



The resilience dividend: A tractable climate-economy model

Andrea Bacchiocchi ^{a,b},^{*} Fabio Tramontana ^a, Federico Favaretto ^{a,c}

^a Department of Economics, Society and Politics (DESP), University of Urbino Carlo Bo, Via Saffi 42, 61029 Urbino (PU), Italy

^b Sustainability Environmental Economics and Dynamics Studies (SEEDS), inter-university research centre, c/o Department of Economics and Management, University of Ferrara, Via Voltapaletto 11, 44121 Ferrara, Italy

^c Centre on Economics, Finance and Regulation (BAFFI), Bocconi University, Via Röntgen 1, 20136 Milan, Italy

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ABSTRACT

This paper develops a quantitative, analytically tractable, closed-form climate-economic model to study mitigation and adaptation policies. We identify a “resilience dividend” under baseline scenarios, where avoided long-run climate damages substantially exceed the economic costs of moderate policy intervention. Conversely, delayed intervention imposes a dual burden: even aggressive late-stage abatement fails to counteract thermal and atmospheric inertia in the medium term, escalating cumulative costs, while inducing sustained temperature overshoot. When incorporating exogenous technological progress or quadratic climate damages, the model aligns with recent empirical evidence on catastrophic risks, demonstrating that highly stringent policy action transitions from a discretionary choice to an economic imperative. Finally, an extension incorporating the structural shift toward intangible capital reveals that digitalization alters long-term decarbonization pathways depending on whether dematerialization gains outweigh computational energy demand. Overall, the findings indicate that decisive, front-loaded climate policy serves as an indispensable foundation for long-term macroeconomic stability and prosperity.

1. Introduction

Climate change represents an existential threat to humanity, with multifaceted consequences, including rising global temperatures, an increased frequency of extreme weather events, and sea-level rise. In line with the Paris Agreement (Conference of the Parties, COP21), governments and international organizations have committed to substantial CO₂ reductions by 2050 to limit global warming to below 2 °C (Boneva et al., 2021). Failing to achieve this objective would expose the economy and society to significant, though uncertain, risks (Stern, 2007; Drudi et al., 2021; Intergovernmental Panel on Climate Change (IPCC), 2021). These risks materialize through both physical channels, e.g., damages from extreme events and gradual climate shifts (Carney, 2015; Alogoskoufis et al., 2021), and transition channels, which stem from asymmetric production cost adjustments across sectors as economies transition toward low-carbon technologies (Dennis, 2023; Gourdel et al., 2024). Ultimately, limiting global warming depends

on policy choices regarding mitigation, which is aimed at reducing greenhouse gas (GHG) emissions generated by economic activity¹ and adaptation, which is aimed at moderating the economic costs of climate damages.² Evaluating the interaction between these two policy levers, along with their associated costs and macroeconomic consequences under alternative scenarios, is therefore essential for policy design and public debate. In this paper, we develop a quantitative, discrete-time, closed-form climate-economic model, related to the Dynamic Integrated Climate–Economy (DICE) framework (Nordhaus, 1991; Barrage and Nordhaus, 2024), to examine mitigation and adaptation policies within a transparent and analytically tractable setting.

The academic literature analyzing the nexus between climate change and the economy builds on seminal contributions by Nordhaus (Nordhaus, 1991, Nordhaus, 1992). The DICE model has gained widespread recognition for its pioneering integration of economic dynamics with climate processes (Nordhaus, 2018a, Nordhaus, 2018b).

^{*} Corresponding author at: Department of Economics, Society and Politics (DESP), University of Urbino Carlo Bo, Via Saffi 42, 61029 Urbino (PU), Italy.

E-mail addresses: andrea.bacchiocchi@uniurb.it (A. Bacchiocchi), fabio.tramontana@uniurb.it (F. Tramontana), federico.favaretto@uniurb.it (F. Favaretto).

¹ Mitigation efforts may reflect a combination of carbon pricing, regulatory measures and subsidies for clean technologies, all aimed at reducing the carbon footprint of production.

² Adaptation expenditures include investments in infrastructure, public health, disaster preparedness, climate damage insurance, and land-use strategies designed to reduce the vulnerability of economic systems to climate shocks.

Our model is quantitative because it reproduces several results of DICE and Integrated Assessment Models (IAMs) in terms of stock of emissions, damage, and temperature increase. Our simulations can be meaningfully mapped into the Shared Socioeconomic Pathways (SSP) framework (Riahi et al., 2017; Meinshausen et al., 2020), allowing for a direct comparison with the IAM literature.

The primary advantage of our approach is the derivation of closed-form solutions via a system of difference equations in two state variables. We achieve this tractability by employing a linear damage function in temperature and a linearized Cobb–Douglas production function. Our simulation exercises demonstrate that this simplified formulation is robust to the more complex specifications used in the DICE-type models without compromising the central findings. Rather, these simplifications enhance analytical transparency, clarifying the core mechanisms and dynamic channels that govern the climate-economy transition.³

We use this framework to conduct comparative statics on alternative mitigation and adaptation policies and assess their long-run economic and environmental consequences. Far from being a limitation, the reduced complexity of the model provides the flexibility required to accommodate further extensions. We illustrate this capability by integrating intangible capital, analyzing how its expansion either accelerates or impedes decarbonization across two opposing structural scenarios.

Specifically, our model departs from standard DICE formulations in several key respects. First, it adopts a discrete-time dynamic structure that simplifies temporal resolution while preserving the underlying economic mechanisms.⁴ Second, it explicitly incorporates adaptation alongside mitigation, thereby broadening the policy space under consideration. Finally, unlike traditional IAMs, our framework does not rely on an explicit intertemporal welfare maximization objective. Instead, it tracks the joint dynamics of two fundamental state variables, the capital stock $K(t)$ and the stock of anthropogenic emissions, $SE(t)$, under alternative policy intensities. The trajectories of these variables are governed by structural relationships and policy rules, facilitating extensive comparative statics that are typically intractable in traditional DICE-type models due to their reliance on optimal-path numerical solutions.

Our model entails three blocks: economy, climate and policy.

The economic block is characterized by a linearized production function in capital and labor. Consumption is defined as a constant share of net output, with the remainder allocated to gross capital investment. While the baseline model focuses purely on capital accumulation, we subsequently introduce exogenous technological progress to capture long-run productivity growth. Net emissions are proportional to gross output, adjusted for mitigation intensity, and drive the accumulation of the atmospheric carbon stock net of natural decay. Global temperature anomaly is modeled as a linear function of accumulated emissions, while climate damages depend jointly on temperature and adaptation efforts. Both mitigation and adaptation reduce net output through convex, quadratic cost functions that capture increasing marginal costs. In its baseline form, the model combines these elements with a linear damage function, yielding a two-dimensional discrete-time system that admits a single economically meaningful, locally stable equilibrium.

Under our baseline parameterization, this steady state serves as a robust long-run attractor,⁵ ensuring the tractability and reliability of both simulation paths and comparative static analyses. Our analysis yields several central findings.

First, we identify a “resilience dividend” associated with moderate climate policy interventions under baseline conditions. In this regime,

climate expenditures act as productive investments rather than pure costs: although initial interventions divert resources from consumption and capital accumulation, the resulting protection of the productive base – achieved through substantial reductions in climate damages – more than compensates for the upfront outlays. Up to a critical threshold, environmental protection and macroeconomic resilience therefore operate as complementary objectives.

Second, the model uncovers a “capital reallocation trade-off” at higher policy intensities. While moderate interventions generate net economic gains through avoided damages, highly stringent policies reintroduce tensions between macroeconomic performance and environmental ambition (Kikstra et al., 2022; Nordhaus, 2023). Beyond the optimal intensity threshold, the marginal costs of policy intervention rise faster than the marginal benefits of damage mitigation, crowding out capital investment and reducing long-run net output.

Third, the dynamic simulations demonstrate that gradual policy ramp-ups driven by reactive climate awareness risk substantial overshoots of environmental targets. Even aggressive late-stage abatement aligned with net-zero emissions targets fails to prevent significant carbon accumulation and warming, causing climate damages to peak at 2.2% of gross domestic product (GDP) in the medium term. Consequently, delaying climate action imposes a dual burden: it increases cumulative economic expenditures while inducing persistent temperature overshoots due to thermal and carbon-cycle inertia.⁶ Postponing the primary burden of climate policy thus introduces path dependencies that lock the economy into protracted damage regimes, underscoring the imperative for early, decisive intervention.

Fourth, relaxing the baseline linearity assumptions confirms the structural robustness of the model. Incorporating a quadratic damage specification and a standard Cobb–Douglas production function preserves our qualitative results, with minimal quantitative divergence. This validates our linearized baseline as an analytically tractable yet highly accurate proxy for more complex climate-economic systems.

Fifth, under the quadratic damage specification, the economic impact of rising temperatures aligns closely with the “planetary boundaries” literature, accounting for irreversible tipping points and nonlinear climate feedback. Specifically, our findings provide a quantitative bridge to recent empirical estimates by Bilal and Känzig (2026), illustrating how even a 1 °C increase in global temperature can trigger systemic output losses as high as 20%. Because marginal benefits escalate nonlinearly under quadratic damages, the optimal policy strategy shifts from moderate intervention to very strong climate action as an essential safeguard for macroeconomic survival.

Sixth, contrasting the baseline with a scenario incorporating exogenous technological growth and cost-efficiency gains in abatement reveals that very strong policy interventions yield the highest long-run economic output. This dynamic generates a self-financing “resilience dividend” rather than an “environmental sacrifice”. Technological maturity enables aggressive climate policies to remain cost-effective even as higher production levels exert upward pressure on gross emissions, effectively decoupling long-run growth from climate risk.

Seventh, relaxing the assumption of a fixed consumption share by allowing for higher initial savings and investment – consistent with an initially high marginal product of capital – demonstrates the robustness of our results. Although temporary investment surges slightly elevate intermediate temperatures and damages, the structural trajectories of the macroeconomic variables remain stable as consumption converges to its steady-state equilibrium share.

Lastly, analyzing the structural shift toward intangible capital reveals that digitalization is not inherently green. Its net environmental impact depends on whether digital assets facilitate dematerialization or

³ This comes at the cost of abstracting from the richer representation of the carbon cycle and climate feedbacks embedded in full-scale IAMs.

⁴ See also similar recent discrete-time approaches such as Antoci et al. (2021); Bacchiocchi et al. (2024, 2025), and Dávila-Fernández et al. (2025).

⁵ Moreover, the system is not close to a bifurcation in parameter space.

⁶ This gradual policy ramp-up generates “hump-shaped” trajectories across key variables, where climate damages rise in the short run and begin to subside only after policy stringency reaches critical levels.

exacerbate energy demand via computational infrastructure. Simulating two polar scenarios indicates that the expansion of intangible capital operates primarily through environmental channels, altering long-term emissions and temperature pathways, while exerting negligible influence on aggregate capital accumulation.

Over recent decades, climate-economic modeling has expanded substantially, both through refinements within the neoclassical framework (Cai and Lontzek, 2019; Dietz and Stern, 2015) and via alternative paradigms that challenge standard assumptions (Lamperti et al., 2018). Our paper aligns with the former tradition, maintaining neoclassical foundations while prioritizing closed-form analytical tractability. In this respect, our work contributes to the literature seeking closed-form solutions for IAMs and climate-growth systems. For instance, Golosov et al. (2014) derive an analytically tractable optimal carbon tax in dynamic general equilibrium, showing that optimal policy depends on discounted marginal damages and carbon persistence. Bretschger and Karydas (2019) analyzed climate change within an endogenous growth framework under fossil fuel depletion, examining how innovation and policy interact to determine long-run emissions and growth. While our baseline model focuses on capital accumulation and exogenous technological shifts to maintain analytical tractability, it connects naturally to the broader endogenous growth literature, where policy-induced innovation and research and development (R&D) are central to structural transitions (Acemoglu et al., 2012; Aghion et al., 2016). Similarly, Traeger (2023) developed an analytically solvable model that generalizes DICE while preserving tractability. Building on these insights, our approach combines reduced-form climate dynamics with an analytically solvable macroeconomic core to clarify the mechanisms governing the climate-economy transition.

Our paper also contributes to the literature examining the joint interaction of adaptation and mitigation in IAMs. De Bruin et al. (2009) integrate adaptation (AD) into the DICE framework (AD-DICE), distinguishing between residual damages, adaptation expenditures, and mitigation costs. In their framework, adaptation effectively mitigates near-term damage, whereas mitigation acts with longer lags by altering emissions and temperature trajectories. However, Patt et al. (2010) cautioned that adaptation is difficult to capture within aggregate IAMs because real-world adaptation is heterogeneous, localized, and constrained by institutional frictions. In line with this caveat, our reduced-form adaptation mechanism represents an aggregate macroeconomic buffer against climate damages rather than a localized adaptation process. More recently, Carlino et al. (2022) demonstrated that mitigation-adaptation interactions become critical under deep uncertainty, where static policies generate trade-offs between protective outlays and emissions abatement. Our contribution is complementary: rather than solving for an optimal policy path numerically, we offer a closed-form, scenario-based framework that makes explicit how mitigation and adaptation jointly determine capital accumulation, net output, emissions, temperature, and climate damage. This enables us to define the precise conditions under which climate policy generates a resilience dividend, when stringent targets reintroduce capital reallocation trade-offs, and why delayed action generates persistent economic losses due to thermal and atmospheric inertia.

The remainder of the paper is organized as follows. Section 2 presents the structural equations of the model. Section 3 characterizes the system equilibria and local stability properties. Section 4 discusses the numerical simulations and comparative statics across policy pathways. Section 5 concludes.

2. Model equations

This section describes the structural components of the model and the governing equations of the climate-economy system. The simulation horizon is set to $H = 300$ years; we use the notation H for the final period to avoid confusion with $T(t)$, which denotes global temperature.

The following subsections distinguish between the model's analytical structure and its empirical grounding. First, we present the equations defining economic growth, carbon accumulation, and climate feedback, emphasizing the functional forms that ensure dynamical tractability. Subsequently, in Section 2.10, we justify the calibration of each parameter, aligning our baseline values with empirical macroeconomic evidence and the IAM literature.

2.1. Gross economic output

The first block of the model follows neoclassical growth models, such as Solow's (Solow, 1956), featuring an aggregate production function dependent on capital and labor inputs. The nonlinear form of the Cobb–Douglas function, $Y(t) = K(t)^\alpha L^{(1-\alpha)}$, complicates the derivation of closed-form dynamic solutions. To preserve analytical tractability while retaining the underlying economic intuition, we adopt a linear approximation of the Cobb–Douglas specification. Specifically, by holding labor L constant over time and evaluating the function at representative values of $K(t)$, the production function is linearized via a first-order Taylor series expansion around a reference point.

The resulting expression for gross output is given by:

$$Y(t) = \alpha K(t) + (1 - \alpha) L \quad (1)$$

where $K(t)$ is the capital stock at time t , representing one of the model's two endogenous state variables; $L > 0$ denotes the labor input, assumed constant over time. This reflects the empirical regularity that population growth and labor force participation rates remain broadly stable in mature economies. In the calibration, labor is set to $L = 25$, representing total labor input in billions of effective hours. The parameter $\alpha \in (0, 1)$ represents the output elasticity of capital, calibrated to $\alpha = 0.3$.

The decision to employ the linearized formulation in Eq. (1) rather than a log-linear transformation stems from our objective to retain closed-form tractability when solving the joint capital accumulation and emissions stock dynamics. For a fixed labor supply, this approximation closely tracks the behavior of the full Cobb–Douglas function over plausible ranges of the capital stock (see Appendix A.2 for numerical validations conducted under the same parameter set, supporting the robustness of the linearized production function).

We abstract from endogenous technical change in the baseline scenario of Section 4.1, where growth dynamics are driven purely by capital accumulation. This maintains analytical clarity when isolating macro-climate linkages across policy intensities. However, in Section 4.4, we introduce exogenous technological progress to combine capital accumulation with ongoing productivity growth, aligning the model more closely with full-scale IAM and DICE models at a modest cost to tractability.

2.2. Capital accumulation

Capital evolves according to the following difference equation:

$$K(t+1) = K(t)(1 - \delta) + Q(t)(1 - c) \quad (2)$$

where $\delta \in (0, 1)$ denotes the effective depreciation rate of productive capital, set to 0.12. This parameter captures physical wear and tear, technological obsolescence, and the accelerated retirement of carbon-intensive capital over the climate transition horizon.

The term $Q(t)$ denotes net output, defined in Section 2.7 as gross output net of climate-related damages and policy expenditures for mitigation and adaptation. The parameter $c \in (0, 1)$ represents the aggregate consumption share of net output,⁷ encompassing both private

⁷ A constant consumption share c aligns with the Kaldor facts of economic growth (Kaldor, 1957), which highlight the long-run empirical stability of the

and public consumption.⁸ We calibrate $c = 0.8$, with the residual share $1 - c = 0.2$ representing the national savings rate, which is fully directed toward investment as in standard Solow-type models.

Eq. (2) captures the fundamental intertemporal trade-off between current consumption and capital formation. Capital stock expansion occurs whenever net savings exceed physical depreciation. Intuitively, this process is governed by the marginal product of capital, $MPK(t)$: as long as the additional net output generated by an incremental unit of capital – after deducting climate damages and policy costs – exceeds the depreciation rate, the economy retains an incentive to accumulate capital. A formal analytical derivation of these growth conditions is presented in Section 3.4.

2.3. Mitigation policy

Net emissions $E(t)$, measured as the anthropogenic flow of CO_2 released in year t from economic production, are given by:

$$E(t) = \sigma (1 - I_m) Y(t) \quad (3)$$

where $\sigma > 0$ denotes the carbon intensity of output—representing emissions (in kg of CO_2) generated per unit of gross economic output $Y(t)$. In the baseline calibration, we set $\sigma = 0.25$. This parameter is treated as exogenous, capturing both direct and indirect emissions associated with production activities. The model abstracts from natural background emissions, which are assumed constant and invariant to economic decisions.

Parameter $I_m \in (0, 1)$ represents the mitigation effort in period t , defined as the fraction of gross emissions abated via climate policy. The boundary $I_m = 0$ corresponds to a zero-abatement baseline, whereas $I_m = 1$ denotes full decarbonization of output. Emission abatement operates proportionally: an effort level I_m directly reduces net emissions by that fraction. This linear structure aligns with standard specifications in IAMs, including DICE frameworks (Nordhaus, 2023). In practice, mitigation efforts reflect a portfolio of carbon pricing mechanisms, regulatory standards, and clean technology subsidies designed to lower the carbon intensity of production.

