdep* package functions - is available on CRAN - https://cran.r-project.org/web/packages/SpatialRegimes/index.html.

[3] For specific aspects of the resolution algorithm and for more details, please see Assuncao et al. (2006); for computational aspects and software implementation, see the *skater* function of the R *spdep* package.

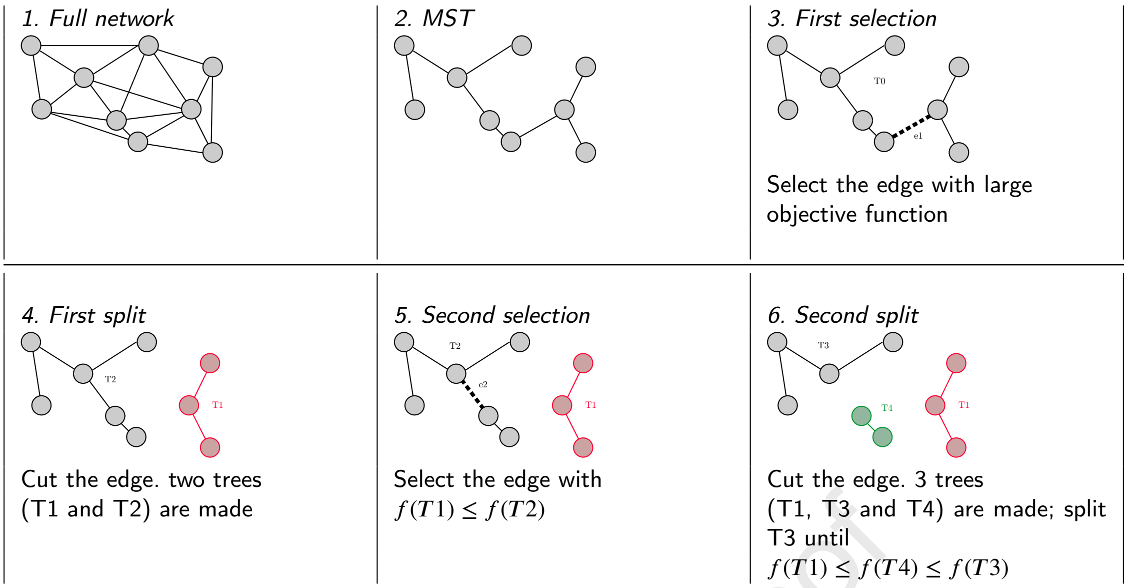


Figure 1: Spatially constrained cluster-wise algorithm *SkaterF*.

of some covariates $\mathbf{x}$, RSS can be defined as usual as:

$$RSS = \sum_{i=1}^{n} (y_i - \widehat{f_k}(\mathbf{x}_i))^2 \tag{1}$$

where i are the units that in a given step of the algorithm belong to the subgraph under consideration[4] and k is the relative estimated spatial regime; this statistic[5], therefore, provides a crucial criterion to identify clusters of adjacent units that are the most similar to each other in terms of distance from a regressive mean estimate (*within*) and, at the same time, the most dissimilar from the other ones (*between*).

The algorithm – stylized in the following Figure 1 – can ideally be divided into two principal phases: *(i)* a first phase of identification of the geography and distances among units, and *(ii)* a second phase in which the effective spatially constrained clustering algorithm is carried on.

In the first phase, the units, described by the functional relation $y = f(\mathbf{x})$ in space, can be represented as a neighbourhood graph (1. Full network) where - in general terms - the distance between points can represent/mimic geographical, economic, etc. distance. This full network is then simplified (2. MST) according to the minimum spanning tree (MST, Pettie and Ramachandran, 2000) algorithm[6].

Starting from this simplified representation of the neighbourhood of the individual units, the aim of the second phase is to identify spatial regimes that are as homogeneous as possible in terms of the estimated functional relationship and heterogeneous between different clusters.

From a general standpoint, at each iteration, a specific edge ($e1$, $e2$ in Figure 1) is removed from the initial MST graph ($T0$, see Step 3 - First selection), containing a set of trees $T1, \dots, Tn$, by comparing the optimum solutions for each of the trees $T1, \dots, Tn$. The solution that best splits (4. First split) the $T0$ graph is the optimum solution S_{e1} according to the objective function, which, in our specification, is the residual sum of squares of the regression residuals:

$$S_{e1} = RSS(T0) - [RSS(T1) + RSS(T2)] \tag{2}$$

Each subsequent step (5. Second selection), then, will aim at finding the edge that maximizes the equation (2) in order to get the greatest improvement of quality (6. Second split), until the desired number of clusters k is achieved. It should be noted that a crucial weakness of this method lies in the recursive research of the edges and in the calculation of all the estimates of the function under examination for each pair of subgraphs identified and for each step. In other terms, "*the exhaustive comparison of all possible values of the objective function is expensive computationally [and it] leads to a combinational explosion.*" (Assuncao et al., 2006).

A heuristic solution to this problem has been proposed by Assuncao et al. (2006) who suggest looking for the edges, candidates to split the graph in two parts, no longer among all the possible nodes, but only among those already calculated; in this way the complexity of the search is considerably reduced.

For other more technical details related - for example - to the search for the desired global maximum instead of the local ones, the choice of the starting points and the performance of the original algorithm, please refer to Assuncao et al. (2006).

[4]Therefore, please read this equation in conjunction with the following equation (2); please also note that the specification of the function $f(\cdot)$ is defined identically for all clusters k, while the estimated parameters of that function change for each spatial regime.

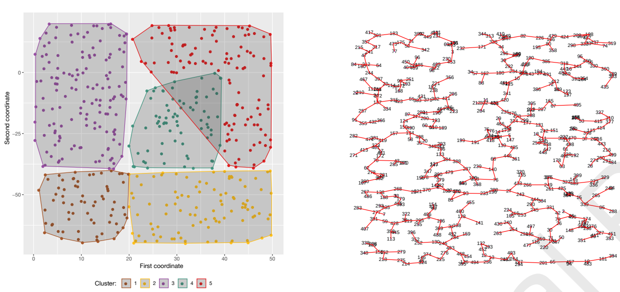
[5]In this paper the OLS estimator has been chosen; this choice can be easily generalized in future research.

[6]For more details, please see Assuncao et al. (2006); Auteri et al. (2019).

4. An application to simulated data

In this section, a simulation exercise is carried out with the aim of evaluating the properties, the goodness of fit and the time complexity of the proposed algorithm.
More in detail, the aim is to construct homogeneous not overlapping areas in which the covariates coefficients of a generic functional form[7] assumes different values. A generic dependent variable, therefore, will be linked to a set of covariates in the same way for all the points belonging to the same area.

In this regard, 500 units (100 units for each of the 5 regimes) are generated and, for each unit, the coordinates[8] are randomly drawn by using two Uniform distributions from 0 to 50 and from -70 to 20, *i.e.* $\mathcal{U}(0,50)$ and $\mathcal{U}(-70,20)$, respectively. Consequently, we set the matrix of covariates which include the constant, A, L and K variables by drawing from $\mathcal{U}(1.5,4)$. Figure 2(a) shows the generated clusters/regimes in space.



(a. Simulation setting) (b. Minimum spanning tree)

Figure 2: Simulation setting and proximity MST graph

Note that spatial regimes can also be not geographically well-defined, *i.e.* points generated by the same functional specification (cluster 3) can also be sparsely distributed in space (in the figure, partially overlapping other points of cluster 5). Figure 2(b), instead, identifies the minimum proximity graph between one unit and another one using the MST (Pettie and Ramachandran, 2000) algorithm.

For each regime, finally, a different (in the coefficients) spatial function is set assuming a linear functional form. More in particular, we set 5 different vectors of parameters (including the intercept): $\beta_1 = (13, 0.5, 0.3, 0.2)$, $\beta_2 = (11, 0.8, 0.1, 0.1)$, $\beta_3 = (9, 0.3, 0.2, 0.5)$, $\beta_4 = (7, 0.4, 0.3, 0.3)$ and $\beta_5 = (5, 0.2, 0.6, 0.2)$ and a normally distributed error term $\epsilon \in \mathcal{N}(0,1)$.

[7]Without loss of generality, the production function has been here considered; clearly the algorithm is generalizable to any functional economic setting.

[8]Please pay attention to the fact that the coordinates in this algorithm are used to identify the neighbourhood and to calculate the cost of each edge based on the distance between it nodes. The distance metric must, therefore, be chosen according to the nature of the coordinates, *e.g.* great circle distances in the case of latitude/longitude.