The economic cost of mitigation is specified in reduced form as a convex quadratic function of abatement effort:

$$C_{Im} = \theta I_m^2 \quad (4)$$

where $\theta > 0$ governs the scale of marginal mitigation costs. This quadratic formulation reflects increasing marginal abatement costs, capturing the exhaustion of low-cost mitigation opportunities prior to deploying capital-intensive technologies. The variable C_{Im} denotes the fraction of gross output diverted from consumption and capital accumulation toward mitigation measures (e.g., renewable energy deployment, industrial decarbonization, low-carbon R&D, and carbon dioxide removal infrastructure).

In Section 4.3, the mitigation effort is modeled dynamically, with $I_m(t)$ rising linearly across the simulation horizon to reflect escalating policy stringency driven by growing environmental awareness. Conversely, in Section 4.4, the mitigation cost parameter $\theta(t)$ declines over

savings rate in mature economies. Behaviorally, this reflects the permanent income hypothesis (Friedman, 1957), wherein aggregate consumption remains proportional to income over long horizons despite transitory fluctuations. Robustness checks under a time-varying consumption share are reported in Appendix A.3.

⁸ Public consumption captures government expenditures on goods, services, and transfers that do not directly expand the productive capital stock. We assume stable expenditure rules across both sectors, maintaining a constant combined absorption share of net output. Climate-related outlays (mitigation and adaptation) are treated separately as policy-driven costs rather than standard public consumption.

time to capture exogenous technological progress, learning-by-doing, and efficiency gains in green innovation.

2.4. Adaptation policy

Climate adaptation policy provides a complementary mechanism to mitigation, focusing on moderating economic damages induced by warming rather than directly abating emissions. Adaptation outlays encompass defensive investments in protective infrastructure, public health resilience, disaster preparedness, climate risk insurance, and land-use planning designed to reduce systemic vulnerability to climate shocks.

The cost of adaptation is modeled in reduced form as a quadratic function of adaptation effort:

$$C_{Ia} = \mu I_a^2 \quad (5)$$

where C_{Ia} denotes the adaptation expenditure in period t , $I_a \in [0; 1]$ is the adaptation effort, and $\mu > 0$ governs the marginal cost of adaptation. In other words, C_{Ia} represents the fraction of gross output allocated annually to adaptation activities, diverting resources from consumption and capital accumulation to attenuate the macroeconomic fallout of climate damages. Consistent with the mitigation framework, the quadratic specification in Eq. (5) captures the principle of increasing marginal costs. While initial resilience-building interventions (such as updating building codes or improving early-warning systems) remain highly cost-effective, comprehensive protection requires progressively larger resource reallocations.⁹

In Section 4.3, adaptation effort is dynamic, with $I_a(t)$ increasing linearly across the simulation horizon. This trajectory represents a proactive policy response to escalating climate risks, capturing the necessity for economies to scale defensive investments as global temperatures rise. In Section 4.4, we let the adaptation cost coefficient $\mu(t)$ decline over time to model technological progress and learning-by-doing across the adaptation sector. Key drivers of these efficiency gains include advanced materials for climate-resilient construction, precision agriculture innovations that mitigate thermal stress, and the integration of AI-driven predictive modeling into disaster management, all of which reduce the economic outlays required to achieve targeted levels of protection.

2.5. Average temperature equation

The evolution of the mean global temperature anomaly is modeled as a linear function of the cumulative anthropogenic emissions stock. Specifically, the temperature anomaly at time t , measured in $^\circ\text{C}$ relative to the pre-industrial baseline (1850–1900 average), is given by:

$$T(t) = \eta SE(t) \quad (6)$$

where $T(t)$ represents the temperature anomaly and $SE(t)$ denotes the atmospheric GHG stock accumulated through period t (defined in Section 2.8).

The parameter $\eta > 0$ denotes temperature-emissions sensitivity, capturing the responsiveness of the climate system to cumulative carbon forcing. This linear mapping provides a reduced-form representation of the Transient Climate Response to Cumulative Emissions (TCRE), determining the warming generated per unit of accumulated emissions. While η does not independently determine the long-run equilibrium temperature – which is pinned down by the terminal level of the emissions stock, SE^* – it governs the pace of thermal adjustment along the transition path.

⁹ This convexity reflects the logistical and engineering challenges of implementing large-scale protective infrastructure, such as extensive sea-level defenses or the structural reconfiguration of agricultural and urban systems.

In line with the climate-economy literature, we assume a relatively small η to account for climate system inertia and the delayed impact of emissions on global warming. As discussed in Section 2.10, this functional form anchors the model to empirical benchmarks while preserving the analytical tractability essential for long-run dynamic analysis.

Under this specification, global temperature continues to rise as long as the emissions stock $SE(t)$ increases. Thermal stabilization at T^* requires emissions stock stabilization at a constant level SE^* . Consequently, the temperature trajectory stabilizes over the simulation horizon only if net emissions $E(t)$ approach near-zero levels through stringent abatement policies or carbon removal technologies.

2.6. Climate damage function

Rising average temperatures amplify the frequency, duration, and severity of extreme weather events, generating economic losses primarily through the degradation of productive capital and infrastructure. Following the established approach of the DICE framework (Nordhaus, 2017), we link climate dynamics to the macroeconomy through a reduced-form damage function mapping global temperature increases to aggregate output losses.

To preserve the analytical tractability of the dynamic system, our baseline specification assumes that climate damages scale linearly with warming. This simplification establishes a transparent benchmark for evaluating the fundamental trade-offs between capital accumulation and climate policy. We subsequently relax this assumption in Section 4.2 to account for nonlinearly escalating damages and tipping points at higher temperature thresholds via a quadratic formulation consistent with the standard DICE consensus. Specifically, climate-related damages are modeled as:

$$D(t) = \psi (1 - I_a) T(t) \quad (7)$$

where $D(t)$ denotes the fraction of gross output lost to climate damages in period t . The variable $T(t)$ represents the temperature anomaly relative to the pre-industrial baseline. The parameter $\psi > 0$ captures the marginal economic cost per additional degree of warming ($\psi = 0.01$, as detailed in Section 2.10), while $I_a \in [0, 1]$ denotes adaptation effort, which moderates effective economic damages. In this formulation, adaptation acts as a protective shield: higher levels of I_a attenuate the share of potential damages that materialize into realized economic losses.

The multiplicative term $(1 - I_a)$ explicitly captures this shielding effect. As I_a increases, a smaller fraction of climate shocks translates into output losses. In the extreme – and purely theoretical – case of full adaptation ($I_a = 1$), climate-induced output losses are fully neutralized within the model, though physical impacts such as extreme weather events and sea-level rise persist. Conversely, in the absence of adaptation ($I_a = 0$), the economy remains fully exposed to climate damages. Adaptation policy thus functions as a macroeconomic buffer: it reduces the vulnerability of output to climate shocks without altering the underlying temperature trajectory.

This distinction highlights the fundamental asymmetry between adaptation and mitigation. Mitigation policies – such as carbon pricing, clean energy deployment, and emission permits – act directly on the climate system by reducing GHG emissions, thereby moderating global temperature increases. In contrast, adaptation policies – such as flood defenses, infrastructure retrofits, and enhanced disaster response – act directly on the economic system, mitigating damages after climate shocks materialize. Adaptation (modeled in Section 2.4) is therefore essential even under ambitious mitigation scenarios, providing critical protection against the committed near- and medium-term warming locked in by historical emissions.

In practice, adaptation expenditures include riverbed maintenance, sea wall and flood barrier construction, urban drainage and cooling upgrades, and the deployment of early-warning systems for heatwaves,

storms, and wildfires. While these interventions do not reduce mean global temperature or the frequency of physical hazards directly, they substantially reduce the realized economic costs of such events.¹⁰

2.7. Net output

The fraction of gross output available for capital accumulation and consumption (hereafter referred to as net output) is determined by subtracting climate-related damages and policy expenditures from gross production. Specifically, net output $Q(t)$ is defined as:

$$Q(t) = Y(t)[1 - D(t) - C_{Im} - C_{Ia}]; \quad (8)$$

where $Y(t)$ represents gross output at time t , $D(t)$ denotes the fraction of output lost to climate damages (defined in Section 2.6), C_{Im} represents the output share allocated to mitigation costs (Section 2.3), and C_{Ia} captures the output share devoted to adaptation expenditures (Section 2.4). This formulation highlights that climate policy interventions divert productive capacity from standard economic uses, while rising temperatures inflict direct output losses. Consequently, the residual $Q(t)$ constitutes the effective output available for investment and consumption, serving as the core driver of the capital accumulation dynamics detailed in Section 2.2.

2.8. Dynamics of emissions stock

The law of motion for the anthropogenic emissions stock is governed by the following difference equation:

$$SE(t+1) = SE(t)(1 - \lambda) + E(t) \quad (9)$$

where $SE(t)$ denotes the accumulated atmospheric stock of CO_2 emissions through period t , $E(t)$ represents the flow of net emissions generated in year t (defined by Eq. (3) in Section 2.3), and $\lambda \in [0, 1]$ is the natural atmospheric decay rate of previously accumulated emissions.

This specification reflects the long atmospheric residence time of carbon: only a minor fraction of accumulated emissions is removed by natural sinks each period, while the remainder persists in the atmosphere. The decay parameter λ is relatively small, capturing the slow pace at which CO_2 is reabsorbed by oceanic and terrestrial sinks. We calibrate $\lambda = 0.004$ per annum.

In steady state, the emissions stock SE^* remains constant over time. This requires annual net emissions to exactly balance natural atmospheric absorption, yielding the equilibrium condition $E^* = \lambda SE^*$, as established in Section 3.4. This steady-state relationship plays a pivotal role in governing long-run temperature trajectories, as the mean global temperature anomaly relative to pre-industrial levels, $T(t)$, scales linearly with $SE(t)$ via the temperature Eq. (6).

2.9. Dynamical system

The dynamic system is derived through a sequential substitution of structural equations. First, the gross output expression (1) is substituted

¹⁰ Mitigation comprises interventions that reduce atmospheric GHG concentrations, slowing global warming and ultimately reducing the probability and intensity of extreme climate events. Adaptation, by contrast, includes measures that lessen the socioeconomic consequences of climate change without addressing its physical drivers. There are a few exceptions: localized adaptation measures that modify microclimates (e.g., urban afforestation or wetland restoration) can yield localized cooling benefits. Similarly, certain adaptation strategies mitigate cascading systemic risks (e.g., forest management to prevent desertification or watershed governance to alleviate drought risk). Globally, however, only mitigation fundamentally alters the long-run trajectory of climate change. Adaptation remains primarily a defensive strategy indispensable for managing residual climate risks over short- and medium-term horizons.

into Eqs. (3), (4), (5), and (8). Next, the net emissions Eq. (3) is inserted into the emissions stock dynamics (9), which in turn is substituted into the temperature Eq. (6). The resulting temperature expression is then embedded into the climate damage function (7). Finally, the mitigation and adaptation cost functions (4) and (5), alongside the climate damage function, are substituted into the net output Eq. (8), with the resulting formulation incorporated into the capital accumulation Eq. (2).

The resulting model constitutes a two-dimensional, nonlinear discrete-time dynamical system in the state variables K and SE .¹¹ This system is presented in Eq. (10a), preserving the original model structure in compact form. For analytical clarity, an algebraically expanded formulation isolating K and SE is provided in Eq. (10b), highlighting the fundamental nonlinearities governing the system dynamics.

$$\begin{cases} K(t+1) = (1-c)(L(\alpha-1) - K\alpha)(\theta I_m^2 + \mu I_a^2 + SE\eta\psi(1-I_a) - 1) + K(1-\delta) \\ SE(t+1) = SE(1-\lambda) + \sigma(1-I_m)(K\alpha + L(1-\alpha)) \end{cases} \quad (10a)$$

$$\begin{cases} K(t+1) = -\alpha\eta\psi(1-I_a)(1-c)KSE + (1-\delta + \alpha(1-c)(1-\theta I_m^2 - \mu I_a^2))K \\ \quad - L\eta\psi(1-I_a)(1-\alpha)(1-c)SE + L(1-\alpha)(1-c)(1-\theta I_m^2 - \mu I_a^2) \\ SE(t+1) = \alpha\sigma(1-I_m)K + (1-\lambda)SE + L\sigma(1-I_m)(1-\alpha) \end{cases} \quad (10b)$$

2.10. Calibration

The complete parameter set is reported in Table 1, which provides detailed descriptions, baseline values, and plausible ranges of variation for each parameter. The model is calibrated at an annual frequency across a 300-year simulation horizon ($H = 300$). This timeframe captures the long-run interaction between capital accumulation, the carbon cycle, and thermal inertia, while avoiding the compounding uncertainties inherent in multi-millennial projections. All parameter values are calibrated to match empirical macroeconomic evidence from advanced economies and established benchmarks from the IAM literature, ensuring the system operates within a physically and economically sound state space.

The climate module is anchored to a 2024 baseline using observational records from the World Meteorological Organization (WMO), the National Oceanic and Atmospheric Administration (NOAA), and Copernicus. As illustrated in Fig. 1, which traces the joint evolution of global temperature anomalies and atmospheric CO₂ concentrations since 1850, the global temperature anomaly has reached +1.2 °C above pre-industrial levels (1850–1900 baseline). Concurrently, atmospheric carbon concentrations have escalated from approximately 285 ppm to 420 ppm, accelerating markedly over the past four decades. In accordance with these observations, we set initial conditions to $T(0) = 1.2$ and $SE(0) = 420$. These values serve as the starting point for simulating future dynamics under alternative policy scenarios. To ensure structural consistency between these initial values and the climate feedback equations, the sensitivity parameter η is calibrated to 0.003 (Table 1). This specification aligns the model with observed warming trajectories and generates a Transient Climate Response to Cumulative Emissions (TCRE) consistent with IPCC estimates.

For the macroeconomic block, parameters are calibrated to replicate the structural features of mature advanced economies. The capital share in the aggregate production function is set to $\alpha = 0.3$, consistent with empirical evidence from OECD economies and the Penn World Tables

indicating a labor share ($1 - \alpha$) between 0.6 and 0.7 alongside a capital share between 0.2 and 0.4.

Labor input (L) is fixed at 25 billion effective hours per year, approximating the aggregate labor supply of major European economies (such as France, Italy, or the UK)—equivalent to roughly 25 million full-time workers at 1000 effective hours annually.

The initial capital stock, $K(0)$, is calibrated alongside L and α to generate plausible dynamic paths for output (Y) and capital intensity (K/Y). We normalize initial capital to $K(0) = 30$ billion units.¹² Under this calibration, capital accumulates to a steady-state value of $K^* \approx 54$ in the baseline scenario and $K^* \approx 73$ under exogenous technological progress (Section 4.4). These trajectories generate an implied capital–output ratio defined by $\frac{K(t)}{Y(t)} = \frac{K(t)}{\alpha K(t) + (1-\alpha)L}$. As capital accumulates, this ratio converges endogenously toward an asymptotic upper bound of $1/\alpha$ (3.3 when $\alpha = 0.3$), in accordance with standard Kaldor facts (Kaldor, 1957) and macroeconomic benchmarks for advanced economies. Across our simulation paths, the capital–output ratio fluctuates stably between 2.0 and 2.5.

The baseline aggregate consumption share is set to $c = 0.8$, encompassing both private and public consumption. Empirical estimates for advanced economies show that private consumption accounts for 0.60 to 0.75 of net output, consistent with the permanent income hypothesis (Friedman, 1957). Public consumption absorbs an additional 0.10 to 0.20 of GDP.¹³ Combined, the total consumption share of 80% ($c = 0.8$) matches OECD empirical averages, leaving a national savings rate of $1 - c = 0.2$. In Appendix A.3, we conduct sensitivity tests relaxing the assumption of a constant c along the transition path to account for potential shifts in social preferences or fiscal requirements. This exercise confirms the robustness of our core qualitative results even under alternative consumption-saving trajectories.

Finally, the capital depreciation rate is set to $\delta = 0.12$. While standard physical depreciation rates in real business cycle models range from 0.05 to 0.08, our higher value captures an “effective depreciation” rate. This accounts for physical wear, technological obsolescence, and the accelerated premature retirement of carbon-intensive capital assets during the green transition, modeling heightened structural replacement requirements within a parsimonious parameterization.

Climate parameters are selected to replicate the empirical magnitudes of temperature anomalies and economic losses established in the IAM literature, including the DICE model (Nordhaus, 2023).

The parameter σ denotes the carbon intensity of production, calibrated to a baseline value of $\sigma = 0.25$ (0.25 kg of CO₂ per euro of gross output). This value aligns with DICE-2023 estimates (Barrage and Nordhaus, 2024) as a representative benchmark for advanced economies. Because carbon intensity varies across industrialized nations according to energy mix and industrial specialization, empirical estimates from the European Central Bank (ECB) and broader IAM studies (Drudi et al., 2021; Alogoskoufis et al., 2021) establish a plausible range between 0.15 and 0.40. Anchoring our baseline to 0.25 captures an intermediate structural benchmark consistent with the IPCC Sixth Assessment Report (Kikstra et al., 2022), reflecting how baseline emissions efficiency dictates the required stringency of decarbonization pathways.

The damage coefficient ψ governs the trade-off between policy expenditure and avoided climate damages. In our baseline calibration, we set $\psi = 0.01$, implying that in the absence of adaptation, a global temperature increase of 2 °C above pre-industrial levels reduces aggregate output by 2%. This aligns with standard IAM benchmarks, such as

¹² Because the model uses a first-order linear approximation of the Cobb–Douglas production function, the capital stock is expressed in normalized units rather than literal national accounting values.

¹³ This measure captures government consumption expenditures on goods and services, excluding public capital formation and redistributive transfers (such as pensions and sovereign debt service), which do not directly reduce the resources available for capital accumulation.

¹¹ For notational simplicity, from this point forward, the state variables $K(t)$ and $SE(t)$, evaluated at time t , are denoted simply as K and SE .