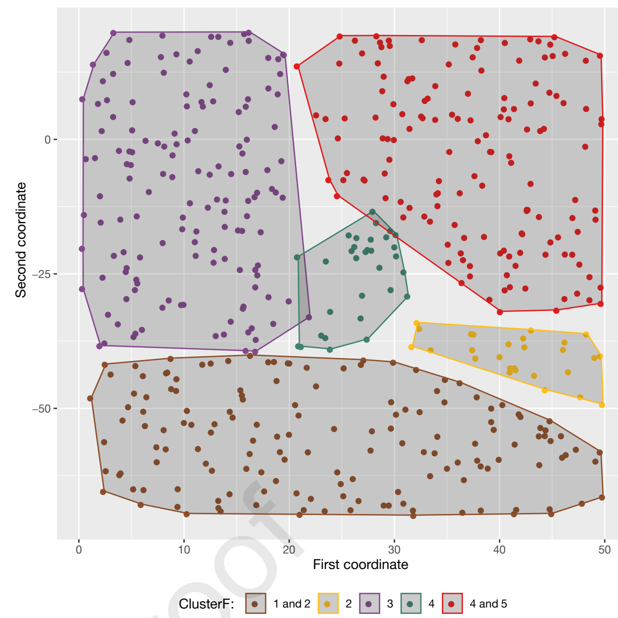


Figure 3: Estimated spatial regimes, $k = 5$

More specifically,

$$y = \begin{cases} 13 + 0.5 * A + 0.3 * L + 0.2 * K + \epsilon, \text{if } i \in \text{cluster 1} \\ 11 + 0.8 * A + 0.1 * L + 0.1 * K + \epsilon, \text{if } i \in \text{cluster 2} \\ 9 + 0.3 * A + 0.2 * L + 0.5 * K + \epsilon, \text{if } i \in \text{cluster 3} \\ 7 + 0.4 * A + 0.3 * L + 0.3 * K + \epsilon, \text{if } i \in \text{cluster 4} \\ 5 + 0.2 * A + 0.6 * L + 0.2 * K + \epsilon, \text{if } i \in \text{cluster 5} \end{cases} \tag{3}$$

Please note that the error ϵ inputted into the simulation (first quartile of the ratio of ϵ/y equal to -6.8%, third quartile 5.2%) can be considered as reasonable and prodromal to the illustration of the properties of the proposed algorithm; further simulations have been added in the A as the random noise varies in the simulated data.

4.1. Estimation and goodness of fit

Given the simulation setting drawn above, the algorithm discussed in section 3 has been applied in order to estimate the spatial regimes both in terms of neighbourhood and in terms of the estimated OLS specification in which the response variable y is expressed as a linear function of the regressors A, L and K.

Figure 3 shows the distribution of the units in the space by estimated spatial regimes. It can be noted the very satisfactory identification of spatial regimes with only some inaccuracy. For example, while in the original clusters 1 and 2 the differences in terms of intercept and the coefficient of the variable A were very pronounced compared to the other 3 clusters (within), but not between, it can happen that the algorithm does not capture this difference. Another critical point can be found where (clusters 4 and 5) the boundary is blurred or overlapped.

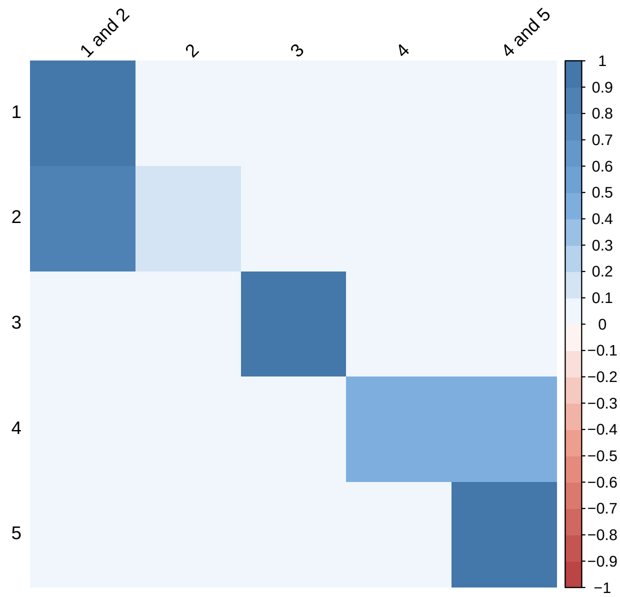


Figure 4: Rank correlation between real and estimated spatial regime membership

Figure 4, however, allows a better visual appreciation of the valuable correspondence between the real and the estimated membership to the different clusters, highlighting a general good coincidence of the two measures. Note the splitting of the original cluster 4 between estimated clusters 1 and 4. The Cohen Kappa (0.29) and the Intraclass correlation coefficient (ICC1k absolute agreement, 0.45) statistics show a good consistency between the real and estimated memberships.

A crucial difference between the *SkaterF* regressive cluster-wise methodology and the standard two-stage (clustering and regression) methodologies consists of a better fit in the estimated regressive model for the obtained regimes. This finding can be appreciated looking at Table 1. In the first column the baseline model estimates are reported (equation (4)), while in the second one (baseline × real, equation (5)) the estimates related to the simulated model are reported distinctly for the real regimes; finally, in the third column (baseline × skaterF, equation (6)), we report the estimates distinctly for the spatial regimes estimated through the *SkaterF* algorithm.

More formally, the three compared specifications can be expressed respectively as:

$$y = \beta_0 + \beta_1 A + \beta_2 L + \beta_3 K + \epsilon \tag{4}$$

$$y = \beta_{0i} + \beta_{1i} A + \beta_{2i} L + \beta_{3i} K + \epsilon_i, \forall i \in \{1, 2 \ldots, 5\} \tag{5}$$

$$\begin{cases} y = \beta_{0i} + \beta_{1i} A + \beta_{2i} L + \beta_{3i} K + \epsilon_i, \forall i \in \{1, 2 \ldots, 5\} \\ i = f(A, L, K) \text{ is an estimate of the true regime code,} \\ \text{obtained through the } f(\cdot) \text{ SkaterF algorithm} \end{cases} \tag{6}$$

Covariate/Regime	(baseline)	(baseline × real)	(baseline × SkaterF)
Intercept	8.329*** (0.830)		
Intercept - CLU1		13.418*** (0.812)	
Intercept - CLU2		11.127*** (0.635)	5.634*** (1.714)
Intercept - CLU1 and 2			11.384*** (0.673)
Intercept - CLU3		7.516*** (0.549)	7.597*** (0.720)
Intercept - CLU4		6.048*** (0.711)	6.640*** (1.397)
Intercept - CLU5		5.504*** (0.563)	
Intercept - CLU4 and 5			4.427*** (0.684)
A	0.296* (0.177)		
A - CLU1		0.310* (0.186)	
A - CLU2		0.742*** (0.138)	0.343 (0.318)
A - CLU1 and 2			0.552*** (0.152)
A - CLU3		0.427*** (0.113)	0.406*** (0.148)
A - CLU4		0.535*** (0.161)	0.485 (0.371)
A - CLU5		0.035 (0.112)	
A - CLU4 and 5			0.200 (0.139)
L	0.300* (0.174)		
L - CLU1		0.305** (0.152)	
L - CLU2		0.074 (0.133)	−0.145 (0.417)
L - CLU1 and 2			0.288** (0.135)
L - CLU3		0.349*** (0.115)	0.328** (0.151)
L - CLU4		0.431*** (0.142)	0.245 (0.271)
L - CLU5		0.422*** (0.128)	
L - CLU4 and 5			0.541*** (0.147)
K	0.520*** (0.175)		
K - CLU1		0.141 (0.170)	
K - CLU2		0.151 (0.132)	1.701*** (0.368)
K - CLU1 and 2			0.326** (0.144)
K - CLU3		0.761*** (0.110)	0.771*** (0.144)
K - CLU4		0.407*** (0.157)	0.496* (0.290)
K - CLU5		0.311** (0.126)	
K - CLU4 and 5			0.589*** (0.150)
Observations	500	500	500
R^2	0.029	0.994	0.989
Adjusted R^2	0.024	0.993	0.988
AIC	2471.353	1397.297	1679.294
BIC	2492.426	1485.804	1767.801
Residual Std. Error	2.847	0.958	1.270
F Statistic	5.019***	3,729.116***	2,111.270***

Note: *p<0.1; **p<0.05; ***p<0.01

Table 1
OLS regressive models comparison.

It can be seen how the estimated spatial regimes allow to perfectly grasp the heterogeneity of the different marginal effects with respect to A, L and K. The spatial mismatch of areas 1 and 2 (or 4 and 5) does not have a deep impact on the estimated coefficients because of both the small number of regimes and of the fact that, in these overlapping areas, the estimated coefficient is an average of the true coefficients of the two areas (as it should be).

A crucial question can be asked about the proposed algorithm: are the estimated spatial regimes "optimal"? Or, better, there are other spatial subdivisions (for equal k) that could better describe the underlying regression model? To answer this question a heuristic approach is proposed here to demonstrate that all other partition of units (within a limited range of possibilities) is less satisfactory in terms of model fitting (measured in terms of R^2). Starting with the estimated spatial partition, the units (respectively in terms of 1,2,… ,8 units) that were on the border of the areas have been switched between groups in order to verify how local permutations of regimes were reflected in the final functional estimate. This procedure has been looped 200 times for each number of permutations. Figure 5 shows how the *SkaterF* spatial regime estimated (red dotted line) is optimal in terms of R^2

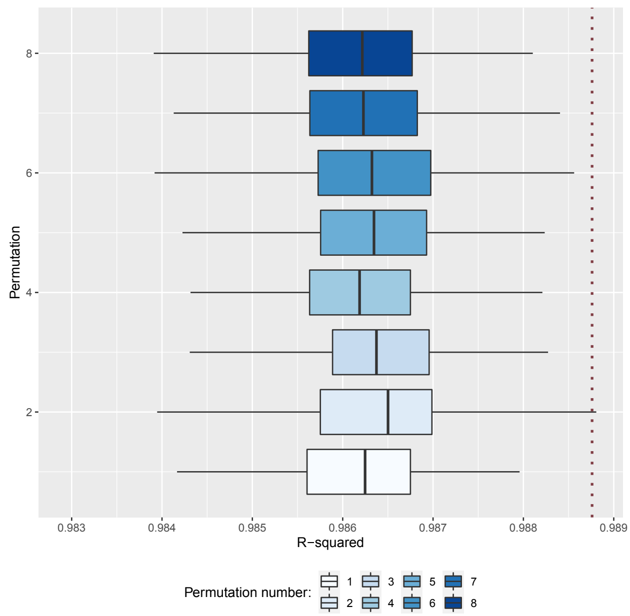
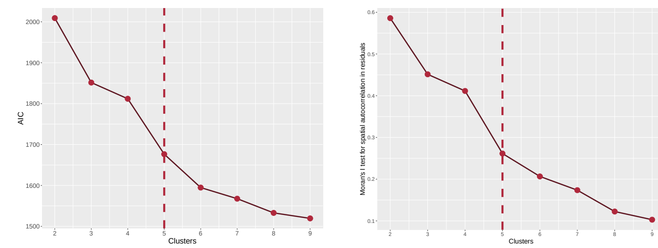


Figure 5: R^2 box-plot by local permutation



(a) (b)

Figure 6: Convergence

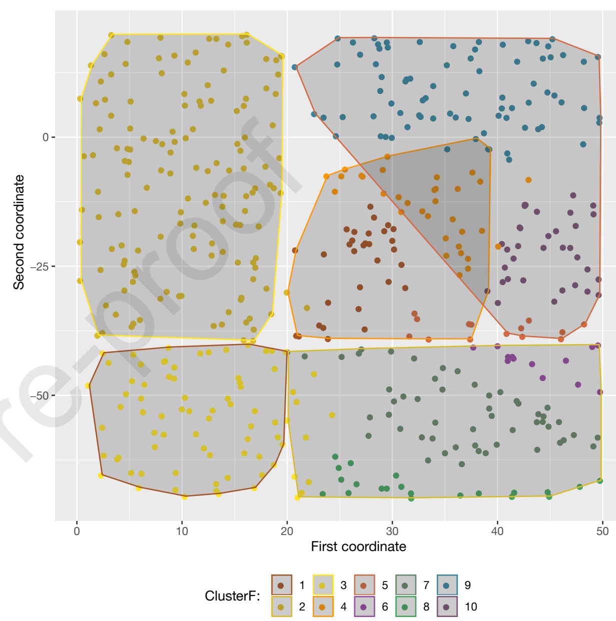


Figure 7: Estimated spatial regimes, $k = 10$ - convex hull related to the division into 5 areas

compared to the distributions of all tested local permutations of 1,2,… ,8 units.

4.2. Identifying the right k number of regimes

In the previous section, the comparison between simulated and estimated regimes has been carried out with the same number $k = 5$, temporarily leaving aside "*a major challenge in cluster analysis, the estimation of the optimal number of cluster*" (Tibshirani et al., 2001).

Even if a detailed discussion of this issue is beyond the scope of this paper, some heuristic criteria for identifying the optimal number k of spatial regimes can be proposed taking into account that - unlike clustering analysis - spatial regimes are identified according to a functional relationship.
For this reason, in addition to the different validation statistics proposed for clustering methods (see *i.e.* Tibshirani et al., 2001; Yan and Ye, 2007), other more specific criteria can be suggested by studying the regression residuals when varying k, and evaluating the additional predictive power even from a spatial residuals autocorrelation point of view. Figure 6 allows to evaluate - although in a very first descriptive form - the decrease of AIC (the lower the better) and of global Moran I for regression residuals (the value tends to zero if there is no spatial autocorrelation) when increasing k.

The basic idea is that, from a number of regimes k forward, in this simulation, $k = 5$, there will no longer be a substantial incremental gain both in the relative quality of statistical model and in more purely spatial terms. But what if, instead, a greater partition than the real one is chosen? What impact this choice have on spatial regimes and on estimates? The answer is displayed in Figure 7.

The spatially hierarchical nature of the algorithm ensures that the new subdivisions (in the example, 10 regimes) are always included in the corresponding higher level areas (in the example, 5 regimes); the regression estimates within these higher level areas will be, by construction, very similar in terms of marginal effects[9].

Finally, the convergence rates of the proposed algorithm varying k and the number of units n are represented in Figure 8. It can be noted that the increase in computational time is proportional to the number of units, but this increase is still negligible and moderate up to a reasonable number of units (approx. 3,000)[10].

4.3. A comparison with other spatial clustering algorithms

A comparison with other spatial regimes estimation algorithms allows a baseline evaluation of the pros and cons of the proposed approach. In particular, the *AWSreg*[11] algorithm proposed by Billé et al. (2018) has been chosen to

[9]Detailed results are available from the authors on request.
[10]The calculation has been carried out on an Intel Core i7 PC @ 2.50 GHz, 2 cores, 16gb RAM; R code have been parallelized and, therefore, it can certainly benefit from increased IT resources.
[11]Even this function is available in the R *SpatialRegimes* package.

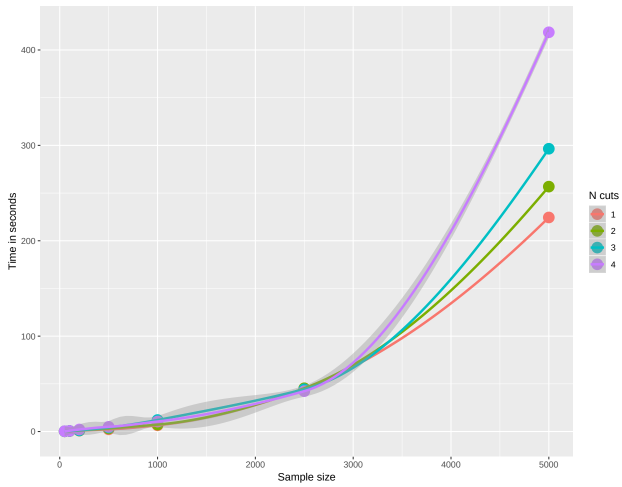


Figure 8: Rate of convergence varying k and n

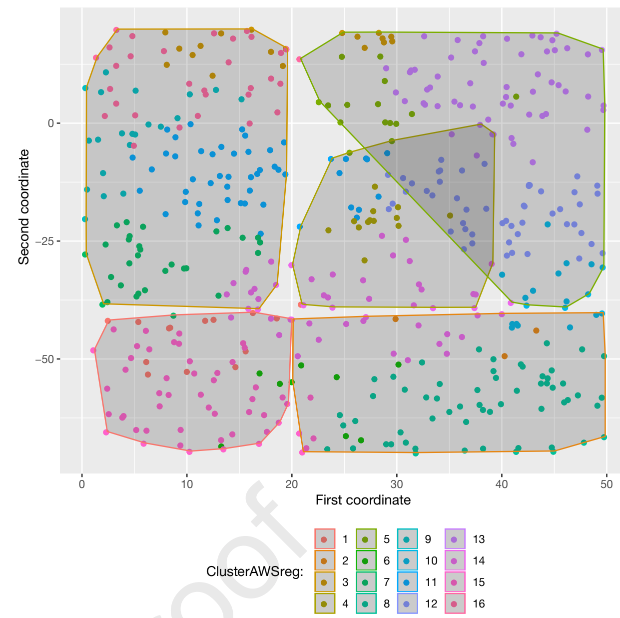


Figure 9: Estimated spatial regimes by *AWSreg* algorithm

evaluate the fitting of the simulated data and the differences with the *SkaterF* algorithm.