Increases in atmospheric carbon dioxide and global temperature (1850-2024)

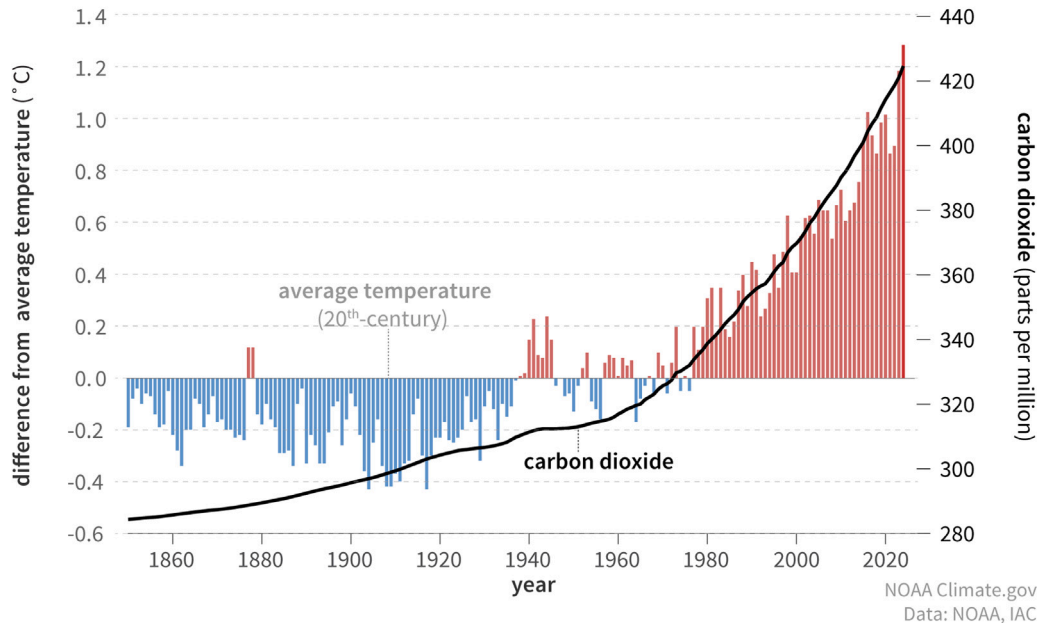


Fig. 1. Carbon dioxide vs. global temperature anomaly 1850-2024.
 Source: NOAA Climate.gov, Increases in atmospheric carbon dioxide and global temperature (1850-2024), <https://www.climate.gov/media/16969>. Data: NOAA and IAC.

DICE-2023 (Barrage and Nordhaus, 2024), which updates (Nordhaus, 2017) based on recent empirical meta-analyses placing output losses from moderate warming in the 1%–3% range. Given substantial uncertainty surrounding damage estimates, we consider an exploratory range for ψ between 0.005 and 0.040. The upper bound reflects growing evidence that traditional IAMs systematically underestimate climate risks, particularly regarding persistent productivity shocks (Dietz and Stern, 2015; Burke et al., 2015). Notably, Bilal and Känzig (2026) employed a global temperature approach, estimating output losses between 12% and 20% per degree of warming—substantially larger than traditional DICE specifications. While our linear baseline retains the mainstream consensus for analytical comparability, we evaluate these catastrophic risks in Section 4.2 through a quadratic formulation that accounts for tipping points and nonlinear planetary feedbacks (Lenton et al., 2019; Armstrong McKay et al., 2022).

The temperature-emissions sensitivity parameter, η , is calibrated to generate temperature paths consistent with benchmark IAM scenarios over the simulation horizon H . In the DICE framework (Nordhaus and Sztorc, 2013; Nordhaus, 2017), equilibrium climate sensitivity is typically parameterized such that a doubling of atmospheric CO_2 yields approximately 3 °C of warming. In our linear formulation, η captures climate system inertia and atmospheric persistence. We anchor the parameter to current climate observations: given the 2024 atmospheric concentration of $SE(0) = 420$ ppm (Fig. 1), setting $\eta = 0.003$ matches the observed warming of $T(0) = 1.2$ °C relative to the pre-industrial baseline. This linear mapping reflects the TCRE, implying an adjustment of approximately 2.8 °C per 1000 ppm (~ 1.6 °C per 1000 GtC). This value lies strictly within the IPCC likely range of 1.0 °C to 2.3 °C per 1000 Gt CO_2 , confirming the physical consistency of the climate module.

The carbon cycle is specified through a reduced-form law of motion with a natural decay rate of $\lambda = 0.004$ per annum. This value captures the long-term atmospheric persistence of CO_2 and is deliberately calibrated toward the conservative bound of IAM specifications (typically ranging between $\lambda = 0.004$ and 0.010) (Dietz and Stern, 2015; Nordhaus, 2017). This lower decay rate accounts for emerging evidence

on climate tipping points, where elevated temperatures degrade the carbon absorption capacity of terrestrial and oceanic sinks (Lenton et al., 2019). In a reduced-form setting, this parameterization serves as a proxy for nonlinear Earth-system feedbacks (e.g., forest dieback and permafrost thawing) that prolong atmospheric residence times under sustained warming.

Mitigation and adaptation outlays are modeled as convex functions, with policy parameters calibrated such that total policy expenditures range between 0% and 5% of GDP depending on effort intensity, consistent with prevailing empirical benchmarks (Kikstra et al., 2022). In the baseline scenario, mitigation effort is set to $I_m = 0.30$, corresponding to a 30% reduction in net emissions relative to the no-policy baseline. With the marginal cost parameter calibrated to $\theta = 0.10$, mitigation expenditures (C_{I_m}) absorb 0.9% of gross output annually. Similarly, adaptation effort is fixed at $I_a = 0.20$, delivering a 20% reduction in climate damages. Given $\mu = 0.05$ – reflecting evidence that adaptation is less capital-intensive than structural mitigation – adaptation expenditures (C_{I_a}) represent 0.2% of gross output per year. While these parameters remain fixed in the baseline to isolate the core analytical mechanisms, they are varied dynamically in subsequent sections to evaluate alternative transition pathways.

3. Equilibrium and local stability analysis

3.1. Equilibrium

Equilibrium is defined by the fixed-point conditions:

$$K(t+1) = K(t) = K^*, \quad SE(t+1) = SE(t) = SE^*.$$

The dynamical system admits up to two equilibrium points, denoted by

$$FP_1 = (K_1^*, SE_1^*), \quad FP_2 = (K_2^*, SE_2^*),$$

which can be expressed in compact analytical form as:

$$K_1^* = \frac{-a_2 - \sqrt{a_2^2 - 4a_1a_3}}{2a_1}, \quad SE_1^* = \frac{\sigma_e \alpha}{\lambda} K_1^* + \frac{I_e}{\lambda},$$

Table 1
Model calibration.

Parameter	Description	Value	Range
α	Input substitution rate in production	0.3	0.2 - 0.4
c	Share of private and public consumption	0.8	0.7 - 0.9
δ	Capital depreciation rate	0.12	0.05 - 0.15
L	Labor employed (in billions hours)	25	
ψ	Damage function coefficient (effect of T on D)	0.010	.005 - .040
η	Temperature-emissions sensitivity (effect of E on T)	0.003	.002 - .005
σ	Carbon intensity of production	0.25	0.15 - 0.40
θ	Marginal cost of mitigation	0.10	0.05 - 0.15
μ	Marginal cost of adaptation	0.05	0.02 - 0.08
I_m	Mitigation effort	0.30	0 - 0.60
I_a	Adaptation effort	0.20	0 - 0.40
λ	Natural decay rate of stock emissions	0.004	.004 - .010

Note: The “Value” column reports baseline simulation values used in Section 4, while the “Range” column shows the variation used for comparative statics analysis.

$$K_2^* = \frac{-a_2 + \sqrt{a_2^2 - 4a_1a_3}}{2a_1}, \quad SE_2^* = \frac{\sigma_e \alpha}{\lambda} K_2^* + \frac{L_e}{\lambda},$$

where

$$a_1 = \frac{\alpha^2 \eta_e \sigma_e}{\lambda}, \quad a_2 = \frac{\delta}{1-c} + \frac{2\alpha \eta_e L_e}{\lambda} - \alpha(1 - \theta_e - \mu_e),$$

$$a_3 = \frac{L^2(1-\alpha)^2 \eta_e \sigma_e}{\lambda} - L(1-\alpha)(1 - \theta_e - \mu_e),$$

and

$$\eta_e = \eta \psi (1 - I_a), \quad \sigma_e = \sigma (1 - I_m), \quad L_e = L \sigma_e (1 - \alpha),$$

$$\theta_e = \theta I_m^2, \quad \mu_e = \mu I_a^2.$$

Under the baseline calibration, the first fixed point, FP_1 , is economically non-viable, as it yields strictly negative values for both state variables. Consequently, subsequent analytical and numerical investigations focus exclusively on the interior, feasible steady state, FP_2 .

3.2. Trace–determinant representation and bifurcation analysis

To analyze local asymptotic stability, we evaluate the Jacobian matrix of system (10b) at the feasible steady state:

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix},$$

where

$$J_{11} = 1 - \delta + \alpha(1-c)(1 - \eta_e SE^* - \theta_e - \mu_e),$$

$$J_{12} = -\eta_e(1-c)[\alpha K^* + (1-\alpha)L],$$

$$J_{21} = \alpha \sigma_e, \quad J_{22} = 1 - \lambda.$$

For a two-dimensional discrete-time system, local stability can be conveniently characterized in terms of the trace and determinant:

$$\tau = \text{tr}(J), \quad \Delta = \det(J).$$

Using the steady-state condition,

$$SE^* = \frac{\sigma_e}{\lambda} [\alpha K^* + (1-\alpha)L],$$

we define:

$$A = \alpha(1-c)(1 - \theta_e - \mu_e), \quad B = \alpha(1-c)\eta_e SE^*.$$

Then,

$$\tau = 2 - \delta - \lambda + A - B, \quad \Delta = (1-\lambda)(1-\delta+A) - (1-2\lambda)B.$$

Proposition 1 (Local Stability and Bifurcation Thresholds). *The steady state (K^*, SE^*) is locally asymptotically stable if and only if*

$$1 - \tau + \Delta > 0, \quad 1 + \tau + \Delta > 0, \quad 1 - \Delta > 0.$$

Moreover:

- $1 - \tau + \Delta = 0$ defines a Fold bifurcation,
- $1 + \tau + \Delta = 0$ defines a Flip bifurcation,
- $\Delta = 1$ and $|\tau| < 2$ define a Neimark–Sacker bifurcation.

Proof. The eigenvalues satisfy $\lambda^2 - \tau\lambda + \Delta = 0$. Stability requires both eigenvalues to lie inside the unit circle, which is equivalent to the Jury conditions. $\square$

Proposition 2 (Economic Interpretation of Stability). *The local asymptotic stability of the dynamical system is governed by the structural interplay between the macroeconomic accumulation factor (A) and the climate feedback mechanism (B).*

- An amplification of climate feedback intensity increases B , exerting downward pressure on both the trace τ and determinant Δ . Under our baseline parameterization, where both eigenvalues remain strictly positive, this shift pushes the steady state away from the Neimark–Sacker bifurcation boundary ($\Delta = 1$), precluding the emergence of limit cycles and closed invariant curves.
- Escalating climate policy expenditures diminish A , thereby suppressing the rate of capital accumulation.
- Elevated capital depreciation δ reduces τ and, beyond a critical threshold, can force an eigenvalue across the unit circle to trigger dynamic instability.

3.3. Discussion and local robustness

Under the baseline calibration, both Jacobian eigenvalues are real, positive, and lie strictly within the unit circle, confirming monotonic asymptotic convergence toward the feasible steady state. The exact numerical coordinates of the fixed points and their associated eigenvalues are detailed in Appendix A.1.

Although the dominant eigenvalue approaches unity, this proximity reflects the deep physical persistence of atmospheric carbon dynamics (specifically, the low natural decay rate λ) rather than underlying structural fragility; the system exhibits slow transitional convergence without bordering a bifurcation boundary. Parameter sensitivity remains tightly constrained because perturbations generate countervailing adjustments across the trace and determinant, preserving the system within the stable domain of the (τ, Δ) plane. Consequently, violating the Jury stability conditions would necessitate severe parameter variations, establishing the local robustness of the equilibrium.

Table 2
Local robustness under one-parameter perturbations.

Parameter	Baseline	Closest bifurcation	Direction	Distance
δ	0.12	Flip	increase	large
α	0.30	Neimark–Sacker	increase	very large
η	0.003	none	–	very large
ψ	0.01	none	–	very large
σ	0.25	none	–	large
I_m	0.30	none	–	large
I_a	0.20	none	–	large

Crucially, climate feedbacks exert a stabilizing influence on the determinant via B , suppressing transient overshoots and oscillatory trajectories. As summarized in Table 2, which presents a qualitative robustness classification under single-parameter perturbations,¹⁴ breaching the Jury stability boundaries requires empirically implausible structural deviations. The feasible steady state $F P_2$ therefore operates as a robust long-run macroeconomic attractor.

3.4. Growth conditions for capital and emissions

The laws of motion governing capital and the atmospheric emissions stock permit a transparent analytical characterization of the structural conditions required for sustained growth in each state variable.

Starting from the capital accumulation difference equation,

$$K(t+1) = K(t)(1-\delta) + (1-c)Q(t),$$

capital grows whenever

$$K(t+1) - K(t) > 0 \iff (1-c)Q(t) > \delta K(t).$$

Substituting the definition of net output,

$$Q(t) = Y(t)[1 - D(t) - C_{Im} - C_{Ia}],$$

alongside the linearized production function and linear damage formulation,

$$Y(t) = \alpha K(t) + (1-\alpha)L, \quad D(t) = \eta\psi(1 - I_a)SE(t)$$

yields the explicit growth condition:

$$(1-c)[\alpha K(t) + (1-\alpha)L] \left(1 - \theta I_m^2 - \mu I_a^2 - \eta\psi(1 - I_a)SE(t) \right) > \delta K(t).$$

Thus, capital expansion is driven by the net marginal return on production, whereas physical depreciation, policy expenditures, and climate damages act as dynamic drags.

These macroeconomic forces directly govern capital accumulation incentives by determining the marginal product of capital in terms of net output:

$$MPK(t) = \frac{\partial Q(t)}{\partial K(t)} = \alpha \left(1 - \theta I_m^2 - \mu I_a^2 - \eta\psi(1 - I_a)SE(t) \right).$$

This expression highlights that climate deterioration, driven by cumulative emissions accumulation over time, amplifies economic damages and erodes capital productivity. In this context, the condition

$$(1-c)MPK(t) \geq \delta$$

establishes a parsimonious reduced-form threshold for capital formation: capital stock expands whenever marginal savings generated by an additional unit of capital exceed physical depreciation. The full dynamic condition also accounts for the baseline contribution of labor, which remains fixed at L in our linearized setting.

Turning to the environmental block, from the law of motion,

$$SE(t+1) = (1-\lambda)SE(t) + E(t),$$

the stock of emissions increases whenever

$$SE(t+1) - SE(t) > 0 \iff E(t) > \lambda SE(t).$$

Substituting the net emissions equation yields

$$\sigma(1 - I_m)[\alpha K(t) + (1-\alpha)L] > \lambda SE(t).$$

Hence, the emissions stock grows whenever current net emissions exceed the natural assimilation capacity of the Earth system. In the long-run steady state, this relation reduces to

$$E^* = \lambda SE^*,$$

which pins down the physical balance between anthropogenic emissions flows and natural sink absorption.

Jointly, these conditions formalize the central feedback mechanism of the model: economic growth increases emissions flows and atmospheric carbon concentrations, while the resulting climate damages feed back negatively into capital productivity, ultimately decelerating capital accumulation along the transition path.

4. Simulations

In this section, using the parameter values reported in Table 1, we trace the joint macroeconomic and climate dynamics generated by the model over a 300-year simulation horizon. This extended horizon is necessary to capture both the thermal and carbon-cycle inertia of the climate system alongside the slow-moving adjustments governing capital accumulation.

The numerical analysis is structured as a sequence of simulation experiments that progressively relax baseline assumptions to assess the robustness of the core findings and evaluate alternative model specifications. Rather than solving an optimizing IAM, our model is designed as a scenario-based simulation tool, making it well-suited for clarifying the structural trade-offs between mitigation and adaptation policies across diverse exogenous policy pathways.

Section 4.1 evaluates a benchmark environment with time-invariant climate policies, comparing five policy intensities – *Null*, *Low*, *Moderate*, *Strong*, and *Very Strong* – mapped directly into the SSP framework. This baseline analysis identifies an optimal interior equilibrium under the *Moderate* scenario: here, the “resilience dividend” fully materializes, as investments in mitigation and adaptation effectively self-finance by insulating the productive capital base from climate damages. However, as an intermediate policy configuration, the *Moderate* scenario fails to achieve the most ambitious climate stabilization targets. Conversely, the *Very Strong* scenario emerges as the only pathway capable of limiting long-run warming to approximately 2 °C. This environmental achievement, however, entails a substantial resource reallocation trade-off: high policy expenditures (exceeding 4% of gross output) crowd out productive investment, resulting in lower steady-state net output relative to the *Moderate* benchmark.

Section 4.2 replaces the linear damage specification with a quadratic formulation in temperature anomalies, aligning the framework with the modern DICE consensus and the planetary boundaries literature. This nonlinear specification captures tipping-point risks and self-reinforcing climate feedbacks. Incorporating accelerating damages fundamentally alters policy evaluation, shifting the economic optimum toward aggressive intervention. Under quadratic damages, the *Null* scenario precipitates catastrophic output losses approaching 20% of gross output. Crucially, the policy hierarchy is reversed: whereas the *Moderate* policy maximized net output under linear damages, the *Very Strong* policy becomes an economic necessity in the quadratic environment. Despite substantial front-loaded costs, aggressive intervention secures the highest long-run net output and averts secular economic stagnation. This comparative exercise demonstrates that when climate risks are nonlinear, ambitious mitigation and adaptation operate not as discretionary environmental preferences, but as essential safeguards for macroeconomic stability.

¹⁴ Table 2 provides a qualitative classification based on one-parameter perturbations, rather than a full global bifurcation analysis.

In Section 4.3, we relax the assumption of constant-policy efforts by allowing mitigation (I_m) and adaptation (I_a) to expand linearly over time, capturing the progressive intensification of climate policy driven by growing environmental awareness. This dynamic path reveals a critical intertemporal asymmetry: while emissions flows respond rapidly to policy stringency, atmospheric concentrations, temperature anomalies, and economic damages adjust with protracted lags due to Earth-system inertia. Under this gradual ramp-up, total policy expenditures peak at over 7% of GDP by year 300. The model generates a pronounced hump-shaped trajectory for climate damages, which escalate over the medium term before receding once policy efforts reach critical thresholds. Consequently, gradualism fails to prevent significant thermal overshoot, leaving global temperature at 2.65 °C at the end of the horizon. This highlights an essential intertemporal trade-off: postponing aggressive mitigation and adaptation increases cumulative climate expenditures while locking the economy into elevated, long-lasting climate damages.