This algorithm, proposed in the context of the estimation of production functions affected by spatial heterogeneity, is mainly based on the iterative estimation of a geographically weighted regression. The weights of this regression are modified according to a test on the difference between the vector of the parameters estimated on each pair of units, until convergence is reached at weights equal to 1 when two units belong to the same cluster and 0 otherwise. The number of groups obtained is, therefore, not predetermined, but depends on the choice of the significance level α of the test.

Being based on a geographically weighted regression, the initial weights of the *AWSreg* are a function of the distance between units only. These will therefore always tend to favour not only spatially contiguous configurations of units but of an approximately circular shape.

Figure 9 shows the spatial regimes classification identified by the *AWSreg* algorithm with respect to the areas simulated in the equation (3); a fair fitting to the simulated data can be seen against a very high number of estimated clusters.

Even if it is based on a different estimation rationale âĂŞ basically, the *SkaterF* method imposes a priori a strong not overlapping constraint while the *AWSreg* method does not âĂŞ the *AWSreg* algorithm can be a valid benchmark in regressive terms. Table 2 reports the fitting estimates (in terms of AIC and BIC, given the different structure of the compared models) for four different regressive specifications of the relationship specified in equation (3).

The first line shows the results for the model with only the covariates A, L and K without the territorial dummies; the second line shows the basic model with the real dummies as simulated previously; the third line shows the basic model with the dummies estimated through the *SkaterF* procedure and, finally, the fourth line shows the results obtained with the *AWSreg* estimates. It can be noted that when the data generating process is strongly heterogeneous among different areas and, therefore, there is a clear separation between different areas in functional terms, the proposed procedure seems to be more suitable to capture these differences. Of course, a more structure simulations set is needed for a better exploration of the strengths and weaknesses of the two approaches.

Specification	AIC	BIC
Only covariates	2471.35	2492.43
Covariates + simulated regimes	1412.12	1450.05
Covariates + *SkaterF* estimated regimes	1676.50	1714.44
Covariates + *AWSreg* estimated regimes	1846.05	1930.34

Table 2
Information criteria by estimation algorithm

5. The estimation of a production function for Italian hospitals

In this section, the spatial algorithm discussed in the previous sections has been applied to the estimation of a production function of the services provided by Italian hospitals, identifying, at the same time, spatially constrained geographic areas in which the production function is maximally homogeneous.

The Italian health care system is based on a National Health Service (NHS), characterized by regional decentralization, with regional governments bearing the political responsibility for the organization of the supply of services and for ensuring that its citizens can actually enjoy a nationally uniform benefits package[12]. Within each region, Local Health Authorities (LHAs) are entrusted with the task of organizing the local supply of services in three different areas of care: hospital, community and public health. As far as hospital care is concerned, it is provided by public and private accredited hospitals. There were, in 2010, 880 hospitals operating for the Italian NHS (502 public, 378 private). There is a quite a variable nature in the governance of public hospitals. Most of them are managed by LHAs (Presidi Ospedalieri), while a few (Aziende Ospedaliere), but the biggest ones, are autonomous institutions. Some of them are managed in partnership with Universities and act as teaching hospitals (Aziende Ospedaliero-Universitarie). In terms of bed capacity, the number of beds for acute inpatient care was 3 per 1,000 inhabitants (2.5 public, 0.5 private). Hospital providers' spending accounted for 45% of the overall healthcare spending in Italy in 2017, a figure larger than the OECD average of 38%.

5.1. The model

The production function estimated specification, using the data for Italian hospitals, is relatively simple and similar to the ones used in most of the papers mentioned in section 2, which pursued the same objective. We use a Cobb-Douglas form, for baseline and conditional models respectively:

$$\begin{aligned} log(DISCH) =& \beta_0 + \beta_1 log(PHYS\_NURS) + \\ & \beta_2 log(BED) + \beta_3 INC + \beta_4 PRIV + \\ & \beta_5 WARD + \epsilon \end{aligned} \tag{7}$$

and for regime and conditional regime inclusive specifications:

$$\begin{cases} log(DISCH) =& \beta_{i0} + \beta_{i1} log(PHYS\_NURS) + \\ & \beta_{i2} log(BED) + \beta_{i3} INC + \beta_{i4} PRIV + \\ & \beta_{i5} WARD + \epsilon_i \\ i =& f_{SkaterF}(DISCH, PHYS\_NURS, \\ & BED, INC, PRIV, WARD) \end{cases} \tag{8}$$

In such a way, we estimate the impact of two inputs on the hospitals' output and we control for some relevant variables. As for the output, we focus on inpatient care and we measure it as the weighted sum of discharged patients for each hospital (DISCH), using the Diagnosis Related Groups (DRG) classification weights at the basis of the hospitals' financing system in Italy. We use the national weights (Ministry of Health Decree of December 18, 2008) so as to offset both inter-and intra-regional differences in tariffs for the same DRG[13]. In such a way, we provide a reasonable standardization of inpatients hospital output across the different hospitals. As for the inputs, we consider two inputs: the number of hospital beds (BED), as a proxy measure of capital; the number of full-time equivalent physicians and nurses (PHYS_NURS), for the labour input. We control for the environment in which a hospital operates and for some of its characteristics. The variable INC measures the average income (standardized between 0 and 1) of the municipalities located within a ray of 5 kilometres from each hospital. PRIV is a categorical variable with value 1 if the hospital is private, and 0 otherwise. Finally, the variable WARD measures the number of wards of a hospital, which is a proxy for its degree of specialization. Finally, ϵ is a zero mean normally distributed error term.

5.2. The data

Data used in the application refer to a sample of 681 public and private accredited Italian hospitals[14] for the year 2010, for which we had comparable data for all the different hospitals of the sample[15]. Data for the output and the inputs, as well as for the number of the wards and the private nature of the hospitals provided by the Italian Ministry of Health (specifically, the Department of Health care). Municipal income data are estimates of the Tax Department of the Italian Ministry of Economy and Finance, for the year 2010. Accordingly, Table 3 presents the descriptive statistics of the variables used in the different specifications of the production function.

5.3. Estimation and results

Before applying the *SkaterF* algorithm for the identification and the estimation of the spatial regimes for the hospital production function, two steps have been taken into account: on the one hand, the specification has been examined – distinguishing between a simpler Cobb-Douglas model and the Translog – and on the other hand, the impact of the covariates on different estimation models has been analysed. As far as the specification of the functional form is concerned, the Table 4 reports the results of the comparison between the basic Cobb-Douglass type specification and Translog, respectively: it can be seen that the simpler model describes very well the relationship between dependent variable and covariates with an additional non negligible advantage: a more parsimonious model is more robust than a more complex one especially when the geographical space is divided among different regimes.

Against this background, a model without interaction between input factors has been chosen estimating a global production function for the entire sample (see Table 5), on the basis of equation (7).
It provides a baseline check for the significance of the

[12] Details of the institutional and financing arrangements of the Italian healthcare system can be found in FerrÃĺ et al. (2014).

[13] Regions are allowed to make variations with respect to this national system.

[14] In 2010, 880 hospitals were operating in Italy, both public and private accredited; the sample has been reduced to 681 hospitals due to duplications - essentially related to the same coordinates (same address) - both for the presence of incorrect data in the inputs and in the contextual variables.

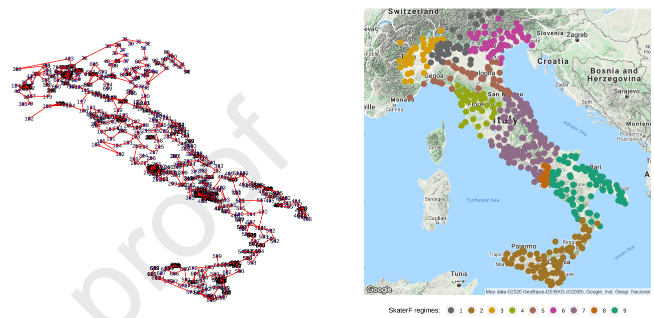
[15] Unfortunately, more recent data, disaggregated at the hospital level and for the different DRG groups of admissions, were not publicly available. Even if the actual results of our exercise may be partially outdated, in terms of what they can suggest for the Italian hospitalsâĂŹ cost function, they are still a good basis for providing an excellent practical example of the application of our methodology.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Discharged patients (log)	706	8.300	1.099	5.556	7.517	9.040	11.123
Capital input (log)	706	4.876	0.926	2.708	4.174	5.475	7.455
Labour input (log)	706	5.427	1.117	2.773	4.627	6.178	8.114
Average income (km5) (std 0-1)	706	0.385	0.092	0.188	0.317	0.438	0.659
Private hospital (dummy)	706	0.365	0.482	0	0	1	1
Ward n.	681	14.369	15.668	1.000	5.000	17.000	156.000

Table 3
Descriptive statistics (year 2010)

	Baseline Cobb-Douglas	Baseline Translog
Capital input	0.759*** (0.027)	1.007*** (0.179)
Labour input	0.344*** (0.022)	0.503*** (0.153)
Capital input^2		−0.001 (0.058)
Labour input^2		0.002 (0.039)
Capital input * Labour input		−0.039 (0.090)
Constant	2.730*** (0.053)	1.686*** (0.252)
Observations	706	706
R^2	0.942	0.944
Adjusted R^2	0.942	0.943
AIC	128.8695	116.9118
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 4
Cobb-Douglas and Translog baseline specifications



(a - MST) (b - Estimated spatial regimes)

Figure 10: MST and estimated spatial regimes

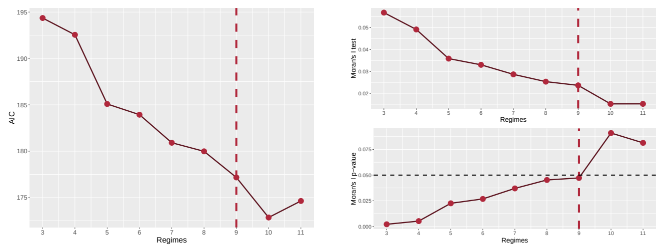
variables included in the model and a benchmark for the subsequent spatial regimes estimates. The global production function is estimated in the baseline version without the control variables (column 1), and in the conditional form, adding the three control variables (column 2)[16]. As for the goodness-of-fit, the R^2 and the AIC indicate an excellent and comparable goodness of fit. The high values of the R^2 should not be surprising for the estimation of a production function for hospital services, when the main regressors are beds and staff. Even if there are few works estimating such production functions they end up with similar results, even with different estimated models (see van Montfort, 1981; Jensen and Morrisey, 1986; Reyes SantÃŋas et al., 2011; Antelo et al., 2017). A reasonable explanation of such a result is that hospital admissions (the output of the production functions) are severely limited by capacity constraints as represented by the two main inputs used in the model, that is beds and staff, which, therefore, "exhaust" the explanation of the variations of output across the different hospitals. Both inputs are significant, even if the magnitude of their output elasticities is substantially different. The control variables too are all significant. The use of capital and labour inputs is less productive in private hospitals and in hospitals with a larger number of wards.

Following the *SkaterF* algorithm, we identify the spatial proximity of the hospitals using the MST algorithm (Figure 10(a)) based on a Euclidean metric[17] and, subsequently, based on the spatial proximity as represented by the MST graph, we estimate the spatial regimes of the hospitals' production function (Figure 10(b)) as defined in equation (8). It should be noted that the minimisation of the weighted distance of the MST algorithm must be balanced by great care to avoid spurious connections, *i.e.* existing not by direct neighbourhood, but by geometric construction, as happens for islands far from the mainland; for this reason, hospitals located in Sardinia and, consequently, their connections have not been considered in the subsequent elaborations within our analysis set. For the second stage of the *SkaterF* algorithm, we impose a minimum size of 35 hospitals for each spatial regime and a number of regimes equal to 9, so as to avoid the presence of small clusters (in terms of number of units) and, at the same time, to avoid a too large number of them. The geographic areas identified for the 9 spatial regimes generally overlap different regions. The only two noticeable exceptions are the spatial regimes 4, which is coincident with one region (Tuscany), and 8, which covers almost half of the hospitals of Campania, mainly in the Naples area.

The choice of the number of estimated regimes has also been validated by the analysis of both the AIC and the Moran I (top the value, bottom the significance) as the number of clusters varies (see Figure 11); the basic idea - already discussed in the previous paragraphs - is to choose the model

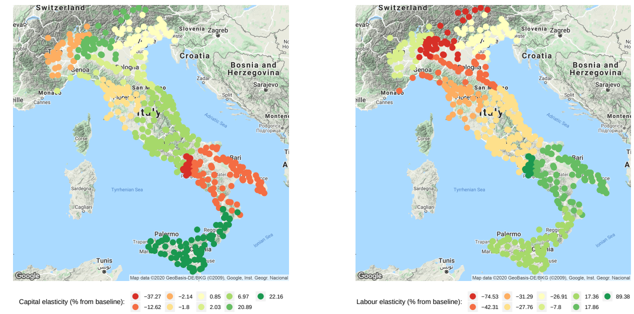
[16] The observations number is slightly different due to the presence of 25 missing data in the WARD variable.

[17] Attention should be paid to the presence of islands in the analysed territories that may create minor inconsistencies in the minimum neighbourhood path; this is, *e.g.* the case of points 185, 186, 192 and 194 which correspond to the Tuscan coast and the Elba island.



(a - AIC) (b - Moran I)

Figure 11: Convergence



(a) (b)

Figure 12: Differences with respect to the baseline conditional model

with a lower AIC, but at the same time presenting a lower, but still significant Moran I.

The unconditional and conditional estimates of the model described in equation (7) for the 9 spatial regimes are reported in Table 5.

A comparison of the global conditional (column 2) and the spatial regime conditional estimates (column 4) reveal that the control variables loose their significance in almost every spatial regime, as if our identification of the different spatial regimes grasps the heterogeneity within the global sample and, therefore, the differences in the values of these variables within each regime become not significant. This result can be interpreted as a signal of the reliability of the identification of the different spatial regimes.

Comparing, instead, the unconditioned spatial regime estimates (column 3) with the conditioned ones (column 4) we notice how the capital input coefficients (see for example clusters 3 to 7) stabilize around the relative mean (column 2); all this is coherent with the role of conditional variables (such as INC and PRIV) in controlling the heterogeneity within each spatial regime. Finally, it should be noted that, unlike pure conditional estimation methods, it is not possible to compare *tout court* conditional with unconditional estimates since, for the properties of the estimation algorithm[18], changing the specification of the model can change the unit's membership to spatial regimes.

Spatial heterogeneity and spatial dependence, as also outlined in section 2, may be the result, jointly or not, of a spatially differentiated impact of omitted variables or intangible factors; for this reason, a robustness analysis has been carried out in order to evaluate if – in this application – these issues were correctly captured by the spatial regimes identified.

Tables 7, 8, 9 and 10 report the spatial simultaneous autoregressive error estimates, showing how the simultaneous autoregressive error coefficient (lambda) is significant in the baseline models, conditional or not, while it is no longer significant when the spatial regimes are identified.

The different spatial regimes of the production function of hospital services are characterized by different values of the output elasticities with respect to capital and labour inputs. They are represented in Figures 12(a) and 12(b), in terms of differences of the coefficients for each spatial regime with respect to the correspondent values of the baseline conditional model.

The size of the differences reveals how, in front of the estimated values of the coefficients of the global production function, the "local" values of the homogenous spatial regimes vary within a quite wide interval (from -74.53% to +89.38% with respect to the baseline value, for the labour elasticity; from -37.27% to +22.16% with respect to the baseline value, for the capital elasticity). There are areas of the country where the hospitals' output is relatively more reactive to labour âĂŞ spatial regimes 2, 8 and 9, covering all the southern Italy; and other areas, where the output is more sensitive to capital âĂŞ this is true especially for spatial regime 1, with spatial regime 2 showing, as already noted, an above average output elasticity for both types of inputs. It also needs to be noticed that while the output elasticity of beds is highly significant for each spatial regime, the output elasticity of labour is not significant or weakly significant, mostly, for those spatial regimes showing a below average value. In these geographic areas, therefore, the hospitals' output tends to change only if there are variations in the number of beds.