Section 4.4 introduces exogenous total factor productivity (TFP) growth alongside learning-by-doing mechanisms that reduce marginal mitigation and adaptation costs over time. In this dynamic setting, the apparent short-run superiority of less stringent policies proves illusory: while the *Null* policy appears advantageous over the first 90 years due to the absence of direct abatement outlays, the ongoing accumulation of climate damages and falling costs of low-carbon technologies radically alter the policy ranking. The *Moderate* policy maximizes net output over the medium horizon, but the *Very Strong* policy establishes decisive economic superiority after 240 years. By generating the highest long-run standard of living, this scenario demonstrates that aggressive climate action is not an economic sacrifice, but the only pathway capable of decoupling long-term growth from climate risk. Endogenous cost reductions make ambitious climate policy self-financing, reinforcing the role of clean R&D as a rational macroeconomic insurance strategy.

Finally, Section 4.5 evaluates how the structural shift toward intangible capital (software, algorithms, and digital infrastructure) alters the climate-economy nexus. We partition aggregate capital to isolate the compositional effect of intangible assets across two opposing scenarios reflecting current debates on the environmental footprint of digitalization. In the first scenario, intangible capital lowers the material intensity of production, driving decarbonization, containing climate damages to 2.6% of output, and stabilizing temperature below baseline levels. In the second scenario, expanding computational and artificial intelligence (AI) infrastructure escalates energy demand, rendering intangible capital a driver of emissions growth and pushing climate damages toward 3% of gross output. These divergent paths demonstrate that digitalization is not inherently green: its net macroeconomic and environmental impact depends on whether efficiency gains from dematerialization exceed the expanding energy requirements of computing infrastructure.

The remainder of this section details these simulation experiments, analyzing the underlying macroeconomic and environmental dynamics using comparative statics.

4.1. Baseline scenario

The baseline scenario assumes that mitigation and adaptation policy parameters remain time-invariant. Table 3 reports the terminal outcomes evaluated at the end of the 300-year simulation horizon H across five policy intensity configurations: *Null*, *Low*, *Moderate*, *Strong*, and *Very Strong*. These scenarios vary systematically in their calibrated mitigation (I_m) and adaptation (I_a) effort levels, generating distinct economic cost profiles and environmental trajectories. While Table 3 documents all five policy regimes, Figs. 2 and 3 illustrate the dynamic transition paths for three representative cases to ensure visual clarity: *Null* ($I_m = 0$, $I_a = 0$), *Moderate* ($I_m = 0.30$, $I_a = 0.20$), and *Very Strong* ($I_m = 0.60$, $I_a = 0.40$), plotted in red, blue, and green, respectively.

These policy configurations map directly into the SSP framework (Riahi et al., 2017; Meinshausen et al., 2020), facilitating direct comparisons with the IAM literature. The *Null* scenario reflects an unmitigated business-as-usual trajectory consistent with high-emissions pathways such as SSP5-8.5 or extreme SSP3-7.0. The *Low* and *Moderate* intensities correspond to intermediate “Middle of the Road” pathways (such as SSP2-4.5), capturing partial and gradual climate policy implementation. The *Strong* scenario aligns broadly with SSP1-2.6, where coordinated international abatement substantially curtails emissions flows. Finally, the *Very Strong* scenario approximates the most ambitious sustainability benchmarks (SSP1-1.9), designed to limit long-run warming to approximately 2 °C or below.¹⁵

Fig. 2 illustrates the dynamic evolution of mitigation costs (C_{I_m}), adaptation costs (C_{I_a}), and climate damages (D) over time, each expressed as a percentage of gross annual output (Y). Fig. 3 traces the trajectories of the primary macroeconomic and environmental variables under the three representative climate policy regimes. Moving sequentially from top-left to bottom-right, the panels depict the 300-year evolution of the capital stock (K), atmospheric emissions stock (SE), gross output (Y), net output (Q), annual net emissions flow (E), and mean global temperature anomaly (T).¹⁶ Policy outlays and climate damages exhibit a clear inverse relationship. As shown in Fig. 2, policy costs scale nonlinearly with effort intensity, reflecting the quadratic formulation of both mitigation and adaptation cost functions.¹⁷ This convexity captures the economic reality of abatement: initial emissions reductions and adaptation measures are relatively inexpensive, whereas deeper decarbonization requires increasingly costly structural adjustments and capital-intensive technologies. In the *Moderate* scenario, total policy expenditures remain modest at 1.1% of gross output, rising to 2.475% in the *Strong* case and reaching 4.4% under the *Very Strong* regime. These expenditure levels match IAM benchmarks for long-term temperature stabilization (Kikstra et al., 2022) and underscore the fundamental trade-off between macroeconomic growth and environmental protection.

Stringent climate policies substantially reduce climate damages. As documented in Table 3, damages decline monotonically from 4.78% of gross output in the *Null* scenario to 3.72% in the *Low* case, 2.77% in the *Moderate* scenario, 1.95% in the *Strong* scenario, and 1.27% under *Very Strong* intervention. This reduction of more than 3.5 percentage points preserves a substantial portion of the productive base, demonstrating the efficacy of joint mitigation and adaptation strategies. In particular, adaptation plays an essential role in containing residual damages from committed warming, complementing mitigation efforts that curb emissions at the source.

While upfront expenditures under the *Very Strong* scenario exceed 4% of GDP, they deliver a substantial long-term economic return. As illustrated in Fig. 2, whereas climate damages accelerate sharply toward the end of the simulation horizon under the *Null* scenario, they remain tightly contained under active policy regimes, generating an expanding “resilience dividend” over time. We define this dividend as the widening wedge between potential unmitigated damages and realized climate losses achieved through defensive investments. Baseline damage estimates align with standard IAM benchmarks, such as

¹⁵ While standard SSP projections extend to 2100, our model simulates dynamics across a multi-century horizon through 2300. Notably, the *Strong* (SSP1-2.6) and *Very Strong* (SSP1-1.9) scenarios represent the only pathways capable of maintaining warming below 2 °C by 2100 (Intergovernmental Panel on Climate Change (IPCC), 2021) (see the green trajectory in the final panel of Fig. 3).

¹⁶ Global temperature anomalies are defined relative to the pre-industrial baseline (in °C).

¹⁷ Although policy efforts remain constant in this baseline, the convex structure of the cost functions implies that more ambitious targets generate disproportionately larger resource diversions. Section 4.3 relaxes this assumption to evaluate dynamic pathways where policy efforts scale up over time.

Table 3
Comparative statics for different mitigation and adaptation efforts.

Policy intensity	Effort and Cost	K_H (b.€)	Y_H (b.€)	Q_H (b. €)	SE_H (ppm)	E_H (kg CO ₂)	T_H (°C)	D_H (%Y)
<i>Null</i>	$I_m = 0, C_{Im} = 0\%$ $I_a = 0, C_{Ia} = 0\%$	53.12	33.44	31.84	1592.04	8.36	+4.78	4.78%
<i>Low</i>	$I_m = 0.15, C_{Im} = 0.225\%$ $I_a = 0.10, C_{Ia} = 0.05\%$	53.93	33.68	32.34	1377.72	7.16	+4.13	3.72%
<i>Moderate</i>	$I_m = 0.30, C_{Im} = 0.9\%$ $I_a = 0.20, C_{Ia} = 0.2\%$	54.04	33.71	32.41	1155.44	5.9	+3.47	2.77%
<i>Strong</i>	$I_m = 0.45, C_{Im} = 2.025\%$ $I_a = 0.30, C_{Ia} = 0.45\%$	53.42	33.53	32.04	929.04	4.61	+2.79	1.95%
<i>Very Strong</i>	$I_m = 0.60, C_{Im} = 3.6\%$ $I_a = 0.40, C_{Ia} = 0.8\%$	52.10	33.13	31.25	702.55	3.31	+2.11	1.27%

Note: Initial conditions at $t = 0$: $K(0) = 30$ b. € and $SE(0) = 420$ ppm, so $T(0) \approx +1.2$ °C over pre-industrial levels. Costs and climate damages are expressed in percent of gross output Y^* per year.

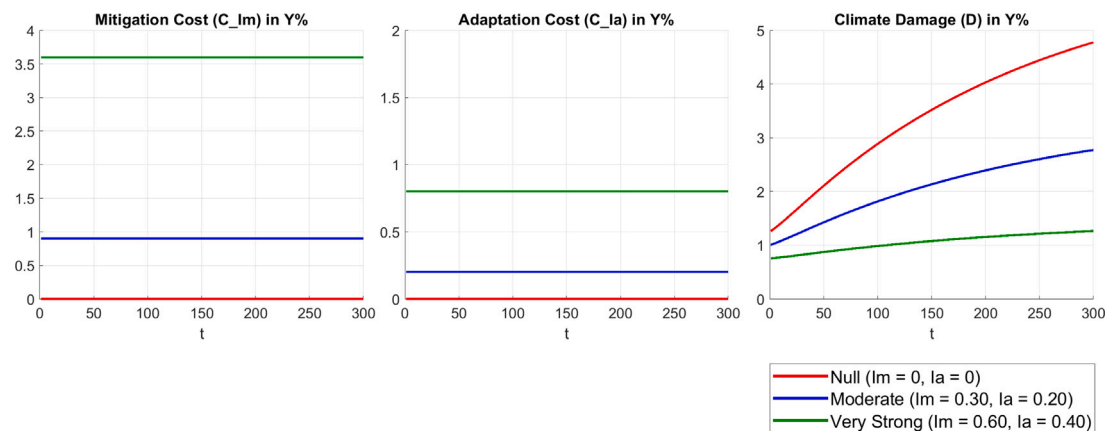


Fig. 2. Evolution of climate-related costs in three policy scenarios.

Note: For readability, we report three representative scenarios out of the five considered in Table 3: *Null* (red), *Moderate* (blue), and *Very Strong* (green). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

DICE-type models (Nordhaus, 2023), where warming of 4–5 °C under unmitigated trajectories generates output losses of 4%–6%. Conversely, recent contributions (e.g., Burke et al., 2015; Dietz and Stern, 2015; Bilal and Känzig, 2026) emphasize that for warming exceeding 3 °C, nonlinear feedbacks and tipping points could trigger double-digit GDP losses (15%–20% annually). To address these risks, Section 4.2 incorporates a quadratic damage function that amplifies losses at elevated temperatures while holding the baseline parameterization of Table 1 constant.

From a climatological perspective, the baseline model demonstrates close consistency with the SSP framework and IPCC projections. The transition dynamics in Fig. 3 provide direct validation at the 2100 policy benchmark: the *Null* scenario exceeds 3 °C with atmospheric concentrations approaching 1000 ppm, whereas the *Very Strong* scenario holds warming to 1.6–1.7 °C at approximately 500 ppm. This 1.4 °C differential represents the threshold between achieving and missing the temperature goals of the Paris Agreement.

Over the full 300-year horizon, these trajectories diverge even more dramatically. The *Null* scenario reaches an atmospheric carbon stock of approximately 1592.04 ppm, inducing catastrophic warming of +4.78 °C. Intermediate policy scenarios achieve progressively stabilized outcomes: the *Low* and *Moderate* policies constrain atmospheric concentrations to 1377.72 ppm and 1155.44 ppm, limiting temperature increases to +4.13 °C and +3.47 °C, respectively. These trajectories correspond closely to SSP2-4.5 pathways, where partial mitigation slows but fails to arrest warming. The *Strong* scenario lowers concentrations to 929.04 ppm and caps warming at +2.79 °C, while the *Very Strong* regime stabilizes atmospheric carbon at 702.55 ppm

and temperature anomalies at +2.11 °C, closely matching SSP1-2.6 pathways. Avoiding nearly 2.67 °C of warming economically justifies ambitious policy outlays as a rational safeguard against biosphere degradation.

Because even the most stringent policy ($I_m = 0.60$) does not achieve full net-zero emissions, the atmospheric emissions stock (SE) continues to grow, albeit at a substantially reduced rate. This behavior captures an intermediate decarbonization regime in which mitigation attenuates emissions intensity without eliminating gross emissions entirely, resulting in gradual atmospheric stabilization rather than active climate cooling.¹⁸ Nonetheless, annual net emissions flows (E) drop sharply with policy stringency, falling from 8.36 kg of CO₂ per euro of output under the *Null* baseline to 5.9 under *Moderate* policy, and reaching 3.31 in the *Very Strong* case.

The interplay between policy costs and avoided damages governs the core trade-off between growth and climate stabilization.

Examining capital dynamics, policy intensity affects long-run accumulation through a resource reallocation mechanism. Steady-state capital K^* rises from 53.12 in the *Null* scenario to a maximum of 54.04 under *Moderate* policy, as the moderate diversion of output into climate protection is more than offset by damage reductions that preserve capital productivity. Beyond this point, further increases in policy

¹⁸ This assumption is relaxed in Section 4.3, where a dynamic policy reaches $I_m = 0.80$ to perform the primary technical abatement, assuming the remaining 20% of residual emissions in hard-to-abate sectors are neutralized via carbon removal technologies outside the model.

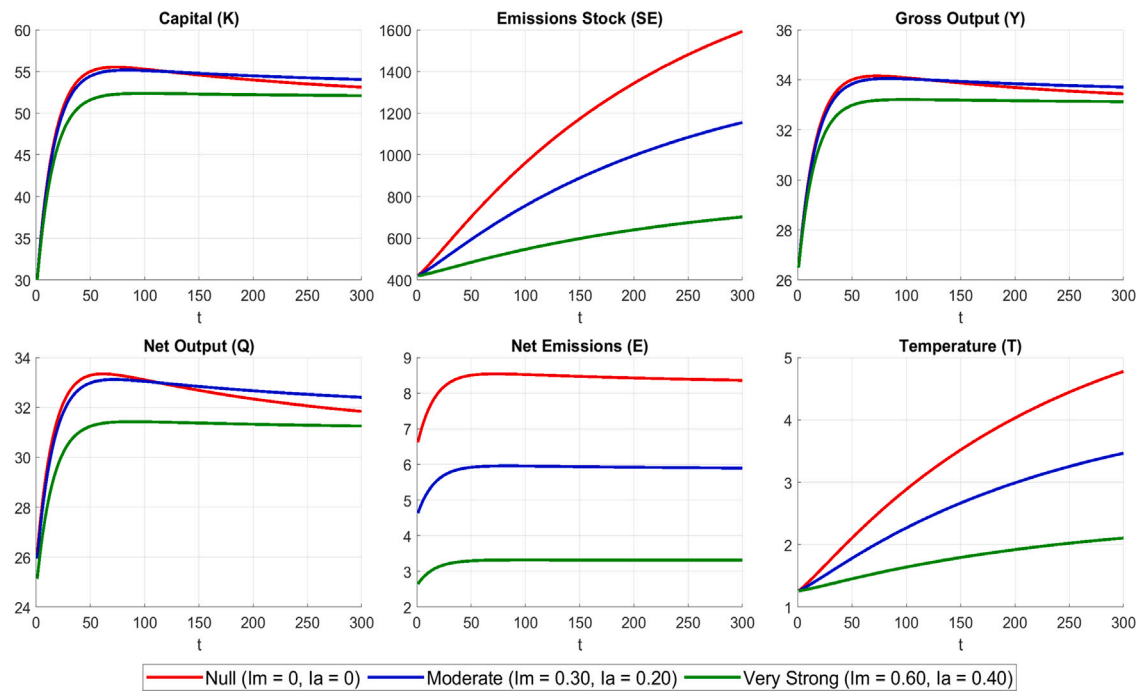


Fig. 3. Macroeconomic and environmental variables in three policy scenarios.

Note: For readability, we report three representative scenarios out of the five considered in Table 3: *Null* (red), *Moderate* (blue), and *Very Strong* (green). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

stringency cause steady-state capital to contract to 53.42 (*Strong*) and 52.10 (*Very Strong*). This inverted-U relationship highlights a “capital reallocation trade-off”: while moderate policies safeguard the productive capital stock, aggressive interventions absorb an excessive share of output, crowding out capital formation.

Gross output (Y) and net output (Q) exhibit similar, more pronounced non-monotonic trajectories. In the long run, steady-state gross output (Y^*) varies within a narrow band: from 33.44 in the *Null* scenario to 33.71 in the *Moderate* case and 33.13 under *Very Strong* policy.

From a dynamic perspective, Fig. 3 demonstrates that capital stock, gross output, and net output expand during the initial phase of the transition – driven by capital accumulation – before converging asymptotically to their steady-state equilibria. In the absence of exogenous technological progress, long-run economic expansion is eventually constrained by the compounding pressures of physical depreciation, policy expenditures, and climate damages. These forces govern capital accumulation incentives: as the marginal product of capital, $MPK(t)$, declines over time under rising atmospheric carbon concentrations, the macroeconomy converges toward the steady-state equilibrium where marginal net savings offset depreciation, satisfying $(1 - c) MPK(t) = \delta$ as established in Section 3.4.

While this stationary outcome differs from standard DICE frameworks – where economic output expands indefinitely due to sustained TFP growth – it establishes a conservative benchmark where climate damages and depreciation offset capital formation. To incorporate sustained long-run growth while retaining analytical tractability, Section 4.4 extends the baseline model by introducing exogenous technological change in line with the broader IAM literature.

Net output (Q) serves as the primary metric for welfare and policy evaluation. The *Moderate* policy scenario attains the highest steady-state level ($Q^* = 32.41$), demonstrating the existence of an interior optimum in policy space. In this regime, combined mitigation and adaptation investments substantially mitigate climate damages while maintaining modest policy costs. This finding aligns with the optimal policy outcome in DICE-type models, where intermediate abatement

targets maximize net economic welfare (Nordhaus and Sztorc, 2013; Nordhaus, 2018a, 2023). Conversely, the *Null* scenario suffers from severe climate damages despite zero policy expenditures, yielding a depressed steady-state net output Q^* (31.84). At the opposite end of the spectrum, the *Very Strong* scenario secures the best environmental trajectory (minimizing emissions, warming, and climate losses) but incurs elevated policy costs that reduce steady-state net output ($Q^* = 31.25$) below the *Moderate* optimum.

This dynamic pattern reflects the increasing marginal cost of climate policy interventions, balanced against their expanding returns in terms of damage mitigation and temperature stabilization. A principal finding emerging from this comparative statics analysis is that *Moderate* climate action generates net macroeconomic gains while simultaneously delivering substantial reductions in climate damages. This synergy constitutes what we term a “resilience dividend”: under this policy regime, climate expenditures operate as productive capital investments rather than pure deadweight costs, as avoided climate damages more than offset initial output diversions.

A pairwise comparison across policy regimes clarifies this economic mechanism. Transitioning from the *Null* to the *Moderate* baseline requires a total policy outlay of 1.1% of Y^* (0.9% for mitigation and 0.2% for adaptation), yet it achieves a 2.0 percentage point reduction in climate damages (falling from 4.78% to 2.77%), delivering an unambiguous net welfare improvement. Conversely, scaling policy stringency from *Moderate* to *Very Strong* necessitates an additional 3.3% of Y^* in policy costs (rising from 1.1% to 4.4%), while securing an incremental damage reduction of only 1.51 percentage points (from 2.77% to 1.27%). Across this higher-intensity interval, the marginal cost of additional policy stringency distinctly outweighs the marginal benefit of avoided damages. Thus, under aggressive policy regimes, the convex escalation of mitigation and adaptation costs outpaces incremental damage reductions, reinstating the classic trade-off between near-term macroeconomic performance and long-run environmental ambition (Stern, 2007; Nordhaus, 2017). Moreover, the baseline model

abstracts from potential co-benefits of mitigation (e.g., induced technological spillovers, localized air quality improvements, and learning-by-doing) that are frequently emphasized in the IAM literature. Incorporating these complementary mechanisms would likely tilt the net benefit calculation further in favor of ambitious abatement.

In sum, these findings establish that the optimal policy corridor balances near-term fiscal outlays against long-term macroeconomic stability. While mitigation and adaptation outlays constrain consumption and capital formation in the short run, they protect the economy from being locked into irreversible, high-damage climate trajectories (Dietz and Stern, 2015; Kikstra et al., 2022). Postponing policy action raises the probability of crossing critical Earth-system thresholds, establishing early, well-calibrated intervention as an indispensable requirement for long-run macroeconomic sustainability.

4.2. Accelerating climate damage function

In the baseline scenario of Section 4.1, a linear climate damage function was adopted to maintain analytical tractability. Under this assumption, the capital accumulation map $K(t+1)$ in Eq. (10b) takes the form of a quadratic polynomial, permitting the closed-form derivation and qualitative characterization of steady-state equilibria.

However, an extensive climate-economic literature indicates that linear damage specifications systematically understate the risks associated with elevated warming trajectories. In this section, we align our framework with standard DICE formulations (Nordhaus and Sztorc, 2013) and recent empirical evidence by specifying climate damages as a quadratic function of temperature anomalies. This nonlinear formulation explicitly captures the disruptive risks of climate change, including accelerated sea-level rise and proximity to irreversible Earth-system tipping points (Lenton et al., 2019; Armstrong McKay et al., 2022)—critical planetary thresholds that could trigger abrupt carbon releases or destabilize the biosphere.¹⁹ Recent empirical estimates by Bilal and Känzig (2026) further suggest that long-run macroeconomic warming impacts may be substantially more severe than previously estimated, with a 1 °C increase in global temperature potentially reducing world GDP by up to 20%. This perspective aligns closely with the planetary boundaries framework (Rockström et al., 2009; Steffen et al., 2020; Richardson et al., 2023), which emphasizes that anthropogenic forcing is pushing the Earth system beyond its safe operating space, precipitating compounding economic losses from extreme weather events.

Accordingly, holding all other structural components constant, we reformulate the climate damage function as

$$D(t) = \psi (1 - I_a) T(t)^2 \quad (11)$$

To maintain consistency and facilitate direct comparisons across specifications, we retain the baseline calibration reported in Table 1 for the linear model in Section 4.1, including the damage parameter $\psi = 0.010$.

Figs. 4 and 5 provide a direct comparative analysis between the linear (blue solid line) and quadratic (red dashed line) damage specifications under the *Moderate* policy regime. In Fig. 4, although both specifications originate from comparable baseline damages (approximately 1% of GDP), their dynamic trajectories diverge sharply over the simulation horizon. Whereas linear damages expand gradually to reach approximately 2.77% of gross output after 300 years, the quadratic specification precipitates an exponential escalation in climate losses, exceeding 9% of gross output by the end of the horizon. As illustrated in these figures, this divergence widens progressively across the simulation timeframe. This structural wedge emerges because climate damages scale disproportionately with thermal forcing; at the elevated

temperatures reached during the latter half of the simulation, nonlinear Earth-system feedback triggers compounding losses that a linear approximation structurally fails to capture.

The resulting contraction across macroeconomic aggregates (Fig. 5) is equally severe: under quadratic damages, steady-state net output (Q^*) contracts to approximately 29, compared to 32.4 under the linear specification—representing a 12% reduction in the long-run standard of living driven solely by the nonlinearity of climate feedback. Crucially, the marginally lower terminal temperature trajectory observed in the quadratic case (3.35 °C versus 3.47 °C) constitutes an endogenous “poverty effect”: severe macroeconomic contraction suppresses the aggregate scale of economic activity, thereby curtailing net emissions flows (E^* contracts from 5.9 to 5.6 kg of CO₂ per euro, lowering SE^* from 1155 to 1120 ppm).

These findings reinforce the macroeconomic rationale for precautionary climate intervention: when damages follow a nonlinear trajectory, the economic penalties of under-abatement escalate substantially. In this regard, comparing linear and quadratic formulations serves not only as a structural sensitivity analysis, but also aligns our parsimonious model directly with mainstream IAM frameworks.

The introduction of quadratic damages fundamentally alters the ranking of policy regimes. Whereas the baseline linear specification established the *Moderate* policy as the interior welfare-maximizing optimum for steady-state net output (Q^*) (Table 3), the quadratic formulation shifts the optimal policy balance decisively toward aggressive intervention.

Figs. 6 and 7 illustrate the comparative dynamics across the *Null* (red), *Moderate* (blue), and *Very Strong* (green) policy regimes under the quadratic specification. The unmitigated *Null* scenario precipitates severe macroeconomic degradation: absent policy intervention, climate damages surge to 20% of gross output by $t = 300$ (Fig. 6). This confirms that under nonlinear damage dynamics, unmanaged climate change constitutes an existential drag on macroeconomic accumulation, corroborating the findings of Bilal and Känzig (2026) and Richardson et al. (2023).

The dynamic transition paths illustrate a critical intertemporal trade-off. Over the near term (approximately the first 50 years), the *Null* and *Moderate* scenarios generate marginally higher paths for K , Y , and Q by circumventing the heavy front-loaded expenditures required for deep decarbonization. However, this early growth advantage proves transient. As warming crosses critical thresholds, compounding damage escalation under the less ambitious scenario chokes capital formation and triggers protracted long-run macroeconomic contraction.

The principal result is the structural reversal of the optimal policy regime. As depicted in the Net Output panel of Fig. 7, the *Very Strong* policy (green trajectory) – despite depressed initial accumulation caused by large policy outlays – stabilizes the macroeconomy and converges to the highest steady-state net output ($Q^* = 30.5$). Conversely, the *Moderate* policy (blue trajectory) converges to a depressed steady state of $Q^* = 28.5$, while the *Null* scenario (red trajectory) precipitates an ongoing economic collapse toward $Q^* = 23.5$. Identical non-monotonic patterns govern capital stock (K) and gross output (Y), demonstrating that the upfront investment demanded by the *Very Strong* regime is not merely compensated, but represents a strict macroeconomic imperative. Under the baseline linear formulation, the marginal reduction in climate damages was insufficient to offset the high costs of the *Very Strong* policy. Under quadratic damages, however, the returns to joint mitigation and adaptation expand dramatically: because losses scale quadratically in temperature anomalies, constraining warming near 2 °C (under *Very Strong* policy) rather than allowing it to reach 4.5 °C (under the *Null* baseline) delivers massive welfare dividends. Specifically, the *Very Strong* policy averts approximately a 17.1 percentage-point escalation in climate damages (curtailing losses from approximately 19.7% to 2.6%, Fig. 6), substantially outweighing its implementation cost of 4.4% of gross output.

¹⁹ Prominent examples include the destabilization of the Greenland and West Antarctic ice sheets or the extensive dieback of the Amazon and boreal forests (Armstrong McKay et al., 2022; Flores et al., 2024).

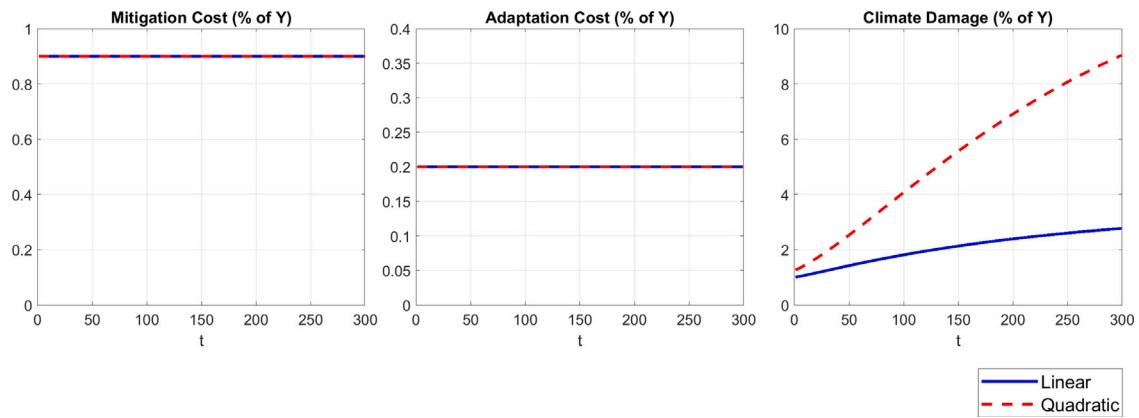


Fig. 4. Evolution of climate-related costs under accelerating damage policy scenarios.

Note: Linear damage function (blue) and quadratic damage function (dashed red). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

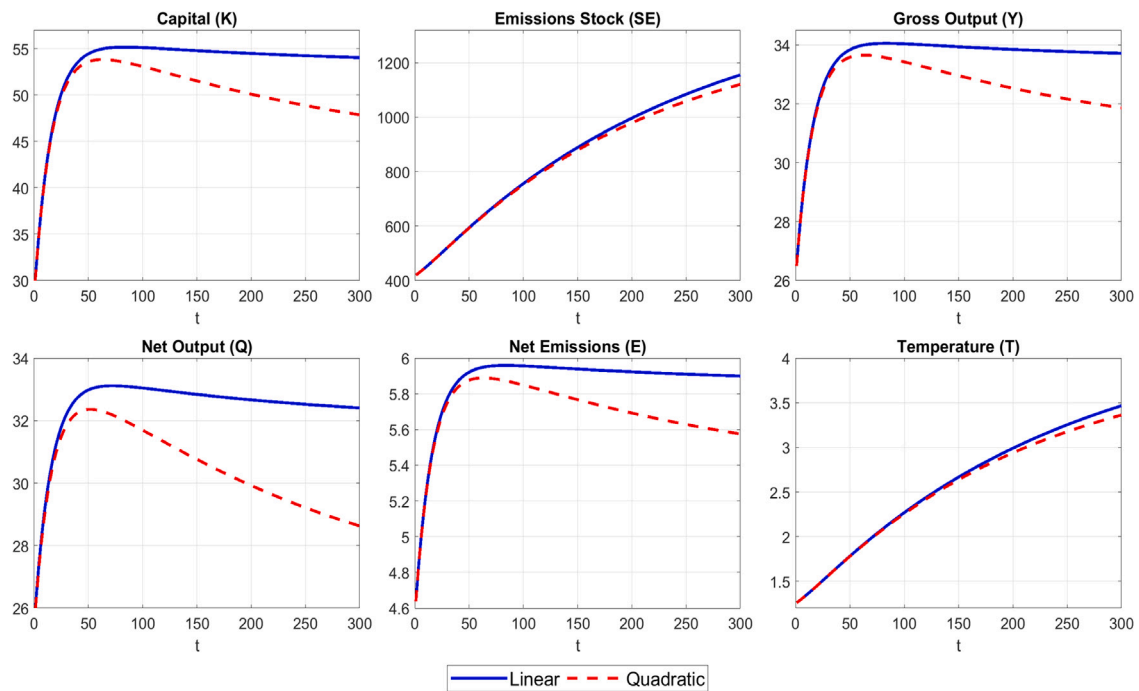


Fig. 5. Macroeconomic and environmental variables under accelerating damage policy scenarios.

Note: Linear damage function (blue) and quadratic damage function (dashed red). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In conclusion, our results demonstrate that optimal climate policy design is fundamentally contingent on the functional form of climate damages. When accounting for planetary boundary dynamics and non-linear tipping risks, ambitious climate action shifts from a discretionary welfare preference into an essential structural safeguard against secular stagnation and systemic economic collapse.

4.3. Time-varying mitigation and adaptation policy

In this section, we relax the assumption of time-invariant policy efforts to reflect escalating climate awareness and the progressive implementation of decarbonization pathways. We specify mitigation (I_m) and adaptation (I_a) to increase linearly across the 300-year simulation horizon, reaching terminal values of 0.80 and 0.50, respectively.

This upward trajectory captures a strategic scaling of mitigation policy in response to intensifying environmental pressures, complemented by an expansion of adaptation programs designed to buffer the

committed warming inherited from historical emissions. The cumulative policy effort over the 300-year horizon is calibrated to represent an intermediate trajectory between the *Moderate* and *Strong* scenarios from Section 4.1,²⁰ yet the temporal distribution of economic costs and environmental benefits shifts fundamentally. The terminal target of $I_m = 0.80$ is particularly relevant as it aligns with empirical net-zero decarbonization strategies for advanced economies (e.g., the European Green Deal). Under such frameworks, an 80% domestic emissions reduction covers the broad base of the economy through electrification, renewable energy deployment, and energy efficiency, while the remaining 20% represents residual emissions from hard-to-abate sectors such as heavy industry, aviation, and agriculture. Because our model

²⁰ Specifically, the mean intensity of these time-varying policies across the simulation horizon H corresponds to $I_m = 0.40$ and $I_a = 0.25$, facilitating a direct comparison with the constant-effort benchmarks.

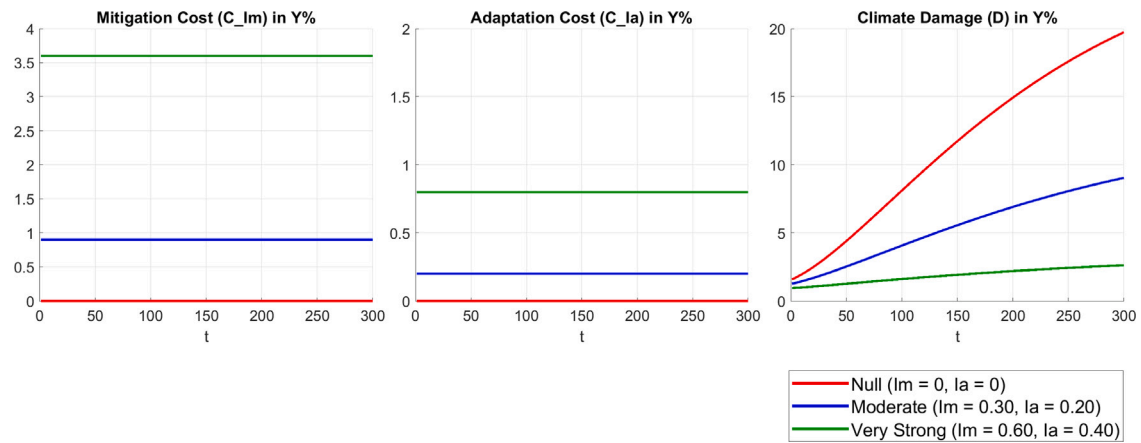


Fig. 6. Evolution of climate-related costs under accelerating damages.

Note: Null (red), Moderate (blue), and Very Strong (green) policy scenarios under a quadratic damage function. **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

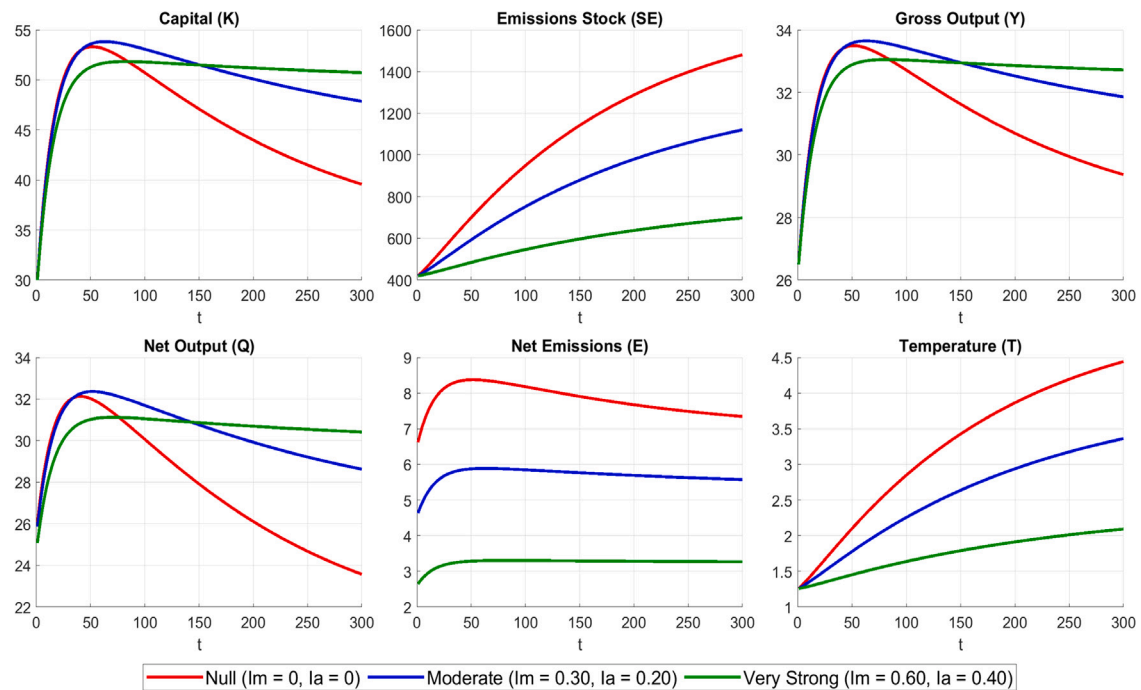


Fig. 7. Macroeconomic and environmental variables under accelerating damages.