As already discussed in section 2.1, the spatial characterization of the hospitals' production function may arise from different sources, related to the nature of health needs and of the connected treatments; to the organization of supply; to the practice styles; up to very exogenous factors as the ecological risk (Brazil, 2021) or to the institutional administrative design (Cavalieri and Ferrante, 2016), all factors for which there can be (observed and unobserved) spatial heterogeneity. It needs to be reminded that, by construction, a hospital's output varies with the volume of its discharges and/or with its case mix, as measured by the DRG weights attached to discharges and reflecting their relative need of resources. Thus, the spatial regimes where the output elasticity of labour is remarkably higher than the global value (2, 9 and, above all, 8) are characterized by a technology where additional units of health professionals bring out a relatively large increase of weighted discharges. A possible explanation of this relatively high return of the labour input is related to some evidence showing that, with a given

[18] And this is, in the view of the authors, precisely the strength of the proposed approach.

number of beds, one potential impact of additional health professionals is a reduction in the length of stay of patients (van Montfort, 1981). There are studies, for instance, that finds a negative relationship between the nurse staffing level and the length of stay (*e.g.* PitkÃďaho et al., 2016). The reasons for this relationship, not only as far as nurse staffing is concerned, may be different. The clinical operations and procedures to be carried out for a patient admitted in a hospital require time and the availability of more staff allows to concentrate them in a shorter time. More staff, above all nurses, may improve the monitoring of patients and, therefore, reduce the risk of complications that extend the length of stay. Whatever the reasons for an increase of health professionals employed by a hospital to impact on the length of stay of its patients, reduction in the length of stay may help the hospital to treat a larger volume of patients. Looking at the regional data of the geographic areas covered by the spatial regimes 2, 8 and 9, for the year 2010 (Ministero della Salute, 2011), it is possible to observe that hospitals within these areas show some critical values related to length of stay. For surgical patients, for instance, the preoperative length of stay is, for all the regions of spatial regimes 2, 8 and 9, above two days (in the range 2.12 âĂŞ 2.38 days), while for many of the other regions the value is around 1.5. Moreover, the percentage of surgical patients treated in one day is, except for one region, well below the national average (from half to a quarter of the national average). It is well possible, therefore, that the significant and positive values of the output elasticity of labour in these spatial regimes may reflect the fact that, within these areas, hospitals with more health personnel can treat more patients, given the number of beds, thanks to an improvement of their length of stay. Of course, increasing the volume of patients by an increase of the staffing levels is not the only technology available where problems with the patients' length of stay may constrain the number of admissions to a hospital (Lewis and Edwards, 2015; PitkÃďaho et al., 2016), but it seems the one adopted to respond to spatially homogenous problems. At the same time, these latter problems may arise from a spatial homogeneity in clinical and managerial governance styles that impact on the organization of the clinical operations and procedures and, therefore, on the length of stay.

The spatial regime 1 shows one of the highest values of output elasticity of the number of beds, which implies that, for a given number of health personnel, an additional bed yields one of the largest increases in the volume of patients and/or in their case mix. This spatial regime characterizes most of the region Lombardia (almost 85% of its hospitals) plus the hospitals of the Bozen area, in Northern Italy. This geographic area, especially Lombardia, is characterized by (potentially) high volumes of demand, as suggested by the average population leaving in a ray of 5 or 30 kilometres from each hospital: about 340,000 for the first measure and about 2.4 million for the second one. Even if in the area of spatial regime 8 these measures are even bigger, hospitals in spatial regime 1 (again, especially the ones in Lombardia) enjoy from the highest volumes of demand from other regions: in 2010 (Ministero della Salute, 2011), the net balance between incoming and outgoing hospitals' patients for Lombardia was positive and of an amount of about 62,000 patients over a national mobility of about 535,000 patients. Campania, whose hospitals fall in spatial regime 8, had a negative net balance of about 42,000 patients. Moreover, the case mix of discharges in 2010 (Ministero della Salute, 2011) was above the national average in Lombardia, and the out-of-region admissions in its hospitals in 2010 (CEIS SanitÃă, 2013) was characterized by the highest average financial value all over the country (3,743 euro per admission, with the second highest value the one in Veneto, 3,331 euro). Therefore, within spatial regime 1, hospitals with more beds meet a (potentially) large demand characterized by a relatively high case mix and, as a consequence, an additional bed creates a larger impact on the hospitals' output than in other spatial regimes. It needs to be pointed out that hospitals in other spatial regimes (particularly 3, 5 and 6) are characterized by an even higher case-mix but their output elasticity of capital is lower than the one in spatial regime 1. This difference can be associated to a different organization of the provision of healthcare services in the regions mostly covered by regimes 3, 5 and 6 with respect to Lombardia. Access to hospitals in those regions is actually more effectively restrained than in Lombardia by the other providers of medical services outside the hospitals. There are several indicators that support these differences: the standardized (by age and sex) hospitalization rate is higher in Lombardia than in the regions covered by spatial regimes 3, 5 and 6 (up to 15% more); different indicators of appropriateness of access to hospital (percentage of daily admissions for diagnostic purposes, hospitalization for diabetes, etc.) are relatively higher too (up to 3-4 times for some of these indicators). The result of the output elasticity of capital for spatial regime 2 is, instead, difficult to be associated to specific characteristics of the geographic areas covered by that regime.

6. Concluding remarks

The objective of this paper was to examine the spatial heterogeneity in the production function of hospital services in Italy. Spatial heterogeneity may offer a wider context for the analysis of the differences among the technologies used by hospitals for the production of their medical services, with respect to other approaches that create a segmentation in different groups by some predetermined characteristics. Our work develops a novel methodological approach to deal with this issue, basically characterized by an endogenous - one stage - determination of the different spatially constrained regimes of the production function.

Our results, besides their methodological value, allow to shed light on the working of the hospital sector in Italy. Even if the geographic segmentation of the healthcare system, at least in the three main macro areas (North, Centre and South) of the country, is widely known and studied, not only for the healthcare sector, our work provides a more refined picture of this segmentation, in line with other studies (*e.g.* Auteri et al., 2019), and draws some unexplored implications, specifically in terms of different production regimes. Traditionally, studies on the production function of healthcare services and on their efficiency of provision have analysed differences across geographic areas delimited by administrative boundaries (*e.g.* countries, regions, etc.) and, consequently, the homogeneity within each area has been strongly linked with the institutional dimension. Our methodology, instead, considers a consistent procedure of endogenous identification of spatial regimes for the production function of hospitals, not on the basis of their administrative affiliation, but of a technological homogeneity. The heterogeneity of the technological regimes identified in our work, in turn, can be associated to a spatial heterogeneity of relevant aspects, like demand, internal organization, clinical and managerial governance, etc. There are different policy implications of our results, and we would like to point out two of them. First, even if technologies can be changed in the long run for improving the economic efficiency of the hospital services production, the identification of the different spatial regimes may provide, in the short run, an accurate information for understanding the production possibilities space of different hospitals as well as for assessing their performance. A global production function, given the variability of the output elasticities shown by our results, would give a biased and distorted picture of the performance and of the impact of input changes for the different hospitals. Second, in the long run, the geographic asymmetry between the political and administrative units and the homogenous spatial regimes can weaken the effectiveness of the policies implemented for improving the extension and the quality of healthcare provision, because of the bias previously mentioned.