Note: Null (red), Moderate (blue), and Very Strong (green) policy scenarios under a quadratic damage function. **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

tracks gross productive emissions without explicitly internalizing carbon dioxide removal technologies – such as carbon capture and storage, direct air capture, or managed terrestrial sinks – an 80% reduction effectively represents a net-zero steady state where residual emissions are neutralized externally.

Fig. 8 depicts the dynamic trajectories of policy efforts and their associated costs. While effort levels I_m and I_a scale linearly, the resulting economic expenditures C_{I_m} and C_{I_a} follow quadratic paths due to the strict convexity of the cost functions. By the end of the simulation ($t = 300$), output diverted toward climate policy exceeds 6% of gross output for mitigation and 1.25% for adaptation.

The most striking finding in Fig. 8 is the dynamic evolution of climate damages (D). Unlike the monotonic paths observed under constant-policy regimes, damages here exhibit a pronounced hump-shaped trajectory. Starting at approximately 1.25% of gross output, damages peak at 2.2% around year 120 before initiating a protracted

decline toward 1.35% at the end of the horizon. This non-monotonic behavior illustrates the profound inertia of the climate system: damages expand for over a century despite intensifying policy intervention because ocean thermal response and atmospheric carbon concentrations adjust to emissions abatements with substantial multi-decadal lags. As emphasized by Folini et al. (2025), the Earth system features intricate feedback mechanisms and thermal lags that standard macroeconomic models frequently understate.

Fig. 9 details the resulting macroeconomic and environmental dynamics, highlighting the staggered temporal peaks that characterize the transition.

Annual net emissions flows (E) respond most rapidly to policy tightening, peaking early at approximately 7.5 kg of CO₂ per euro around year 35 before falling steeply to approximately 1.7 kg at the terminal horizon. This residual flow constitutes the structural decarbonization floor noted above.

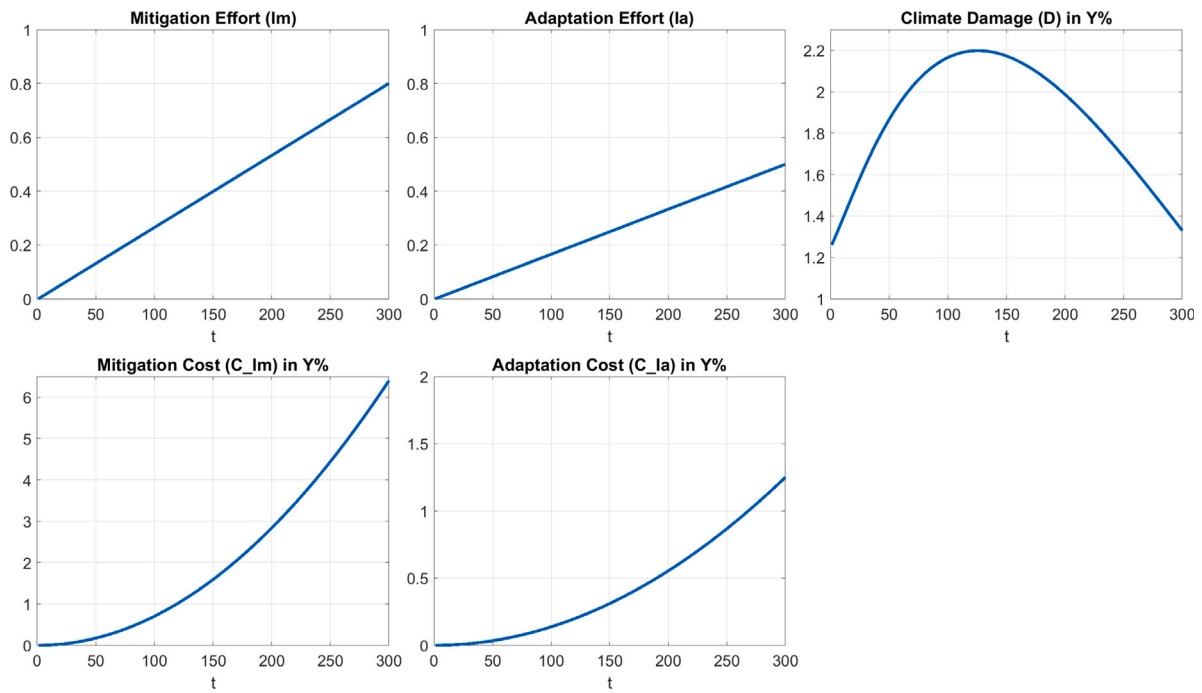


Fig. 8. Evolution of climate-related costs in an increasing policy scenario.

Note: Mitigation and adaptation efforts (I_m , I_a) increase linearly over $t \in [0, H]$, from 0 to 0.80 and 0.50, respectively. **Parameters:** Table 1.

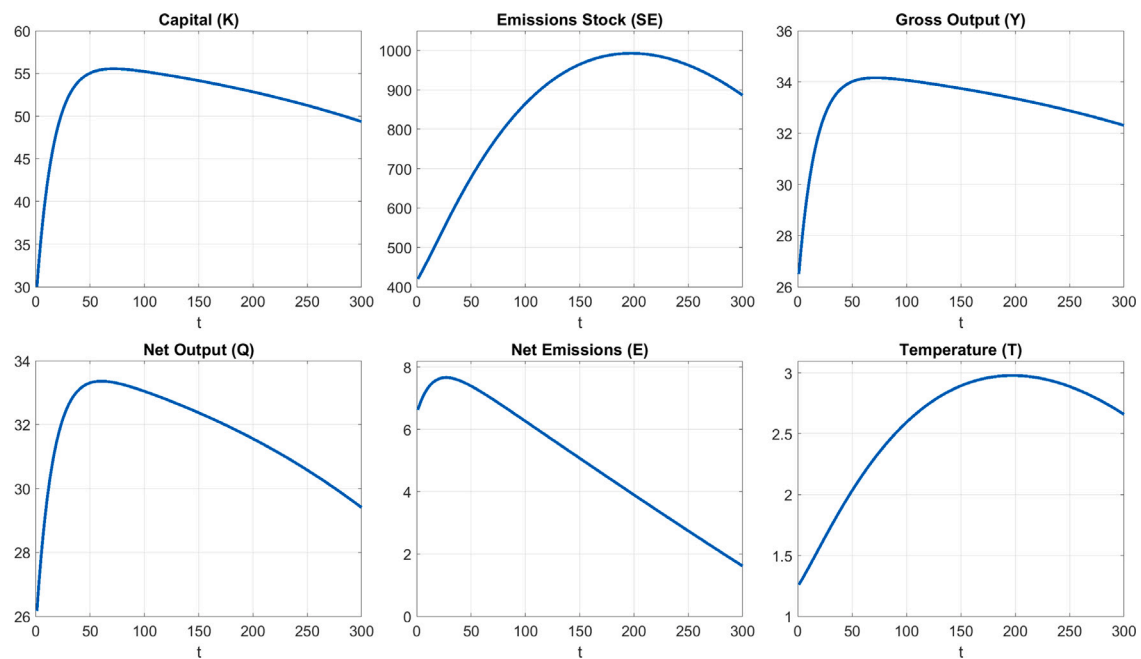


Fig. 9. Macroeconomic and environmental variables in an increasing policy scenario.

Note: Mitigation and adaptation efforts (I_m , I_a) increase linearly over $t \in [0, H]$, from 0 to 0.80 and 0.50, respectively. **Parameters:** Table 1.

Despite this early decline in net emissions, environmental state variables exhibit protracted inertia. The atmospheric carbon stock (SE) peaks near 1000 ppm, and global temperature anomaly (T) reaches its maximum of 3.0 °C around year 200 before entering a slow downward trajectory toward 2.65 °C at horizon H . This temporal wedge between policy stringency and realized climate stabilization is an established property of climate-economic dynamics, driven by the atmospheric residence time of GHGs (Dietz and Stern, 2015; Lenton et al., 2019). It underscores that delayed or back-loaded mitigation and

adaptation policies cannot instantly arrest global warming. As Folini et al. (2025) observe, this atmospheric carbon persistence structurally constrains the timing of economic payoffs, acting as a primary driver of macroeconomic transition risk.

From a macroeconomic perspective, capital stock (K) and gross output (Y) peak around years 60 and 70, respectively ($K \approx 56$, $Y \approx 34$). Beyond this threshold, the macroeconomy enters a phase of gradual contraction driven by capital crowding out: as policy stringency escalates, the mounting costs of I_m and I_a (absorbing nearly 7.65%

of GDP combined by the end of the horizon) crowd out productive investment. Consequently, net output peaks earlier at $t \approx 60$ ($Q \approx 33.4$) and declines visibly to 29.5 by year 300.

These findings establish a critical policy insight: while a gradualist transition pathway may be politically expedient, it incurs severe climate overshoot across both temperature anomalies and realized damages. Our simulations confirm that even an ambitious terminal mitigation target of 80% fails to prevent damages from peaking at 2.2% if the policy ramp-up originates from an unmitigated baseline.

The intertemporal trade-off is clear: the initial economic expansion in Q and Y over the first 60 years is effectively borrowed from future output, enabled solely by negligible initial abatement outlays. However, these early savings are repaid in subsequent periods through elevated temperature anomalies and a permanent diversion of output into late-stage mitigation and adaptation expenditures. This reinforces the macroeconomic superiority of early, front-loaded climate action – such as the *Very Strong* scenario in Section 4.1 – over a delayed linear escalation. Decisive early intervention stabilizes the climate system at lower peak temperatures before convex policy costs and nonlinear damage feedback impose heavy structural drags on long-run economic growth. This highlights the critical importance of early and sustained intervention: postponing climate policy expenditures not only elevates the cumulative economic cost of achieving targeted environmental outcomes, but also substantially raises the probability of medium-term temperature overshoot.

4.4. Technological progress and cost-efficiency gains

In the baseline specification of Section 4.1, long-run macroeconomic growth was driven exclusively by capital accumulation, which eventually converged to a stationary steady state as physical depreciation, policy expenditures, and climate damages balanced new investment. However, following standard foundations in the DICE framework (Nordhaus, 2017), real-world growth trajectories feature ongoing exogenous technological change. In this section, we relax the stationary benchmark by incorporating dynamic TFP growth alongside secular cost-efficiency improvements in climate policy instruments. Specifically, the aggregate production function evolves as $Y(t) = A(t)[\alpha K(t) + (1 - \alpha)L]$, where $A(t)$ denotes aggregate TFP. In accordance with neo-classical growth formulations, $A(t)$ expands at a constant annual growth rate $g_A = 0.05\%$:

$$A(t) = A_0(1 + g_A)^{t-1} \quad (12)$$

where $A_0 = 1$.

This specification transforms the framework into an extended Solow-type growth model where technological change augments aggregate factor productivity. Although g_A is calibrated conservatively,²¹ its interaction with capital formation sustains economic expansion well beyond the stationary baseline shown in Fig. 3, where capital-driven growth plateaued after approximately 70 years. Consequently, steady-state net output (Q) reaches approximately 45 billion euros by year 300, representing a substantial gain over the 32.4 billion euros recorded in the stationary baseline.

Simultaneously, we model green innovation and learning-by-doing dynamics by specifying the marginal cost coefficients for mitigation (θ) and adaptation (μ) to decline linearly over time:

$$\theta(t) = \theta_0 - q \cdot t, \quad \mu(t) = \mu_0 - p \cdot t \quad (13)$$

where $q = 0.00025$ and $p = 0.0001$.

²¹ This calibration reflects empirical long-run TFP trends for advanced industrial economies over a multi-century horizon through 2300. In mature economies, frontier productivity growth stabilizes at modest rates, aligning our simulation path with long-term balanced growth and secular stagnation dynamics.

Originating from the baseline values in Table 1 ($\theta_0 = 0.10, \mu_0 = 0.05$), marginal mitigation costs decline by 75% and marginal adaptation costs fall by 60% over the 300-year horizon. These trajectories represent reduced-form, cumulative efficiency improvements across the simulation horizon rather than point estimates of technology-specific learning curves. Expressed as constant compound annual equivalent rates, the trajectories for $\theta(t)$ and $\mu(t)$ correspond to annual cost declines of approximately 0.46% and 0.31%, respectively. These rates are conservative relative to observed historical cost trajectories for clean technologies, yet they align with standard IAM representations of carbon-saving technical progress via declining backstop costs (Barrage and Nordhaus, 2024). Furthermore, they are consistent with empirical evidence documented by the IPCC regarding cost deflators in renewable power, battery storage, and clean manufacturing (Kikstra et al., 2022). Because adaptation technologies remain spatially heterogeneous and institutionally contingent, we treat the decline in $\mu(t)$ as an optimistic long-run efficiency frontier, reflecting advances in resilient infrastructure, precision agriculture, hydrological forecasting, and AI-driven disaster risk management rather than uniform cost reductions across all adaptation measures.

Figs. 10 and 11 illustrate the macroeconomic and environmental implications of these long-run technological and cost-efficiency gains. A fundamental distinction exists between this scenario and the transition framework analyzed in Section 4.3. Whereas the previous section evaluated a policy-driven escalation in abatement stringency (I_m, I_a) in response to mounting environmental pressures, policy efforts here remain fixed at their baseline intensity levels—*Null* (red), *Moderate* (blue), and *Very Strong* (green). Consequently, dynamic adjustments in this setting are driven exclusively by secular TFP growth and cost deflation across the clean technological frontier.

Fig. 10 highlights a pronounced secular decline in policy implementation expenditures across all active regimes. Under the *Very Strong* scenario, mitigation costs – initially a substantial burden absorbing 3.6% of gross output – decrease steadily across the simulation horizon. By year 300, annual mitigation outlays fall to 0.9% of GDP, effectively converging to the initial cost profile of the *Moderate* baseline. A parallel dynamic governs adaptation outlays: under the *Very Strong* regime, adaptation expenditures decline from 0.8% of gross output to 0.32% by the end of the simulation horizon.

A compelling finding emerges when comparing climate damages against the stationary baseline. In the unmitigated *Null* scenario, damages are slightly higher than in Section 4.1 (reaching approximately 5.6% of GDP); this divergence occurs because TFP-driven economic expansion enlarges the scale of gross production and consequently amplifies absolute emissions flows. Conversely, under the *Very Strong* scenario, climate damages remain tightly contained near baseline levels (approximately 1.5% of gross output), demonstrating that decisive policy intervention successfully insulates an expanding macroeconomy from compounding climate risks.

The macroeconomic trajectories in Fig. 11 reveal a structural realignment in the long-term policy hierarchy. In the baseline stationary model (Table 3), the *Moderate* scenario emerged as the interior welfare optimum for net output because the heavy costs of the *Very Strong* regime were not fully offset by incremental avoided damages ($Q^* = 31.25$ versus 32.41 under *Moderate* policy). Incorporating endogenous technological progress and secular cost-efficiency gains fundamentally reverses this outcome.

Analyzing the dynamic transition paths demonstrates how truncated evaluation horizons generate misleading policy conclusions. Over the initial phase ($t < 90$ years), the *Null* scenario (red trajectory) appears superior across K , Y , and Q because the economy avoids near-term policy outlays while cumulative climate damages remain modest. Across the intermediate window ($90 \leq t \leq 240$), the *Moderate* policy (blue trajectory) establishes transient macroeconomic dominance. However, these intermediate advantages represent a myopic artifact of short-run evaluation horizons that fail to account for long-run technological maturity and cumulative Earth-system feedback.

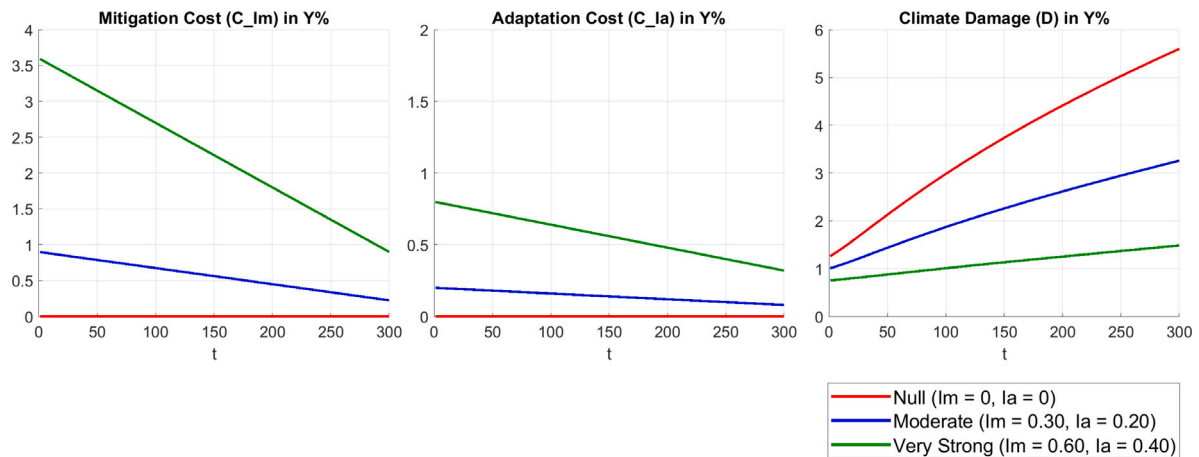


Fig. 10. Evolution of climate-related costs with technological progress.

Note: Null (red), Moderate (blue), and Very Strong (green) scenarios under technological progress and cost-efficiency gains with $g_A = 0.05\%$, $q = 0.00025$, and $p = 0.0001$. **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

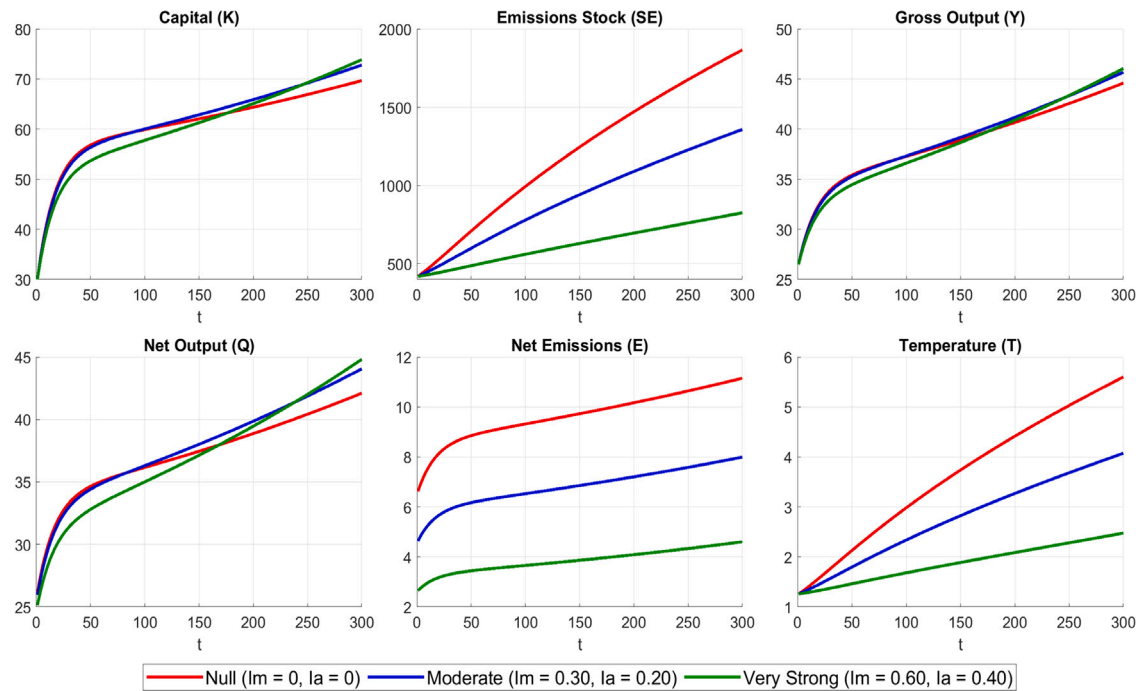


Fig. 11. Macroeconomic and environmental variables with technological progress.

Note: Null (red), Moderate (blue), and Very Strong (green) scenarios under technological progress and cost-efficiency gains with $g_A = 0.05\%$, $q = 0.00025$, and $p = 0.0001$. **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The long-run macroeconomic superiority of the *Very Strong* policy (green trajectory) emerges decisively in the terminal phase of the simulation ($t > 240$ years). As clean technological progress deflates the unit costs of mitigation and adaptation, the drag of aggressive policy outlays diminishes secularly. By the end of the horizon, the *Very Strong* scenario delivers the highest net output ($Q \approx 45$ billion euros, compared to $Q \approx 44$ billion under *Moderate* policy and $Q \approx 42$ billion in the *Null* case).

From an environmental perspective, this simulation proves that sustained economic expansion and climate stabilization are structurally compatible. Although TFP-driven output expansion (Y exceeding 45 billion euros) exerts persistent upward pressure on gross emissions flows, stringent policy intensity successfully constrains cumulative environmental degradation. Long-run warming remains stabilized around

2 °C under the *Very Strong* regime, whereas under the *Null* baseline, temperatures continue climbing toward levels that threaten systemic macroeconomic stability.

The core policy implication is that technological progress fundamentally redefines the economic feasibility of climate action. In this dynamic framework, ambitious mitigation and adaptation cease to operate as a permanent drag on growth, functioning instead as foundational investments for long-term macroeconomic resilience. Comparative analysis across policy scenarios underscores the hazard of myopic planning: an evaluation horizon restricted to the first century favors lower policy stringency, as the immediate costs of aggressive decarbonization are front-loaded while its long-run benefits accumulate with protracted lags.

Over a multi-century horizon, technological maturity and cost-efficiency gains across both mitigation and adaptation establish the

highest policy intensity as the most economically robust and sustainable pathway. By the end of the simulation horizon, the *Very Strong* scenario not only achieves superior environmental stabilization but also generates the highest level of steady-state net output. Accounting for induced technical change thus confirms that aggressive, front-loaded climate policy is an economically rational strategy that preserves both ecological integrity and long-run living standards.

4.5. Intangible capital and emissions

The expanding role of intangible assets is transforming the structural composition of modern economies and reshaping the climate-economy interface. The fourth industrial revolution, driven by the rapid diffusion of AI-based platforms and digital services, has accelerated a secular transition from tangible to intangible capital (Corrado et al., 2009; Haskel and Westlake, 2017). This structural shift elevates the economic weight of software, algorithms, data architectures, and organizational routines relative to physical machinery and industrial facilities.

To evaluate the environmental implications of this transition while preserving closed-form tractability, we partition aggregate capital into tangible/physical capital (K_P) and intangible capital (K_I), such that $K(t) = K_P(t) + K_I(t)$. This formulation is fully consistent with our production block under two standard assumptions that preserve a parsimonious two-dimensional dynamical system. First, tangible and intangible assets are treated as perfect substitutes in the production of gross output, sharing an identical output elasticity α . Under the linearized production function (1), aggregate output is expressed as:

$$Y(t) = \alpha(K_P(t) + K_I(t)) + (1 - \alpha)L, \quad (14)$$

which reduces directly to our baseline aggregate specification without loss of generality. Second, both capital components share the same effective depreciation rate δ and are financed from aggregate national savings $(1 - c)$.²² This structure preserves the aggregate capital accumulation law of motion, (2), avoiding the introduction of additional state variables that would impede analytical tractability.

Maintaining this unified macroeconomic core allows us to isolate the compositional effect of the capital stock on atmospheric emissions. While K_P and K_I are productively equivalent, they differ fundamentally in their emissions footprints.

From an environmental standpoint, the literature offers no consensus on whether structural dematerialization mitigates or exacerbates climate pressures. On the one hand, substituting physical assets with intangible capital reduces the material intensity of production, enhances energy efficiency, and lowers transport-related emissions. Furthermore, a dematerialized economy exhibits lower vulnerability to extreme weather disruptions (e.g., wildfires, flooding, and thermal stress) relative to fixed manufacturing plants and transport infrastructure. On the other hand, scaling digital platforms and AI systems requires intensive computational and data center infrastructure, generating substantial electricity demand that may erode these dematerialization gains (Shehabi et al., 2024).²³

²² Modern growth accounting supports the empirical validity of these assumptions. At the macroeconomic level, intangible assets (such as software, R&D, and organizational design) exhibit marginal productivities comparable to, or even exceeding, physical capital (Corrado et al., 2009). While digital assets often depreciate faster than physical structures, the aggregate parameter δ reflects an economy-wide weighted average. This treatment aligns with international accounting frameworks (e.g., the System of National Accounts [SNA 2008]), which classify intangibles as gross fixed capital formation rather than intermediate consumption, recognizing their role as enduring stores of productive capacity.

²³ By 2028, annual electricity consumption by data centers is projected to absorb between 6.7% and 12% of total U.S. power generation, up from 1.9% in 2018.

Given the ambiguous net sign of this structural shift, we evaluate two opposing scenarios capturing the polar bounds of digital transition pathways. To integrate this mechanism into our model, we introduce a time-dependent multiplier, $\beta(t) > 0$, which scales the carbon intensity of production, reflecting the shifting balance of intangible assets in the macroeconomic production process.

To calibrate $\beta(t)$, we draw on recent micro-econometric literature linking digital adoption and AI deployment to energy intensity. Using firm-level data from German manufacturing, Axenbeck and Niebel (2025) estimate a slight negative elasticity of energy intensity with respect to software capital (between -0.002 and -0.003). Matthess et al. (2023) report heterogeneous effects in European manufacturing: while digital capital intensity correlates with a modest increase in energy intensity ($+0.025$), industrial robot density reduces it (-0.059). Concurrently, Li et al. (2025) show that enterprise AI adoption systematically lowers energy intensity via efficiency gains and process optimization.

While empirical micro-data suggest that $\beta(t)$ remains close to unity, we explore wider parametric bounds ($0.90 < \beta(t) < 1.10$) across our simulations. This wider range accounts for cumulative, multi-century structural transformations that exceed the marginal variations captured in short- and medium-term empirical panels.

Scenario A: More intangible capital reduces net emissions

In the first scenario, the growing weight of intangible assets decreases the emissions intensity of output. Production increasingly relies on digital and AI-driven services rather than energy-intensive physical inputs, thereby reducing net emissions per unit of output. This mechanism is captured by modifying the net emissions equation as follows:

$$E(t) = \sigma\beta(t)(1 - I_m)Y(t) \quad (15)$$

Here, through a linear reduction of $\beta(t)$ from 1 at time $t = 0$ (matching the baseline scenario of Section 4.1) to 0.90 at terminal horizon H , we simulate the macroeconomic and environmental effects of an expanding intangible capital share over time. Net emissions flows are scaled down by intangible capital accumulation, which attenuates the carbon footprint of production.

Scenario B: More intangible capital increases net emissions

In the opposing scenario, the expansion of intangible capital escalates net emissions, primarily due to the surging power requirements of large-scale AI infrastructure and computing data centers (Shehabi et al., 2024). Although production becomes less material-intensive, the aggregate electricity demand of computational architectures outpaces efficiency gains, increasing the emissions generated per unit of GDP. We model these dynamics via a linear escalation of $\beta(t)$ from 1 at time $t = 0$ (matching the baseline scenario of Section 4.1) to 1.10 at horizon H .

The simulations in Figs. 12 and 13 illustrate the divergent trajectories generated by intangible capital across macroeconomic and environmental variables. The realized trajectory depends on whether the transition toward a digital and AI-driven macroeconomy operates as a catalyst for decarbonization (Scenario A) or introduces a power-intensive demand channel that overwhelms efficiency gains (Scenario B).

In Scenario A, where $\beta(t)$ declines from 1 to 0.90 (dashed red trajectory), the shift toward intangible assets reduces emissions intensity while policy expenditures remain constant at baseline levels. As shown in Fig. 12, steady-state climate damages decline to 2.6% of gross output, compared to 2.8% under the baseline (solid blue trajectory). This improvement stems from slower atmospheric carbon accumulation, with the emissions stock stabilizing at approximately 1095 ppm relative to 1155 ppm in the benchmark. Consequently, warming is attenuated, reaching approximately 3.3 °C compared to 3.5 °C in the baseline. This reduction in climate damages yields marginally higher steady-state net output over time, while capital accumulation and gross output paths remain broadly aligned with baseline trajectories.

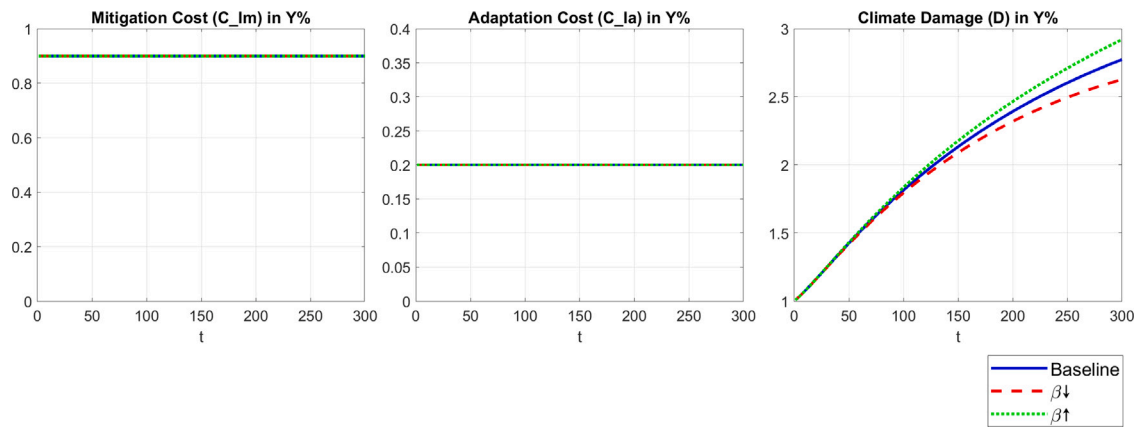


Fig. 12. Evolution of climate-related costs in a scenario with intangible capital.

Note: Moderate baseline ($\beta = 1$, solid blue); Scenario A (β from 1 to 0.90, dashed red); Scenario B (β from 1 to 1.10, dashed green). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

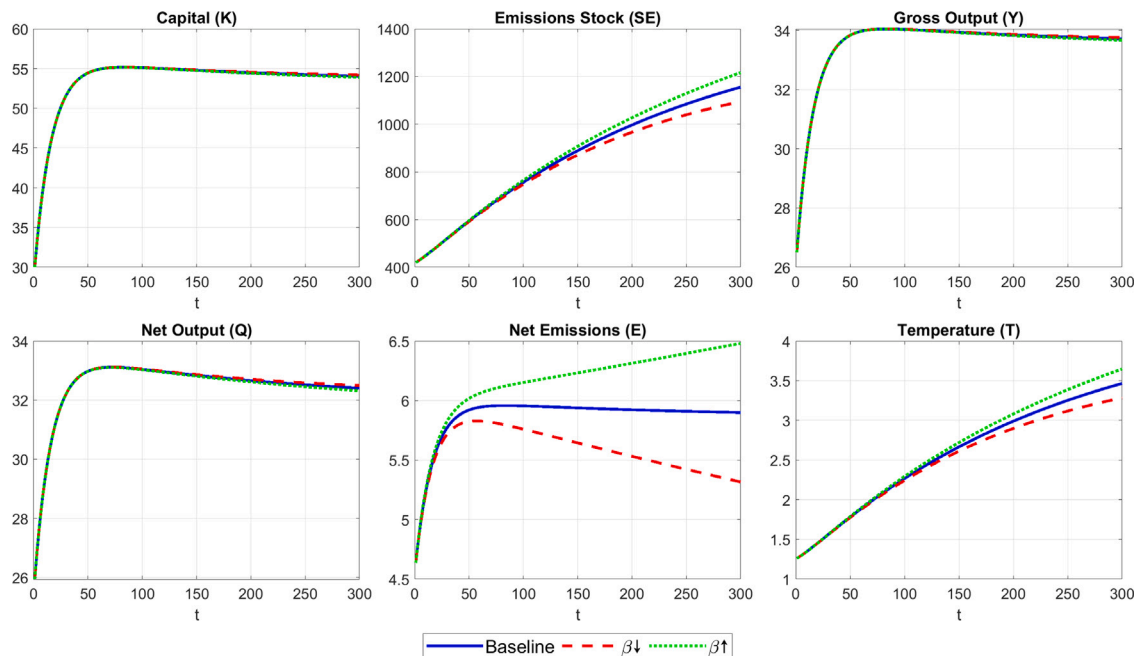


Fig. 13. Macroeconomic and environmental variables in a scenario with intangible capital.

Note: Moderate baseline ($\beta = 1$, solid blue); Scenario A (β from 1 to 0.90, dashed red); Scenario B (β from 1 to 1.10, dashed green). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In Scenario B, where $\beta(t)$ rises from 1 to 1.10 (dashed green trajectory), intangible capital drives substantial electricity demand, raising net emissions intensity. While mitigation and adaptation expenditures remain constant, climate damages escalate more rapidly, approaching 3% of gross output over the long run. This deterioration is driven by faster atmospheric emissions growth, which exceeds 1200 ppm, and a steeper thermal trajectory reaching above 3.6 °C. Annual net emissions remain persistently higher than baseline levels, reflecting the heightened carbon intensity of digital production. Despite these environmental impacts, capital stock and gross output trajectories deviate only modestly, confirming that the primary macroeconomic impact of intangible capital operates through the environmental feedback loop rather than direct factor accumulation.

Jointly, these findings formalize the structural ambiguity of intangible capital expansion: it can serve as a powerful engine of sustainability if it dematerializes production, but it can equally act as a compounding multiplier of climate damages if it accelerates power demand from carbon-intensive energy systems.

5. Conclusion

This paper develops a streamlined climate–economy model that revisits the DICE tradition within a discrete-time dynamical setting. By focusing on the joint trajectories of capital and atmospheric carbon accumulation, the framework bypasses the computational opacity of intertemporal optimization, offering closed-form steady-state solutions and transparent comparative statics. This analytical tractability does not compromise substantive realism: the model retains the core feedback mechanisms linking capital accumulation, GHG flows, and temperature anomalies, while providing explicit roles for both mitigation and adaptation policies.

The analysis yields several robust, policy-relevant conclusions. First, mitigation and adaptation operate as complementary yet structurally asymmetric instruments: mitigation intervenes directly in the production system by curbing emissions flows and decelerating global warming, whereas adaptation operates on the macroeconomy to mitigate

realized output damages without modifying the underlying temperature trajectory. Second, the model establishes a nonlinear trade-off between policy expenditures and avoided climate damages. Under baseline parameterizations, intermediate policy intensity generates a “resilience dividend”, wherein avoided climate damages more than offset policy outlays, rendering climate protection and capital accumulation mutually reinforcing.

Third, optimal policy stringency depends fundamentally on the functional specification of climate damages. Under a linear damage benchmark, moderate intervention maximizes steady-state net output; however, under quadratic damages that account for planetary boundary risks and tipping points, this policy hierarchy reverses, establishing strong or very strong interventions as strict economic necessities. In nonlinear settings, ambitious climate policy is not merely an environmental preference, but an indispensable safeguard against severe output contraction and long-run secular stagnation.

Fourth, the intertemporal sequencing of climate policy is paramount. Gradual or back-loaded implementation schedules generate severe transitional inefficiencies, producing hump-shaped damage trajectories and protracted temperature overshoots due to atmospheric thermal inertia. Decisive, front-loaded policy intervention proves systematically more effective and less costly over multi-century horizons than gradual ramp-up strategies.

Fifth, exogenous technological progress and learning-by-doing fundamentally alter long-run policy rankings. As secular cost-efficiency gains deflate the unit costs of mitigation and adaptation over time, aggressive decarbonization scenarios become self-financing, ultimately dominating less stringent regimes in terms of long-run net output and standard of living metrics.

Finally, the extension to intangible capital demonstrates that structural transformations alter aggregate climate exposure. The secular reallocation toward intangible assets can either accelerate decarbonization or amplify emissions, depending on whether dematerialization gains outweigh the expanding power demands of AI and computational data centers. This underscores the necessity of coupling digital industrial policy with clean energy expansion. Jointly, these findings reinforce the macroeconomic rationale for early, sustained, and balanced climate policy. While mitigation remains indispensable for curbing long-run temperature anomalies, adaptation provides an essential defensive shield against committed near-term warming. Beyond these immediate findings, the tractable two-dimensional framework developed here provides a versatile benchmark for future extensions incorporating endogenous technical change, financial frictions, or higher-order Earth-system tipping dynamics.

CRedit authorship contribution statement

Andrea Bacchiocchi: Conceptualization, Methodology, Formal analysis, Software, Validation, Investigation, Writing – original draft, Writing – review & editing, Visualization, Project administration. **Fabio Tramontana:** Formal analysis, Methodology. **Federico Favaretto:** Conceptualization, Methodology, Validation, Writing – original draft, Writing – review & editing.