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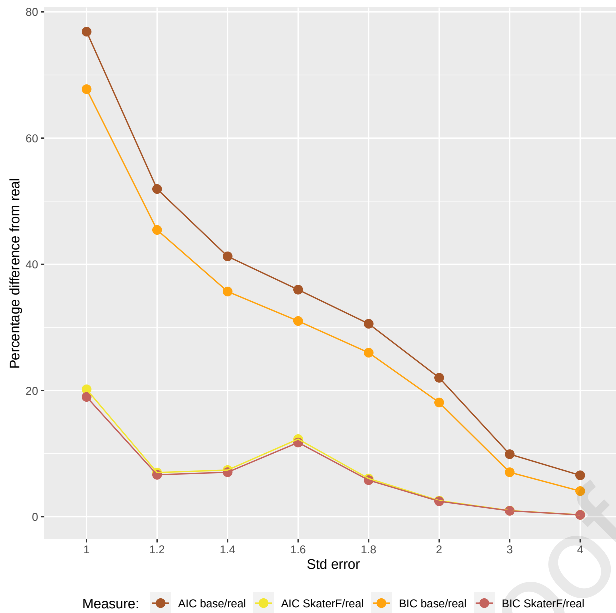

Figure 13: Percentage difference between fit measures as the ϵ standard error changes

A. Additional simulations

In Section 4, the properties of the SkaterF algorithm has been analysed by highlighting the ability to capture spatial regimes in the linear model parameters when these spatial regimes exist. But how does this methodology perform when such spatial regularities dampen or come to decay? The simulations carried out – we report only a brief summary (further details are available from the authors) – show that the advantage over a non-spatial model always persists and decreases as the random noise in the data increases.

Resuming DGP outlined in equation (3), the standard error of the error term ϵ has been increased so that the differences, in terms of the production functions, between the spatial regimes became less and less pronounced. The percentage ratio between the error term ϵ and the dependent variable y has, therefore, increased (see Table 6) from about $\pm 5\%$ in the case of standard deviation equal to 1 (the example presented in the section 4 simulations) up to more than $\pm 20\%$ when standard deviation became equal to 4. Thus, for each setting, the linear specifications presented in the equations (4), (5) and (6) have been estimated verifying the ability of the non-spatial model (“base”) and of SkaterF to capture the model generator of the data, that is the one called "real"; This evaluation has been measured in terms of percentage difference between the "real" model and the other two in terms of AIC and BIC.

Figure 13 shows how the SkaterF estimation algorithm outperforms the linear model by showing on average 10% difference in terms of both AIC and BIC with respect to the reference model; this difference, as it should be, tends to dampen as the added random error grows eliminating, *de facto*, the presence of underlying spatial regimes.

B. Spatial simultaneous autoregressive error estimates

	(Baseline)	(Conditional)	(Spatial regime)	(Conditional spatial regime)
Capital input	0.759*** (0.027)	0.823*** (0.031)		
Labour input	0.344*** (0.022)	0.318*** (0.028)		
Average income (km5)		−0.252** (0.117)		
Private hospital dummy		−0.080*** (0.030)		
Ward n.		−0.003*** (0.001)		
Spatial Regime1:Capital input			1.005*** (0.113)	0.995*** (0.126)
Spatial Regime2:Capital input			0.832*** (0.053)	1.005*** (0.070)
Spatial Regime3:Capital input			0.777*** (0.117)	0.805*** (0.132)
Spatial Regime4:Capital input			0.567*** (0.102)	0.808*** (0.123)
Spatial Regime5:Capital input			0.837*** (0.109)	0.840*** (0.110)
Spatial Regime6:Capital input			0.700*** (0.143)	0.830*** (0.157)
Spatial Regime7:Capital input			0.839*** (0.059)	0.880*** (0.062)
Spatial Regime8:Capital input			0.499*** (0.116)	0.516*** (0.135)
Spatial Regime9:Capital input			0.727*** (0.064)	0.719*** (0.075)
Spatial Regime1:Labour input			0.074 (0.095)	0.081 (0.102)
Spatial Regime2:Labour input			0.390*** (0.039)	0.373*** (0.062)
Spatial Regime3:Labour input			0.407*** (0.106)	0.293** (0.134)
Spatial Regime4:Labour input			0.533*** (0.083)	0.218* (0.122)
Spatial Regime5:Labour input			0.161 (0.102)	0.183 (0.112)
Spatial Regime6:Labour input			0.411*** (0.128)	0.232 (0.149)
Spatial Regime7:Labour input			0.285*** (0.047)	0.229*** (0.057)
Spatial Regime8:Labour input			0.483*** (0.083)	0.601*** (0.116)
Spatial Regime9:Labour input			0.336*** (0.051)	0.374*** (0.066)
Spatial Regime1:Average income (km5)				−0.770** (0.364)
Spatial Regime2:Average income (km5)				−1.039** (0.486)
Spatial Regime3:Average income (km5)				−0.624 (0.689)
Spatial Regime4:Average income (km5)				0.091 (0.745)
Spatial Regime5:Average income (km5)				0.257 (0.496)
Spatial Regime6:Average income (km5)				−1.607** (0.691)
Spatial Regime7:Average income (km5)				−0.495* (0.253)
Spatial Regime8:Average income (km5)				−0.647 (0.764)
Spatial Regime9:Average income (km5)				0.297 (0.466)
Spatial Regime1:Private hospital dummy				−0.111 (0.073)
Spatial Regime2:Private hospital dummy				−0.065 (0.076)
Spatial Regime3:Private hospital dummy				−0.389*** (0.108)
Spatial Regime4:Private hospital dummy				−0.517*** (0.159)
Spatial Regime5:Private hospital dummy				0.093 (0.116)
Spatial Regime6:Private hospital dummy				−0.225* (0.136)
Spatial Regime7:Private hospital dummy				−0.121** (0.057)
Spatial Regime8:Private hospital dummy				0.186 (0.144)
Spatial Regime9:Private hospital dummy				0.046 (0.077)
Spatial Regime1:Ward n.				−0.002 (0.006)
Spatial Regime2:Ward n.				−0.014*** (0.004)
Spatial Regime3:Ward n.				−0.003 (0.004)
Spatial Regime4:Ward n.				−0.001 (0.003)
Spatial Regime5:Ward n.				0.0004 (0.003)
Spatial Regime6:Ward n.				0.0004 (0.005)
Spatial Regime7:Ward n.				−0.0003 (0.002)
Spatial Regime8:Ward n.				−0.006 (0.004)
Spatial Regime9:Ward n.				−0.002 (0.005)
Spatial Regime1 Constant			3.023*** (0.191)	3.483*** (0.437)
Spatial Regime2 Constant			2.157*** (0.144)	1.999*** (0.278)
Spatial Regime3 Constant			2.200*** (0.215)	3.205*** (0.573)
Spatial Regime4 Constant			2.500*** (0.197)	3.191*** (0.427)
Spatial Regime5 Constant			3.515*** (0.184)	3.201*** (0.517)
Spatial Regime6 Constant			2.594*** (0.206)	3.688*** (0.637)
Spatial Regime7 Constant			2.711*** (0.122)	3.039*** (0.193)
Spatial Regime8 Constant			3.173*** (0.237)	2.664*** (0.428)
Spatial Regime9 Constant			2.964*** (0.145)	2.711*** (0.293)
Constant	2.730*** (0.053)	2.739*** (0.095)		
Observations	706	681	706	681
R^2	0.942	0.939	0.999	0.999
Adjusted R^2	0.942	0.938	0.999	0.999
Akaike Inf. Crit.	128.869	126.672	81.109	68.321
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 5
Production function OLS estimates

Std. error	Percentage ratio ϵ/y 1^{st} quartile	3^{rd} quartile
1.00	-6.88	5.17
1.20	-8.54	6.35
1.40	-9.33	6.85
1.60	-10.94	8.58
1.80	-11.41	8.62
2.00	-13.42	11.41
3.00	-23.39	16.22
4.00	-31.96	19.51

Table 6
First and third quartiles of the percentage ratio ϵ/y as the standard error of ϵ varies

	Estimate	Std.Error	z value	Pr(> $\|z\|$)
Intercept	2.730184	0.054587	50.015	<2.20E-16
Capital input	0.770559	0.02726	28.267	<2.20E-16
Labour input	0.333754	0.022555	14.797	<2.20E-16

Lambda: 0.18527, LR test value: 9.682, p-value: 0.0018608
Log likelihood: -55.5937 for error model
ML residual variance (sigma squared): 0.068125, (sigma: 0.26101)
Number of observations: 706
Number of parameters estimated: 5
AIC: 121.19, (AIC for lm: 128.87)

Table 7
Baseline production function - Spatial simultaneous autoregressive error estimator

	Estimate	Std.Error	z value	Pr(> $\|z\|$)
Intercept	2.768436	0.099005	27.9626	<2.20E-16
Capital input	0.829315	0.030717	26.9988	<2.20E-16
Labour input	0.306695	0.028234	10.8628	<2.20E-16
Average income (km5)	-0.26219	0.128985	-2.0327	0.042084
Private hospital dummy	-0.08381	0.030135	-2.7813	0.005414
Ward n.	-0.00298	0.001068	-2.7939	0.005208

Lambda: 0.16762, LR test value: 8.0052, p-value: 0.0046642
Log likelihood: -52.33347 for error model
ML residual variance (sigma squared): 0.067929, (sigma: 0.26063)
Number of observations: 681
Number of parameters estimated: 8
AIC: 120.67, (AIC for lm: 126.67)

Table 8
Conditional production function - Spatial simultaneous autoregressive error estimator

	Estimate	Std.Error	z value	Pr(> $\|z\|$)
Spatial Regime1	3.025314	0.188161	16.0783	<2.20E-16
Spatial Regime2	2.172931	0.141518	15.3545	<2.20E-16
Spatial Regime3	2.196837	0.211767	10.3739	<2.20E-16
Spatial Regime4	2.486767	0.191708	12.9717	<2.20E-16
Spatial Regime5	3.506814	0.179974	19.4851	<2.20E-16
Spatial Regime6	2.609185	0.202154	12.9069	<2.20E-16
Spatial Regime7	2.695234	0.119358	22.5811	<2.20E-16
Spatial Regime8	3.145838	0.233001	13.5014	<2.20E-16
Spatial Regime9	2.969838	0.142015	20.9121	<2.20E-16
Spatial Regime1: Capital input	1.010502	0.111181	9.0888	<2.20E-16
Spatial Regime2: Capital input	0.835222	0.052574	15.8865	<2.20E-16
Spatial Regime3: Capital input	0.776052	0.116335	6.6708	2.54E-11
Spatial Regime4: Capital input	0.58636	0.099985	5.8645	4.51E-09
Spatial Regime5: Capital input	0.838464	0.107053	7.8322	4.89E-15
Spatial Regime6: Capital input	0.685516	0.140952	4.8635	1.15E-06
Spatial Regime7: Capital input	0.843707	0.057344	14.713	<2.20E-16
Spatial Regime8: Capital input	0.513949	0.114093	4.5046	6.65E-06
Spatial Regime9: Capital input	0.729692	0.062729	11.6325	<2.20E-16
Spatial Regime1: Labour input	0.06882	0.093281	0.7378	0.460654
Spatial Regime2: Labour input	0.383936	0.038873	9.8766	<2.20E-16
Spatial Regime3: Labour input	0.407918	0.104858	3.8902	0.0001
Spatial Regime4: Labour input	0.518937	0.081357	6.3786	1.79E-10
Spatial Regime5: Labour input	0.161128	0.099921	1.6125	0.106842
Spatial Regime6: Labour input	0.421251	0.12673	3.324	0.000887
Spatial Regime7: Labour input	0.28459	0.046399	6.1336	8.59E-10
Spatial Regime8: Labour input	0.473804	0.081138	5.8395	5.24E-09
Spatial Regime9: Labour input	0.332747	0.050348	6.6089	3.87E-11

Lambda: 0.077993, LR test value: 1.4709, p-value: 0.22521
Log likelihood: -11.81884 for error model
ML residual variance (sigma squared): 0.060481, (sigma: 0.24593)
Number of observations: 706
Number of parameters estimated: 29
AIC: 81.638, (AIC for lm: 81.109)

Table 9
Spatial production function - Spatial simultaneous autoregressive error estimator

	Estimate	Std.Error	z value	Pr(> $\|z\|$)
Spatial Regime1	3.490042	0.420975	8.2904	2.22E-16
Spatial Regime2	2.005129	0.266469	7.5248	5.29E-14
Spatial Regime3	3.228588	0.548451	5.8867	3.94E-09
Spatial Regime4	3.183113	0.409746	7.7685	7.99E-15
Spatial Regime5	3.20196	0.495504	6.462	1.03E-10
Spatial Regime6	3.683254	0.611553	6.0228	1.71E-09
Spatial Regime7	3.032268	0.186157	16.2888	<2.20E-16
Spatial Regime8	2.659654	0.411845	6.4579	1.06E-10
Spatial Regime9	2.703152	0.283104	9.5483	<2.20E-16
Spatial Regime1: Capital input	0.994183	0.121447	8.1861	2.22E-16
Spatial Regime2: Capital input	1.003881	0.067025	14.9777	<2.20E-16
Spatial Regime3: Capital input	0.804664	0.126688	6.3515	2.13E-10
Spatial Regime4: Capital input	0.813682	0.118093	6.8902	5.57E-12
Spatial Regime5: Capital input	0.841115	0.105691	7.9582	1.78E-15
Spatial Regime6: Capital input	0.827846	0.15108	5.4795	4.27E-08
Spatial Regime7: Capital input	0.882148	0.059949	14.7149	<2.20E-16
Spatial Regime8: Capital input	0.52039	0.129579	4.016	5.92E-05
Spatial Regime9: Capital input	0.719445	0.072463	9.9284	<2.20E-16
Spatial Regime1: Labour input	0.079933	0.098353	0.8127	0.41638
Spatial Regime2: Labour input	0.37243	0.060067	6.2003	5.64E-10
Spatial Regime3: Labour input	0.289714	0.128195	2.26	0.023824
Spatial Regime4: Labour input	0.21409	0.116536	1.8371	0.066192
Spatial Regime5: Labour input	0.181916	0.107777	1.6879	0.091433
Spatial Regime6: Labour input	0.234456	0.143821	1.6302	0.10306
Spatial Regime7: Labour input	0.229016	0.054401	4.2098	2.56E-05
Spatial Regime8: Labour input	0.597464	0.111995	5.3348	9.57E-08
Spatial Regime9: Labour input	0.373548	0.06378	5.8568	4.72E-09
Spatial Regime1: Average income (km5)	-0.76697	0.354857	-2.1614	0.030668
Spatial Regime2: Average income (km5)	-1.03557	0.469309	-2.2066	0.027344
Spatial Regime3: Average income (km5)	-0.63767	0.667288	-0.9556	0.339264
Spatial Regime4: Average income (km5)	0.098277	0.715817	0.1373	0.890799
Spatial Regime5: Average income (km5)	0.253292	0.479416	0.5283	0.597267
Spatial Regime6: Average income (km5)	-1.59918	0.665518	-2.4029	0.016266
Spatial Regime7: Average income (km5)	-0.49551	0.246144	-2.0131	0.044105
Spatial Regime8: Average income (km5)	-0.63169	0.740665	-0.8529	0.393733
Spatial Regime9: Average income (km5)	0.327458	0.451767	0.7248	0.468551
Spatial Regime1: Private hospital dummy	-0.11224	0.070396	-1.5944	0.110855
Spatial Regime2: Private hospital dummy	-0.06527	0.07327	-0.8908	0.37304
Spatial Regime3: Private hospital dummy	-0.38926	0.104027	-3.7419	0.000183
Spatial Regime4: Private hospital dummy	-0.51827	0.152754	-3.3929	0.000692
Spatial Regime5: Private hospital dummy	0.092955	0.110968	0.8377	0.402213
Spatial Regime6: Private hospital dummy	-0.22639	0.130644	-1.7329	0.083119
Spatial Regime7: Private hospital dummy	-0.1204	0.054735	-2.1997	0.02783
Spatial Regime8: Private hospital dummy	0.183569	0.138231	1.328	0.184182
Spatial Regime9: Private hospital dummy	0.046513	0.074215	0.6267	0.530836
Spatial Regime1: Ward n.	-0.00186	0.00569	-0.3265	0.74405
Spatial Regime2: Ward n.	-0.01424	0.004106	-3.4671	0.000526
Spatial Regime3: Ward n.	-0.00269	0.003628	-0.7422	0.457938
Spatial Regime4: Ward n.	-0.00064	0.002771	-0.2296	0.818414
Spatial Regime5: Ward n.	0.000424	0.003064	0.1385	0.889859
Spatial Regime6: Ward n.	0.000326	0.004889	0.0667	0.946817
Spatial Regime7: Ward n.	-0.0003	0.001794	-0.1697	0.865274
Spatial Regime8: Ward n.	-0.00584	0.003682	-1.5869	0.112544
Spatial Regime9: Ward n.	-0.00194	0.004708	-0.413	0.679628

Lambda: 0.02905, LR test value: 0.19202, p-value: 0.66124
Log likelihood: 20.93523 for error model
ML residual variance (sigma squared): 0.05505, (sigma: 0.23463)
Number of observations: 681
Number of parameters estimated: 56
AIC: 70.13, (AIC for lm: 68.322)

Table 10
Conditional spatial production function - Spatial simultaneous autoregressive error estimator

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