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Declaration of competing interest

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Appendix

A.1. Numerical values of equilibria and stability

This section provides the numerical counterparts to the analytical derivations established in Section 3. Applying the baseline parameterization (Table 1), we solve the fixed-point conditions of the dynamical system. The values reported below are the stationary equilibria of the model, whereas Table 3 reports the corresponding outcomes at the end of the finite simulation horizon ($H = 300$), obtained under the same parameterization. The small differences between the two reflect the gradual convergence of the system toward its long-run equilibrium, particularly for the climate variables.

The model admits two stationary equilibria, of which only one is economically viable:

- **Non-feasible Equilibrium** (FP_1): $(-3379.38, -43588.73)$. This fixed point is characterized by strictly negative values for both the capital stock and atmospheric carbon concentrations, and is therefore discarded.
- **Feasible Equilibrium** (FP_2): $(53.19, 1463.73)$. This interior steady state represents the unique long-run attractor of the dynamical system, pinning down the mutual balance between capital accumulation and atmospheric carbon concentrations.

Evaluating the Jacobian matrix, J , at the feasible equilibrium, FP_2 , yields the characteristic eigenvalues:

$$\lambda_1 = 0.9959, \quad \lambda_2 = 0.9374.$$

Because $|\lambda_i| < 1$ for $i \in \{1, 2\}$, the steady state is locally asymptotically stable. The proximity of the dominant eigenvalue λ_1 to unity is a direct mathematical consequence of the slow natural decay rate of atmospheric carbon (λ), indicating that shocks to the emissions stock exert persistent, long-lasting effects along the transition path. The robustness of this asymptotic stability is confirmed by the wide parameter distance separating baseline values from the bifurcation boundaries (Table 2). As established in Proposition 2, the climate feedback coefficient B stabilizes the determinant Δ well below the Neimark–Sacker threshold ($\Delta = 1$). Consequently, the monotonic convergence observed in the simulation experiments (Section 4) reflects a structural property of the dynamical system rather than an artifact of baseline calibration choices.

A.2. Cobb–Douglas production function robustness check

To address potential concerns regarding the empirical validity of the linearized production function over extended horizons, this section provides a sensitivity analysis by simulating the model under a standard neoclassical Cobb–Douglas technology:

$$Y(t) = K(t)^\alpha L^{1-\alpha}. \quad (16)$$

The linear formulation employed in the main text, $Y(t) = \alpha K(t) + (1 - \alpha)L$, represents a first-order Taylor approximation around the reference expansion point. Standard approximation theory establishes that the accuracy of this linearization depends on the bounded variation in state

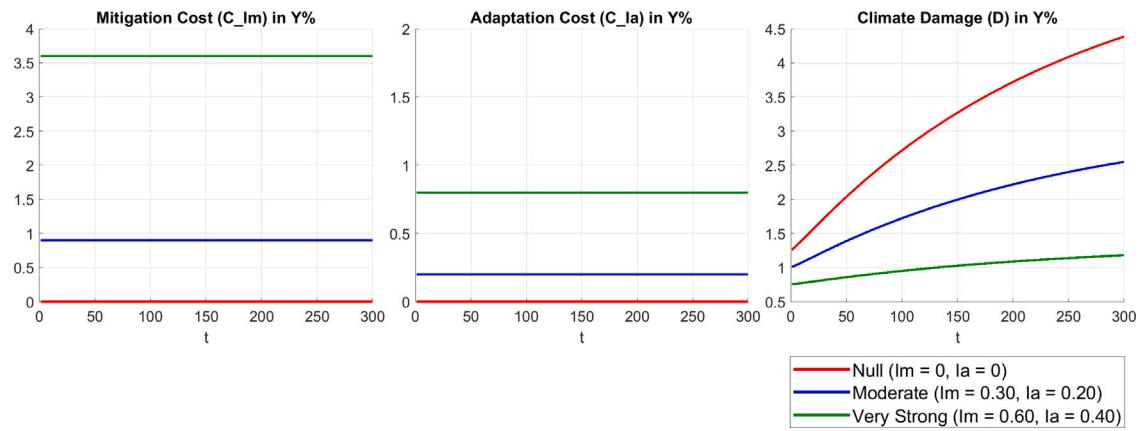


Fig. 14. Evolution of climate-related costs with a Cobb–Douglas production function

Note: *Null* (red), *Moderate* (blue), and *Very Strong* (green) policy scenarios under a Cobb–Douglas production function. **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

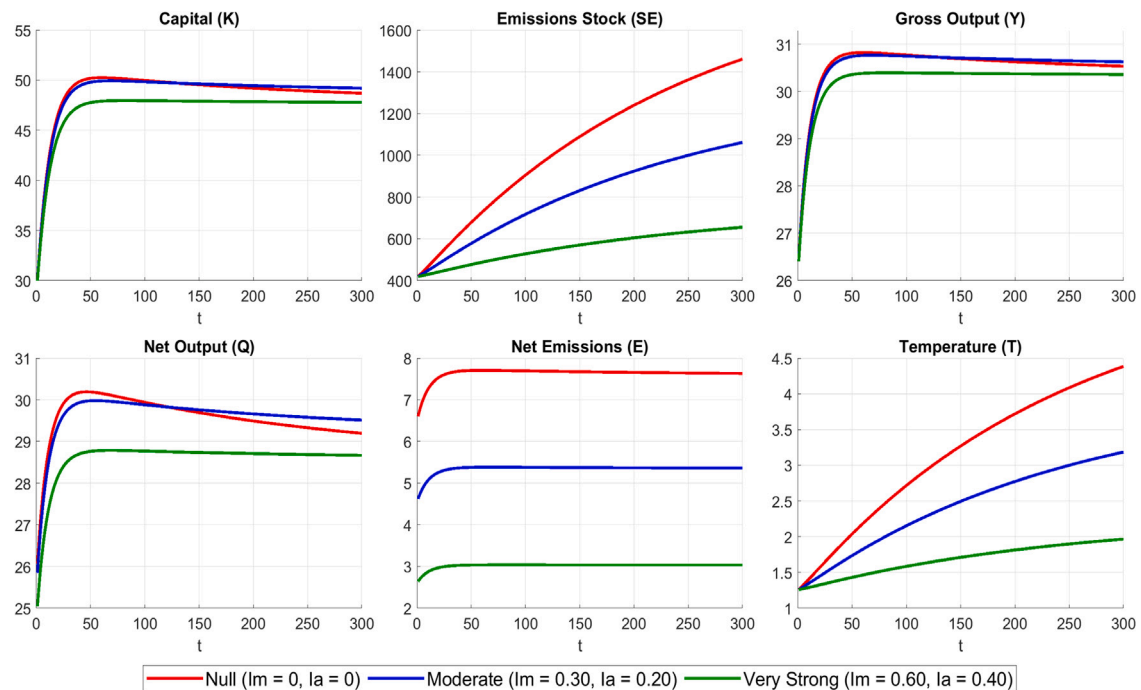


Fig. 15. Macroeconomic and environmental variables with a Cobb–Douglas production function.

Note: *Null* (red), *Moderate* (blue), and *Very Strong* (green) policy scenarios under a Cobb–Douglas production function. **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

variables relative to the expansion base. In our framework, the capital stock $K(t)$ transitions from an initial value of $K(0) = 30$ to steady-state levels K^* ranging between 48 and 51 depending on policy stringency—representing a modest cumulative expansion of approximately +60% across a 300-year horizon. Because the economy does not undergo explosive growth or radical structural shifts in the capital-to-labor ratio, the linearized technology serves as a reliable approximation of the underlying nonlinear production frontier.

Figs. 14 and 15 depict the transition dynamics under Cobb–Douglas technology across the *Null* (red), *Moderate* (blue), and *Very Strong* (green) scenarios, holding all parameters (Table 1) and initial conditions identical to the baseline scenario.

Comparing these trajectories with the linearized benchmark in Figs. 2 and 3 yields several consistent insights:

First, the primary structural distinction lies in the higher concavity of the transition paths for the capital stock (K) and gross output

(Y). This property stems directly from diminishing marginal returns to capital, which are more pronounced under Cobb–Douglas technology than under the linearized specification. As a consequence, transitional convergence toward the steady state accelerates slightly, yielding marginally lower steady-state output levels (Y^* stabilizes around 30.5 billion euros rather than 33.5 billion euros).

Second, the fundamental hierarchy across policy scenarios is strictly preserved. Net output (Q) displays an identical relative ordering, reaffirming the *Moderate* scenario (blue trajectory) as an interior welfare-maximizing optimum that balances climate protection against capital investment outlays. The “capital reallocation trade-off” identified in the baseline model is therefore fully invariant to the functional form of production.

Third, the environmental state variables – atmospheric emissions stock (SE), annual net emissions (E), and mean global temperature anomaly (T) – exhibit dynamics virtually indistinguishable from the

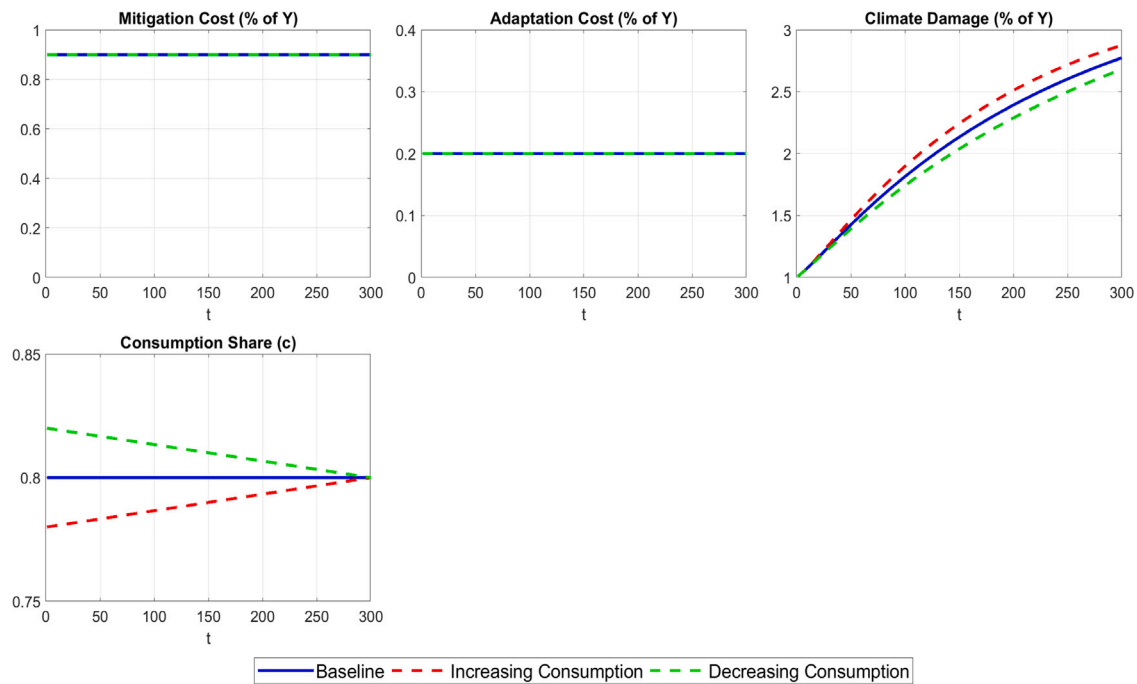


Fig. 16. Evolution of climate-related costs in three consumption path scenarios.

Note: Moderate baseline ($c = 0.80$, solid blue); Increasing consumption (c from 0.78 to 0.80, dashed red); Decreasing consumption (c from 0.82 to 0.80, dashed green). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

baseline model. For example, long-run warming under the *Very Strong* scenario stabilizes just below 2°C , whereas the unmitigated *Null* scenario leads to severe climate degradation exceeding 4°C . This confirms that the macro-climate feedback loop is qualitatively and quantitatively robust to the linearization of aggregate technology.

In sum, while the Cobb–Douglas formulation introduces expected neoclassical curvature, the quantitative divergences remain minimal and the qualitative policy conclusions remain fully consistent with the primary text. These robustness exercises confirm that the linearized production function serves as an analytically tractable, numerically reliable foundation for integrated climate–economy policy analysis.

A.3. Consumption transition path results

In this section, we relax the assumption of a constant consumption share (c) to evaluate the robustness of our core findings. In standard intertemporal optimization frameworks, such as the DICE model (Nordhaus, 2017), consumption is determined endogenously via an Euler equation, enabling forward-looking agents to smooth intertemporal utility. To assess the sensitivity of our baseline results to the absence of dynamic optimization, we examine two alternative exogenous transition trajectories for c toward the steady state (Figs. 16 and 17).

In the first scenario (dashed red trajectory), we simulate a transition path where the consumption share originates at a lower level (0.78) and rises asymptotically toward 0.80. This dynamic mirrors the “investment sacrifice” characteristic of the neoclassical Ramsey model (Cass, 1965), which is economically rationalized by an elevated initial marginal product of capital (MPK). As established in Section 3, the MPK is highest during early periods when the atmospheric emissions stock (SE) remains low; as carbon concentrations accumulate and climate damages intensify, the marginal return on capital systematically declines. This high initial return creates strong incentives for early saving and capital deepening to establish the productive base (Gollier, 2012). This behavior aligns with standard predictions from optimizing IAMs, where economies front-load investment in response to high real returns before increasing consumption as they converge toward the steady state.

In the second scenario (dashed green trajectory), we specify a path where c begins at an elevated level (0.82) and declines gradually toward 0.80. This scenario captures the macroeconomic dynamics of economies constrained by structural development barriers and limited initial fiscal space (Besley and Persson, 2013). Mobilizing domestic savings is inherently challenging at earlier stages of economic development, as a larger fraction of aggregate output is committed to nondiscretionary public expenditures, essential social services, and structural institutional mandates.²⁴ In such environments, generating surplus savings is constrained by subsistence requirements and structural budget rigidities (Acemoglu et al., 2005). Moreover, underdeveloped domestic financial systems and binding credit constraints impede the efficient intermediation of early savings into productive capital accumulation (Aghion and Bolton, 1997; Matsuyama, 2007).

The simulation results in Fig. 17 confirm that the trajectory of consumption primarily modulates the rate of transitional growth. Under the Ramsey-type trajectory (dashed red), an elevated initial saving rate accelerates the accumulation of physical capital (K) and gross output (Y). This expanded economic base temporarily scales up net emissions flows and accelerates the accumulation of the atmospheric carbon stock (SE). Conversely, when initial consumption is elevated (dashed green), macroeconomic expansion proceeds more gradually, dampening intermediate environmental pressures along the transition path.

Crucially, while these alternative consumption paths alter the speed of macroeconomic convergence, their long-run climatological impacts remain exceptionally stable. Because both scenarios asymptotically converge to the benchmark steady-state consumption share ($c = 0.80$), steady-state net emissions flows fully realign. The resulting cross-scenario discrepancies in terminal carbon concentrations and temperature anomalies are minor (approximately $\pm 0.15^\circ\text{C}$), particularly when contrasted with the multi-degree divergences driven by alternative policy regimes (such as transitioning from *Null* to *Very Strong* policy). This confirms that in our model, long-run warming is fundamentally

²⁴ Recall that c encompasses both private and public consumption outlays.

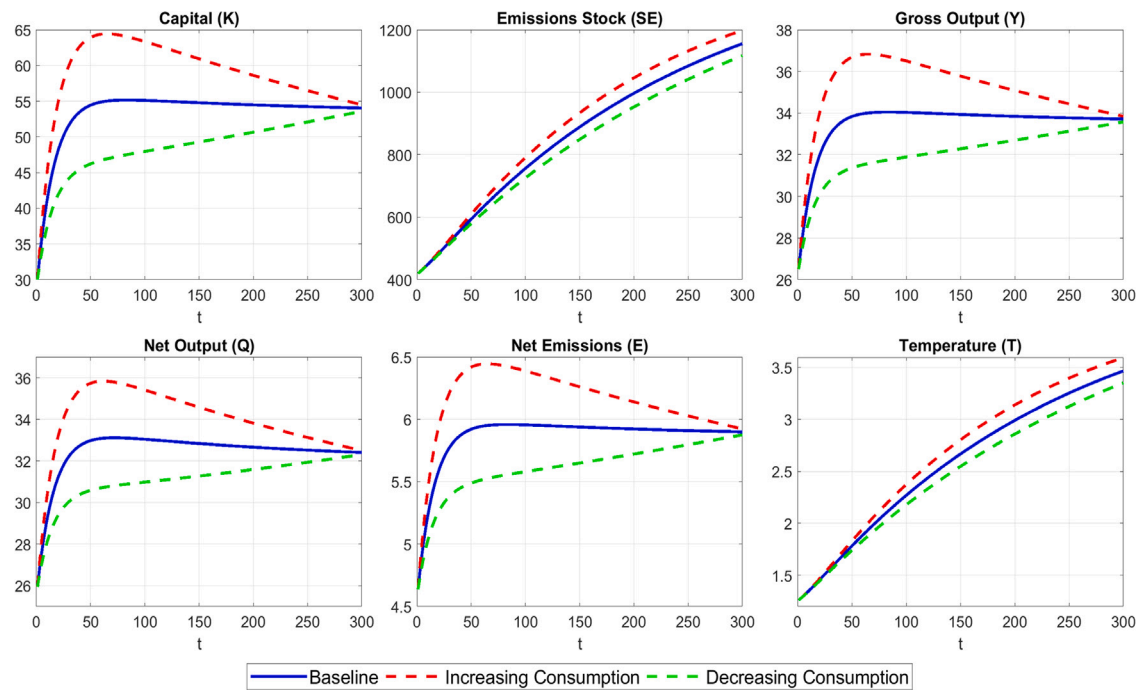


Fig. 17. Macroeconomic and environmental variables in three consumption path scenarios.

Note: Moderate baseline ($c = 0.80$, solid blue); Increasing consumption (c from 0.78 to 0.80, dashed red); Decreasing consumption (c from 0.82 to 0.80, dashed green). **Parameters:** Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

governed by the structural carbon intensity of technology and the stringency of climate policy, rather than the intertemporal timing of consumption. Consequently, adopting a time-invariant consumption share constitutes an analytically tractable simplification that preserves the validity of the model's core policy implications.

Data availability

The MATLAB replication package (Bacchiocchi et al., 2026), containing the code required to reproduce the benchmark equilibrium and stability calculations, Table 3, and Figs. 2–17, is publicly available in Mendeley Data: <https://doi.org/10.17632/m98njvj92j.1>.

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